Non-Abelian dualities and quantum criticality in square lattice anitferromagnets

SudipFest
A workshop in honor of
Sudip Chakravarty's
70th birthday
UCLA, April 5, 2019

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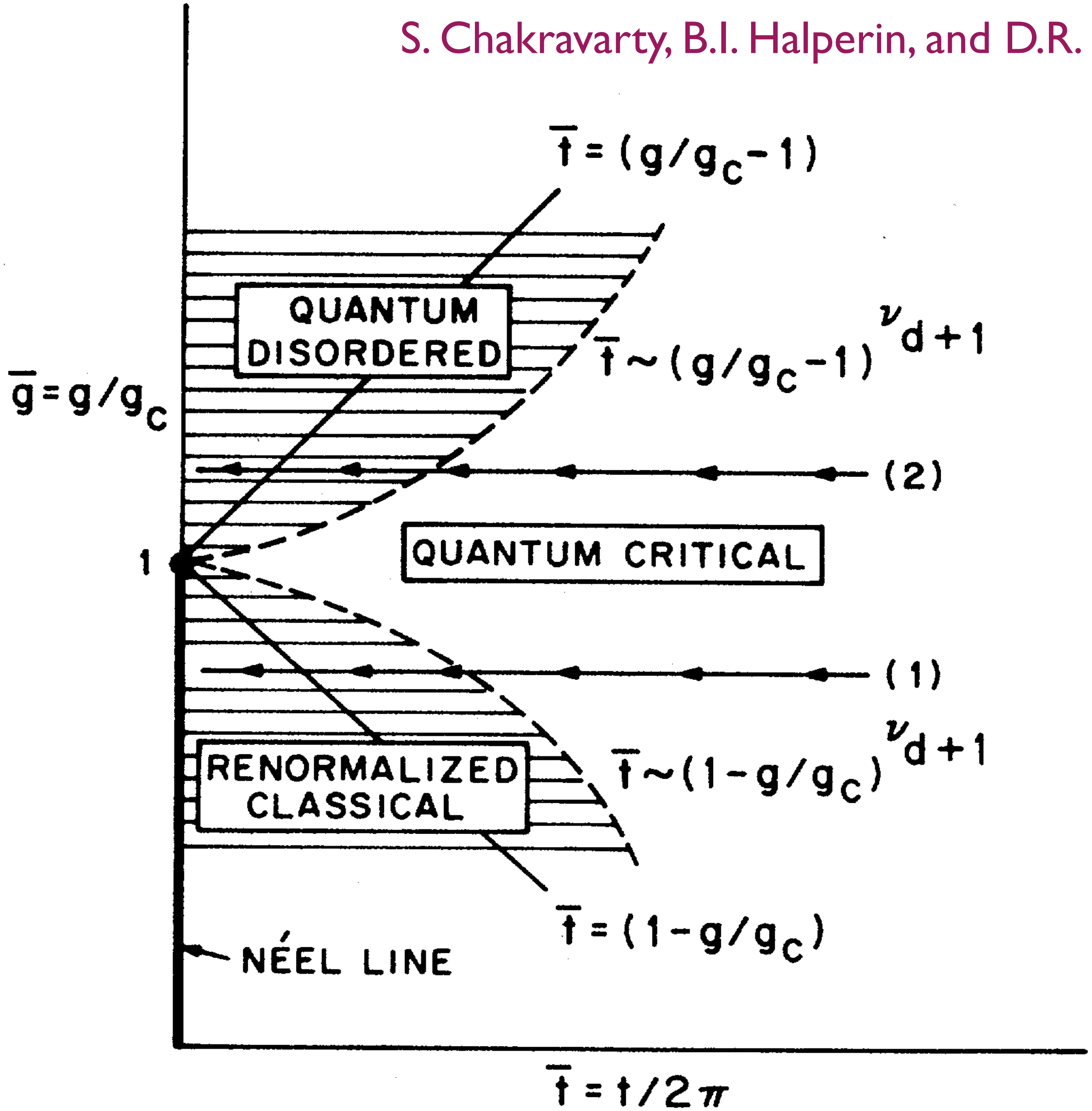
Subir Sachdev

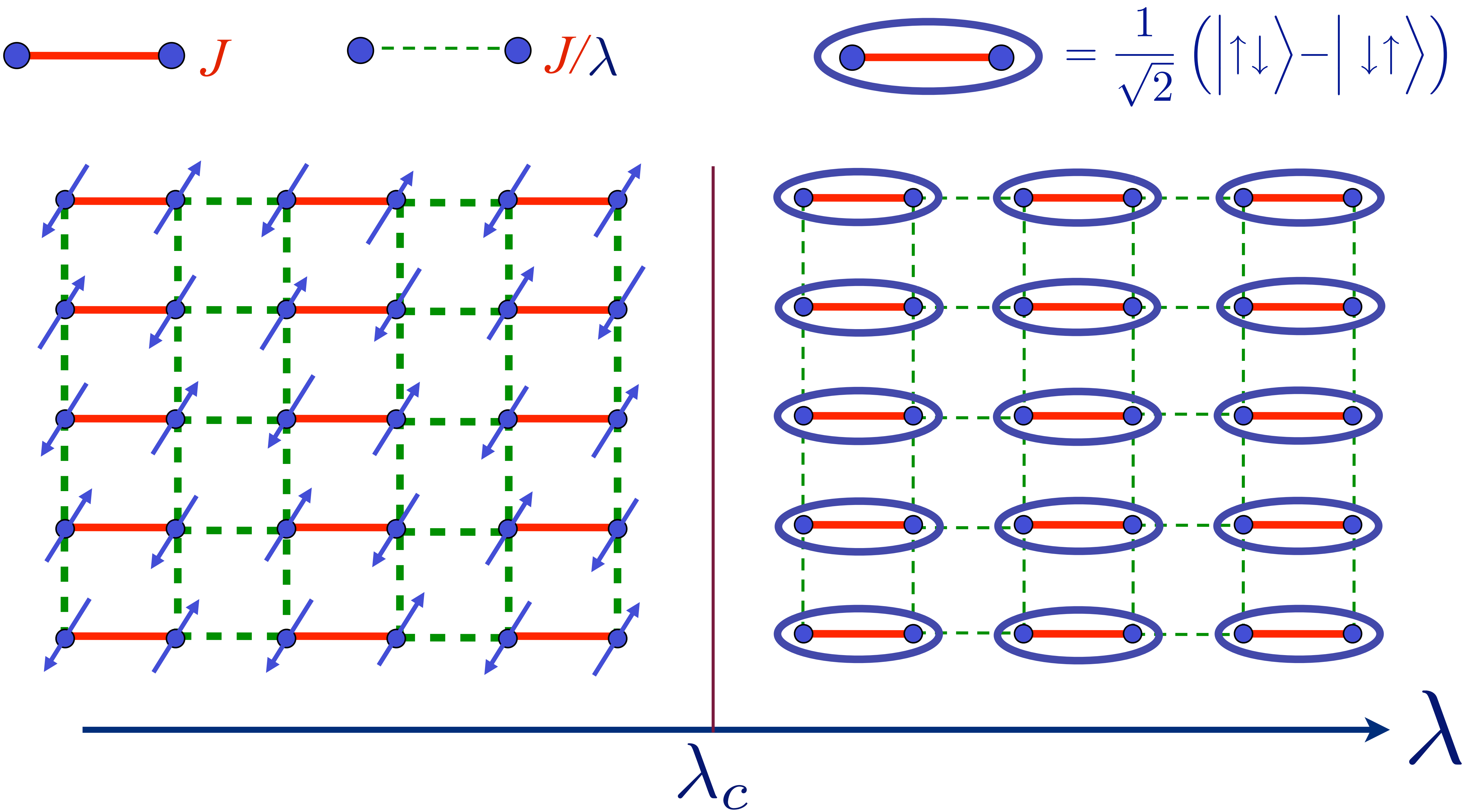
Talk online: sachdev.physics.harvard.edu

S. Chakravarty, B.I. Halperin, and D.R. Nelson, PRB **39**, 2344 (1989)



Quantum criticality of LGW theory of Néel order, N_a, $a = x, y, z$

$$\mathcal{S}_N = \int d^2 r d\tau \left[(\partial_\mu N_a)^2 + s N_a^2 + u (N_a^2)^2 \right]$$



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1. Neel-VBS criticality in square lattice antiferromagnets

2. Recent experimental and numerical results

3. Critical theory for onset of semion topological order

4. More non-Abelian dualities

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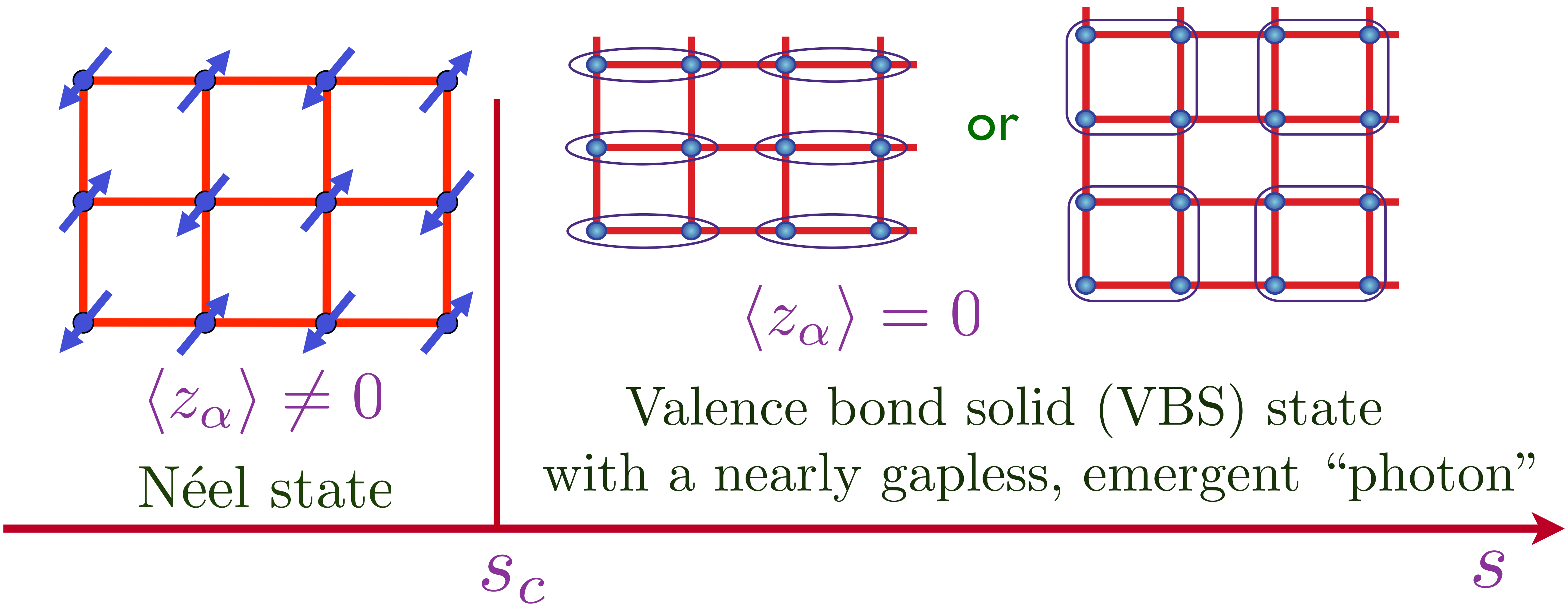
Quantum criticality in a frustrated square lattice antiferromagnet

N. Read and S. Sachdev, PRL **62**, 1694 (1989)



Critical $\mathbb{CP}^1$ theory for photons and deconfined spinons:

$$\mathcal{S}_z = \int d^2 r d\tau \left[|(\partial_\mu - i a_\mu) z_\alpha|^2 + s|z_\alpha|^2 + u(|z_\alpha|^2)^2 + \frac{1}{2e_0^2}(\epsilon_{\mu\nu\lambda}\partial_\nu a_\lambda)^2 \right]$$

O.I. Motrunich and A. Vishwanath, *Phys. Rev. B* **70**, 075104 (2004).
T. Senthil, A. Vishwanath, L. Balents, S. Sachdev and M.P.A. Fisher, *Science* **303**, 1490 (2004).

Quantum criticality in a frustrated square lattice antiferromagnet

SU(2) gauge theory of rotating reference frame in spin space (similar to Schwinger bosons):
Write the lattice electron operator $c_{i\alpha}$ as

$$C_i = \begin{pmatrix} c_{i\uparrow} & -c_{i\downarrow}^\dagger \\ c_{i\downarrow} & c_{i\uparrow}^\dagger \end{pmatrix} \quad , \quad C_i = R_{si}\Psi_i$$

$$\Psi_i = \begin{pmatrix} \psi_{i+} & -\psi_{i-}^\dagger \\ \psi_{i-} & \psi_{i+}^\dagger \end{pmatrix} \quad , \quad R_{si} = \begin{pmatrix} z_{i\uparrow} & -z_{i\downarrow}^* \\ z_{i\downarrow} & z_{i\uparrow}^* \end{pmatrix}$$

Ψ are fermionic 'chargons', R_s is a SU(2) rotation. Spin rotations are *left* multiplication of R_s, while *right* multiplication is an emergent SU(2) gauge symmetry:

$$\Psi \to U\Psi, \quad R_s \to R_s U^\dagger .$$

We Higgs the SU(2) down to U(1) by condensing $(-1)^i \psi_i^\dagger \sigma^z \psi_i$, and the chargons are then gapped, fully filling the lower band. The resulting low energy theory is a U(1) gauge theory for the bosonic spinons z_α.

S. Sachdev, M.A. Metlitski, Y. Qi, and C. Xu, PRB **80**, 155129 (2009)

Quantum criticality in a frustrated square lattice antiferromagnet

SU(2) gauge theory of rotating reference frame in spin space
(similar to Schwinger bosons):

The valence bond solid (VBS) order parameter is the monopole operator, $\mathcal{M}_1$ of the U(1) gauge theory. Lattice symmetries allow source terms only for quadrupled monopoles, $\mathcal{M}_4$ in the action. Condensation of monopoles in the confining phase of the U(1) gauge theory breaks a $\mathbb{Z}_4$ lattice rotation symmetry.

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N. Read and S. Sachdev, *Phys. Rev. Lett.* **62**, 1694 (1989)
O.I. Motrunich and A. Vishwanath, *Phys. Rev. B* **70**, 075104 (2004).
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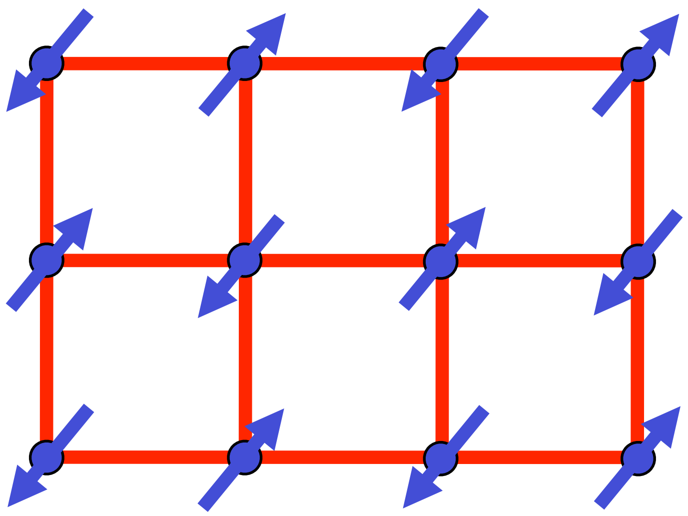
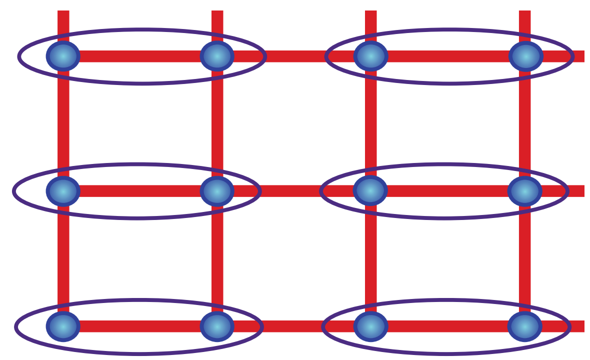
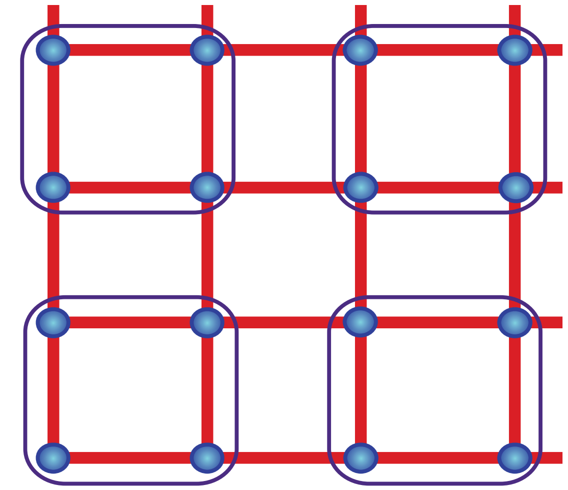


$\langle z_\alpha \rangle \neq 0$

Néel state



or



$\langle z_\alpha \rangle = 0$

Valence bond solid (VBS) state
with a nearly gapless, emergent "photon"

s_c $\quad$ s

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S. Sachdev, M.A. Metlitski, Y. Qi, and C. Xu, PRB **80**, 155129 (2009)

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The F spinons move in a π-flux background, while the $b_{1,2}$ are gapped chargons. The low energy theory is a SU(2) gauge theory of $N_f = 2$ Dirac fermions, f

M. Scheurer and S. Sachdev, PRB **98**, 235126 (2018)

A non-Abelian duality

Critical U(1) gauge (a_μ) theory of $N_b = 2$ relativistic bosons
is dual to
SU(2) gauge (A_μ) theory of $N_f = 2$ Dirac fermions.

$$\mathcal{S}_z = \int d^2 r d\tau \left[|(\partial_\mu - i a_\mu) z_\alpha|^2 + s|z_\alpha|^2 + u(|z_\alpha|^2)^2 + \frac{1}{2e_0^2}(\epsilon_{\mu\nu\lambda}\partial_\nu a_\lambda)^2 \right]$$

$$\mathcal{S}_f = \int d^2 r d\tau \left[\overline{f} \gamma^\mu (\partial_\mu - i A_\mu) f \right]$$

Chong Wang, A. Nahum, M.A. Metlitski, Cenke Xu, and T. Senthil, PRX **7**, 031051 (2017)

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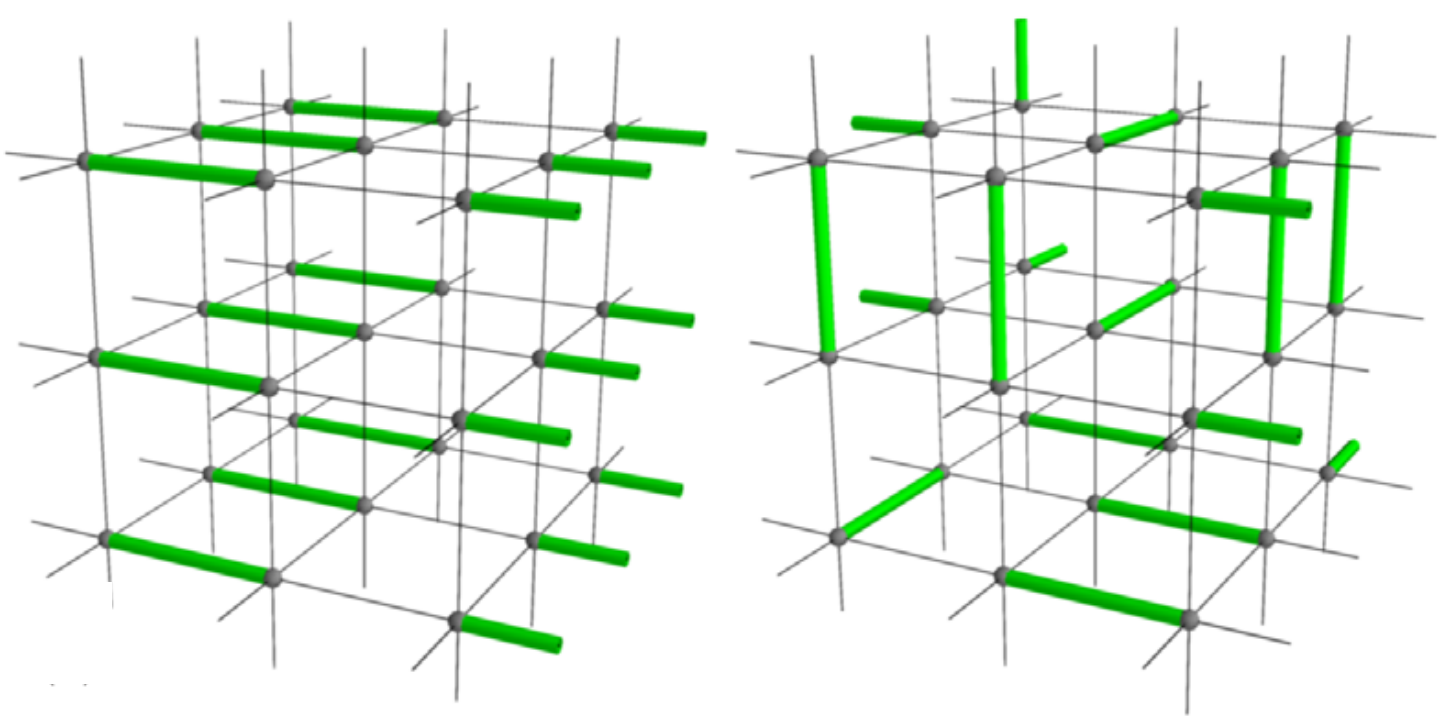
The fermion theory has a SO(5) global flavor symmetry, and the gauge-invariant fermion bilinears form a SO(5) vector which transforms as the Néel and VBS order parameters!

$$(N_x, N_y, N_z, \mathrm{Re}(\mathcal{M}_1), \mathrm{Im}(\mathcal{M}_1))$$

Akihiro Tanaka and Xiao Hu, PRL. **95**, 036402 (2005).
T. Senthil and M.P.A. Fisher, PRB **74**, 064405 (2006)
Chong Wang, A. Nahum, M.A. Metlitski, Cenke Xu, and T. Senthil, PRX **7**, 031051 (2017)

Emergent SO(5) Symmetry at the Columnar Ordering Transition in the Classical Cubic Dimer Model

"Studying linear system sizes up to L=96, we find that this symmetry applies with an excellent precision, consistently improving with system size over this range. It is remarkable that SO(5) emerges in a system as basic as the cubic dimer model, with only simple discrete degrees of freedom. Our results are important evidence for the generality of the SO(5) symmetry that has been proposed for the noncompact CP^1 field theory. We describe an interpretation for these results in terms of a consistent hypothesis for the renormalization-group flow structure, allowing for the possibility that SO(5) may ultimately be a near-symmetry rather than exact."



G.J. Sreejith, Stephen Powell, and Adam Nahum
Phys. Rev. Lett. **122**, 080601 (2019)

1. Neel-VBS criticality in square lattice antiferromagnets

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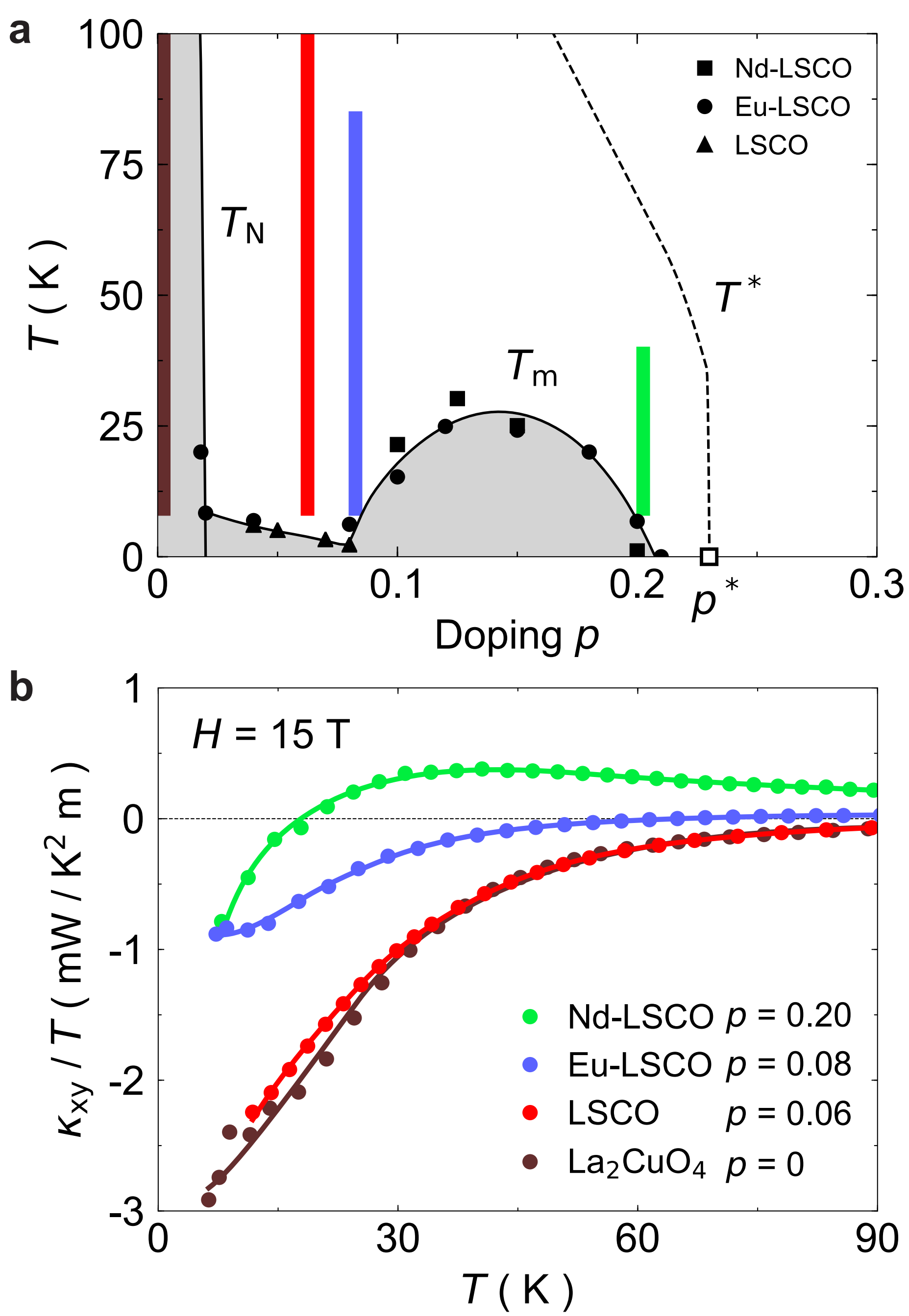
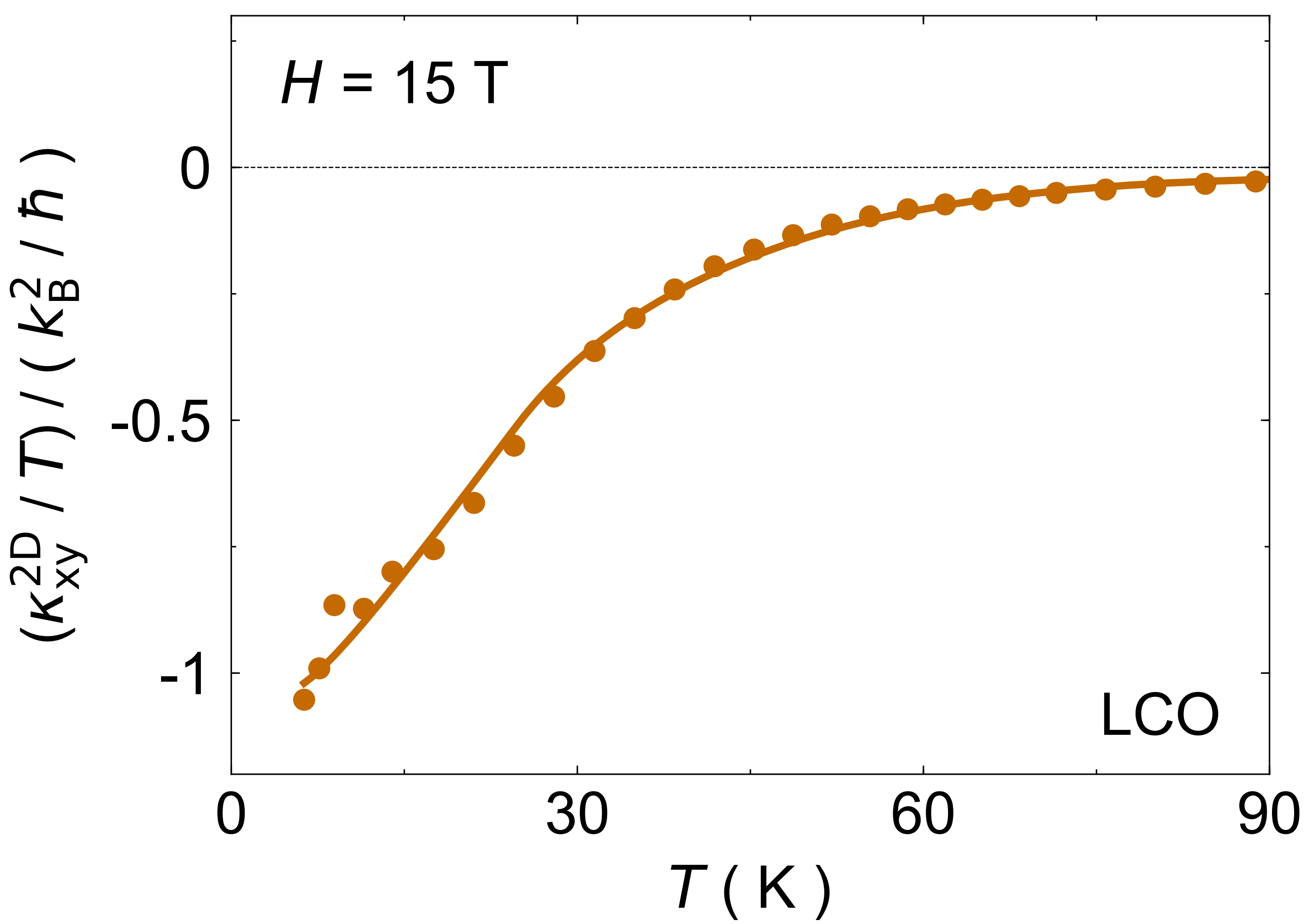
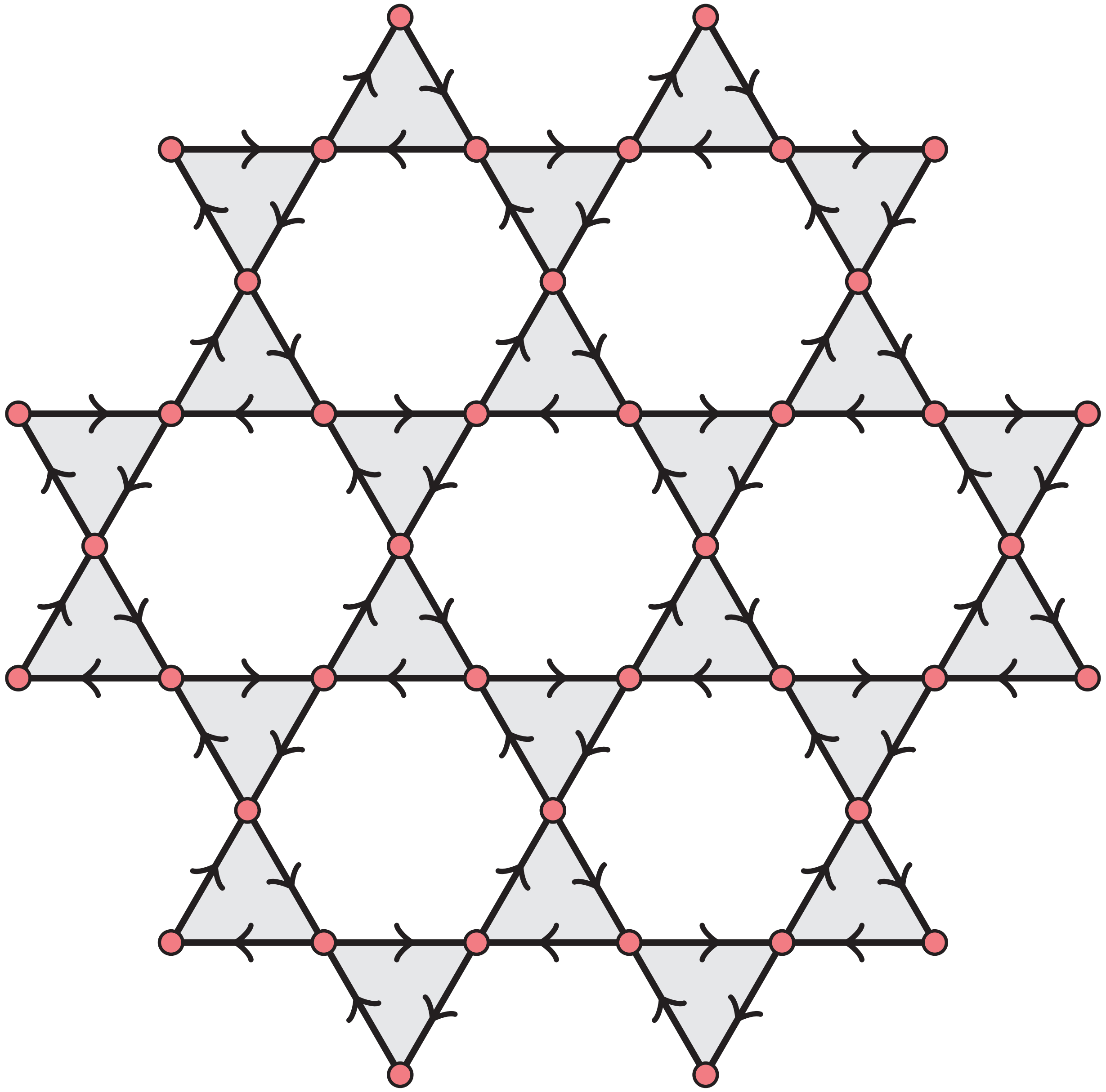
Giant thermal Hall conductivity from neutral excitations in the pseudogap phase of cuprates

G. Grissonnanche, A. Legros, S. Badoux, E. Lefrancois, V. Zatko,
M. Lizaire, F. Laliberte, A. Gourgout, J. Zhou, S. Pyon, T. Takayama,
H. Takagi, S. Ono, N. Doiron- Leyraud, and L. Taillefer, arXiv:1901.03104







$$H = H_1 + H_B$$

$$H_1 = \sum_{i<j} J_{ij} \boldsymbol{S}_i \cdot \boldsymbol{S}_j + \ldots$$

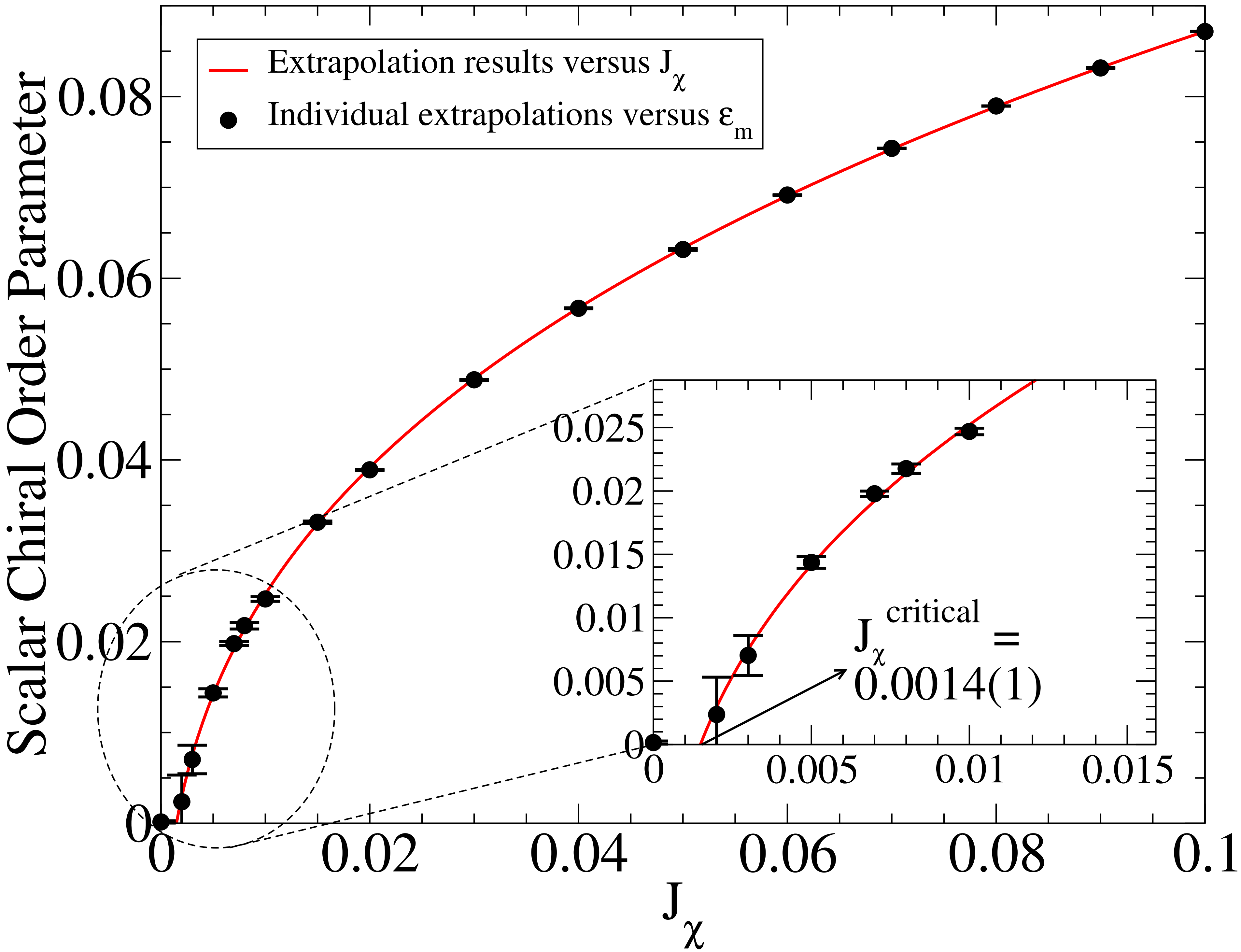
$$H_B = J_\chi \sum_{\triangle} \boldsymbol{S}_i \cdot (\boldsymbol{S}_j \times \boldsymbol{S}_k) - \sum_i \boldsymbol{B}_Z \cdot \boldsymbol{S}_i \,.$$

B. Bauer, L. Cincio, B.P. Keller, M. Dolfi, G. Vidal, S. Trebst and A.W.W. Ludwig, Nature Communications **5**, 5137 (2014)

Semion topological order,
i.e. the Kalmeyer-Laughlin chiral spin liquid,
appears for $J_\chi/J > 0.01$ ($\boldsymbol{B}_Z = 0$).

R. Haghshenas, Shou-Shu Gong, and D.N. Sheng, arXiv:1812.11436

Triangular lattice antiferromagnet

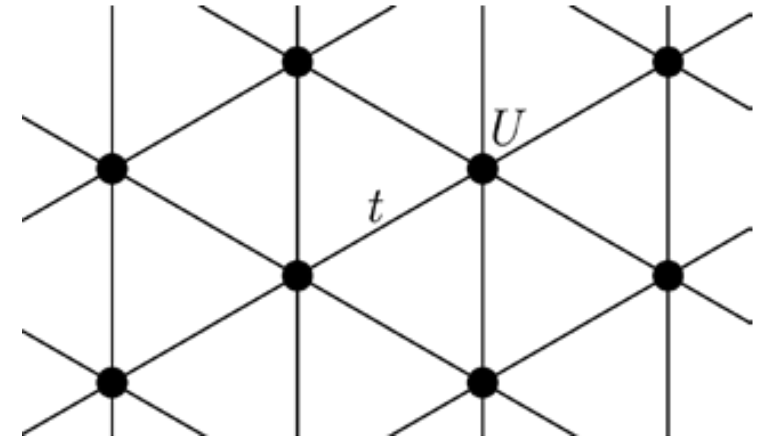


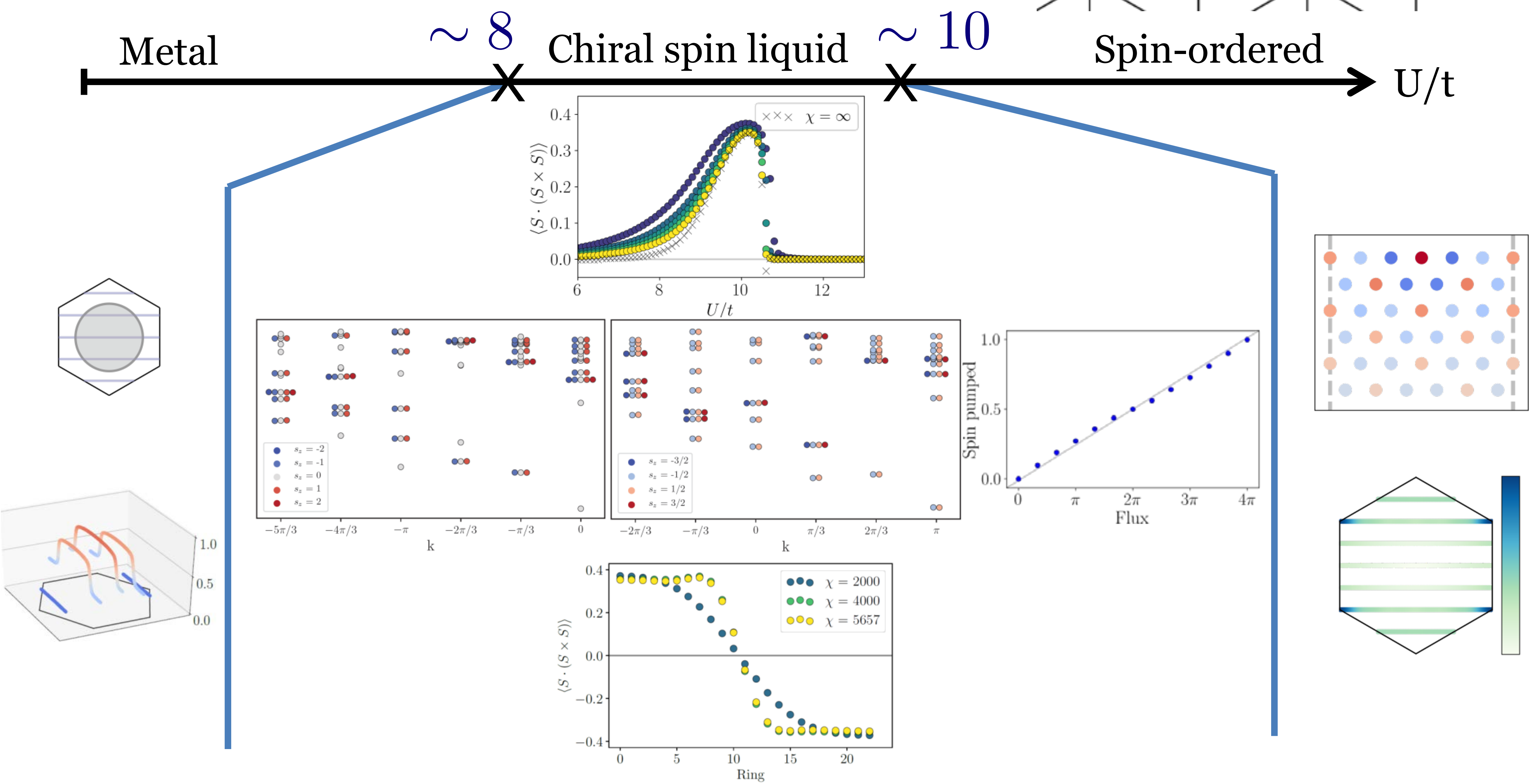
$$J_2/J_1 = 1/8; \text{ critical } J_\chi = 0.0014$$

S. N. Saadatmand and I. P. McCulloch, PRB **96**, 075117 (2017)

Hubbard model:

$$H = -t \sum_{\langle i,j \rangle, \sigma} \left(c_{i\sigma}^\dagger c_{j\sigma} + \text{h.c.} \right) + U \sum_i n_{i\uparrow} n_{i\downarrow}$$

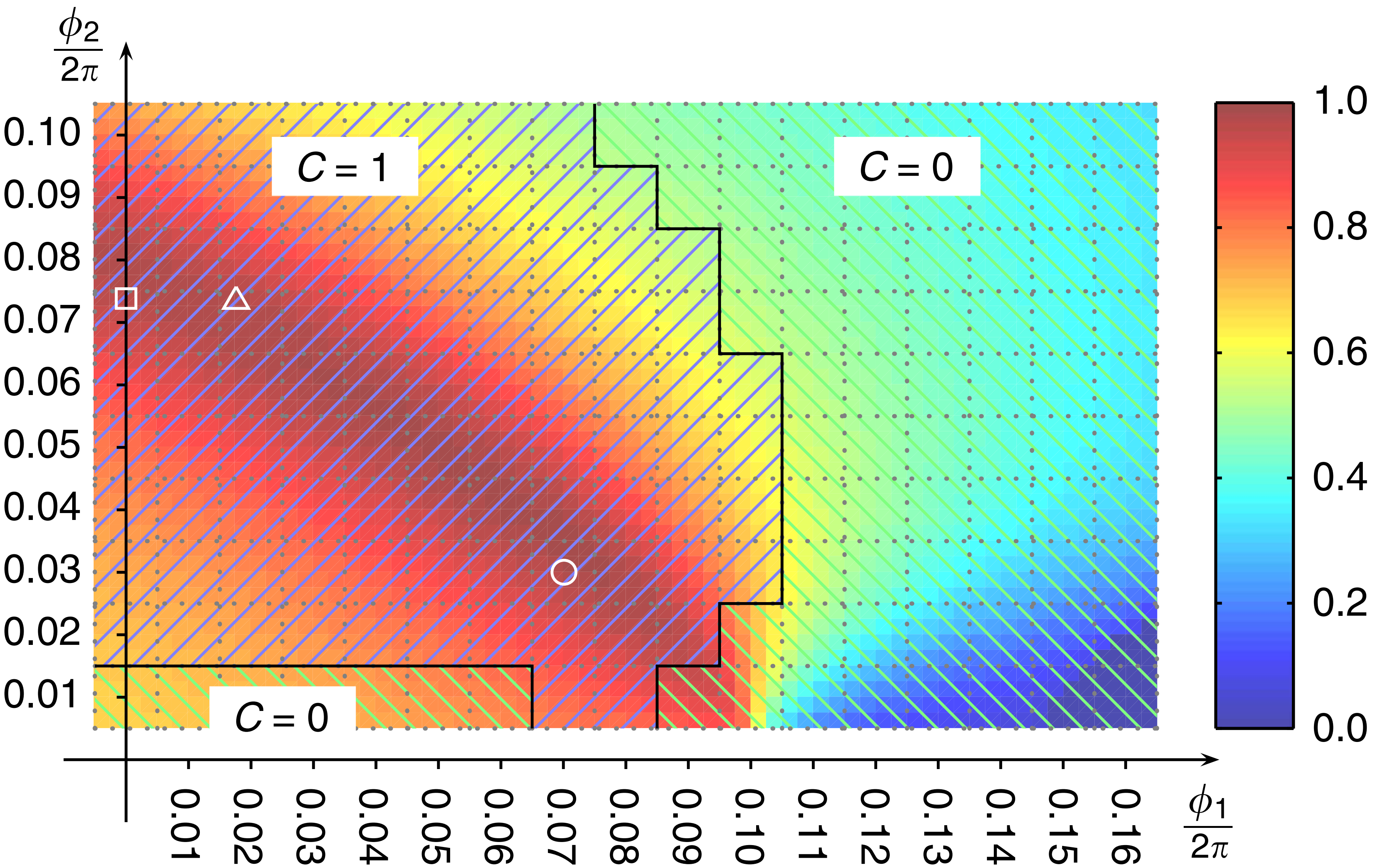




A. Szasz, J. Motruk, M. P. Zaletel, and J. E. Moore, arXiv: 1808.00463

(from slides by Aaron Szasz)

Square lattice antiferromagnet



$$J_1 = 2J\cos(\phi_1)\cos(\phi_2)$$
$$J_2 = 2J\sin(\phi_1)\cos(\phi_2)$$
$$J_\chi = 4J\sin(\phi_2)$$

Anne E.B. Nielsen, German Sierra, and J. Ignacio Cirac, Nature Communications **4**, 2864 (2013)

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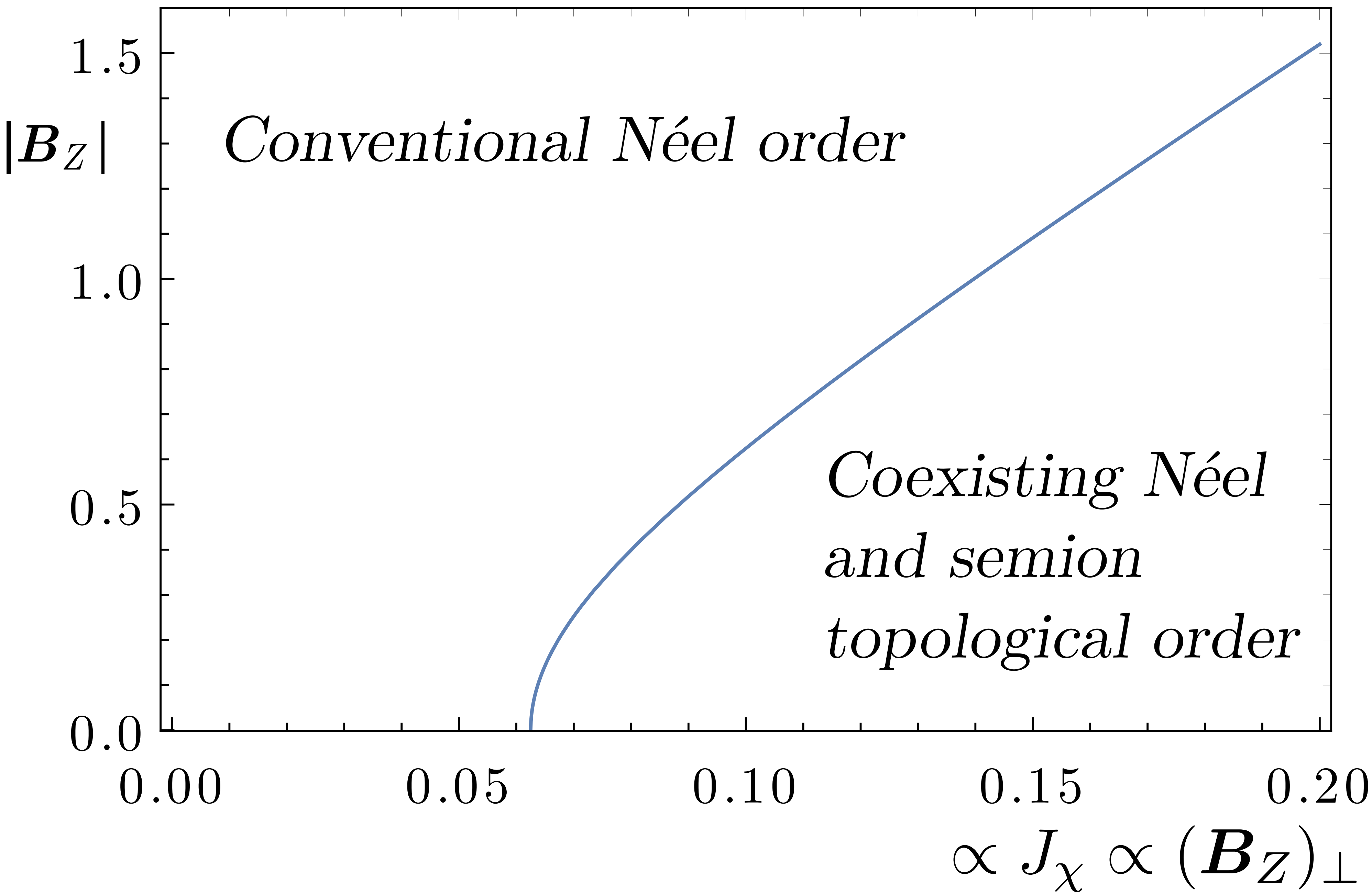
$$- \sum_i \boldsymbol{B}_Z \cdot \boldsymbol{S}_i \,.$$



S. Chatterjee, Haoyu Guo, S. Sachdev, R. Samajdar, M. S. Scheurer, N. Seiberg, and Cenke Xu, arXiv:1903.01992

SU(2) gauge theory of rotating reference frame in pseudospin space (similar to Schwinger fermions):
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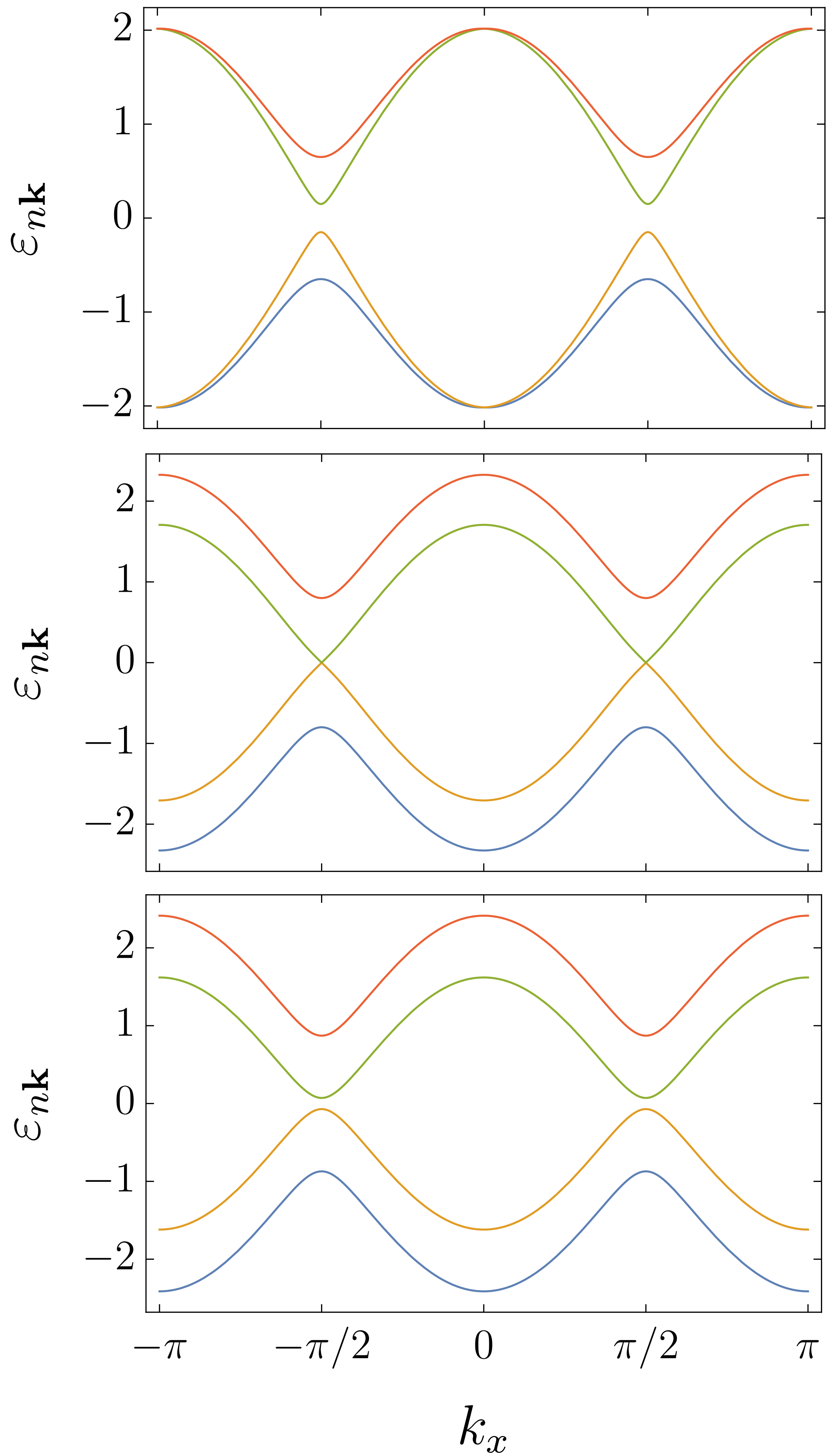
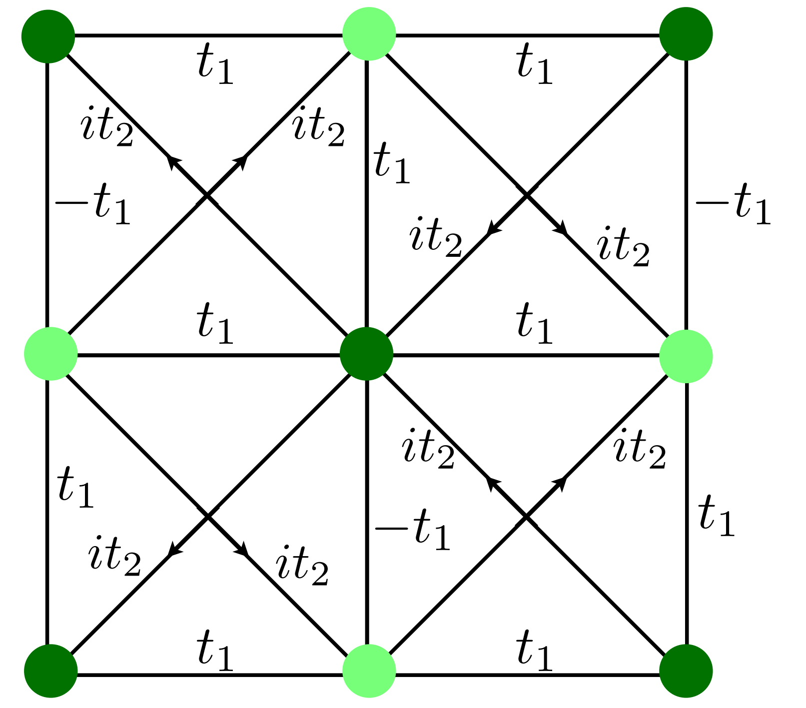
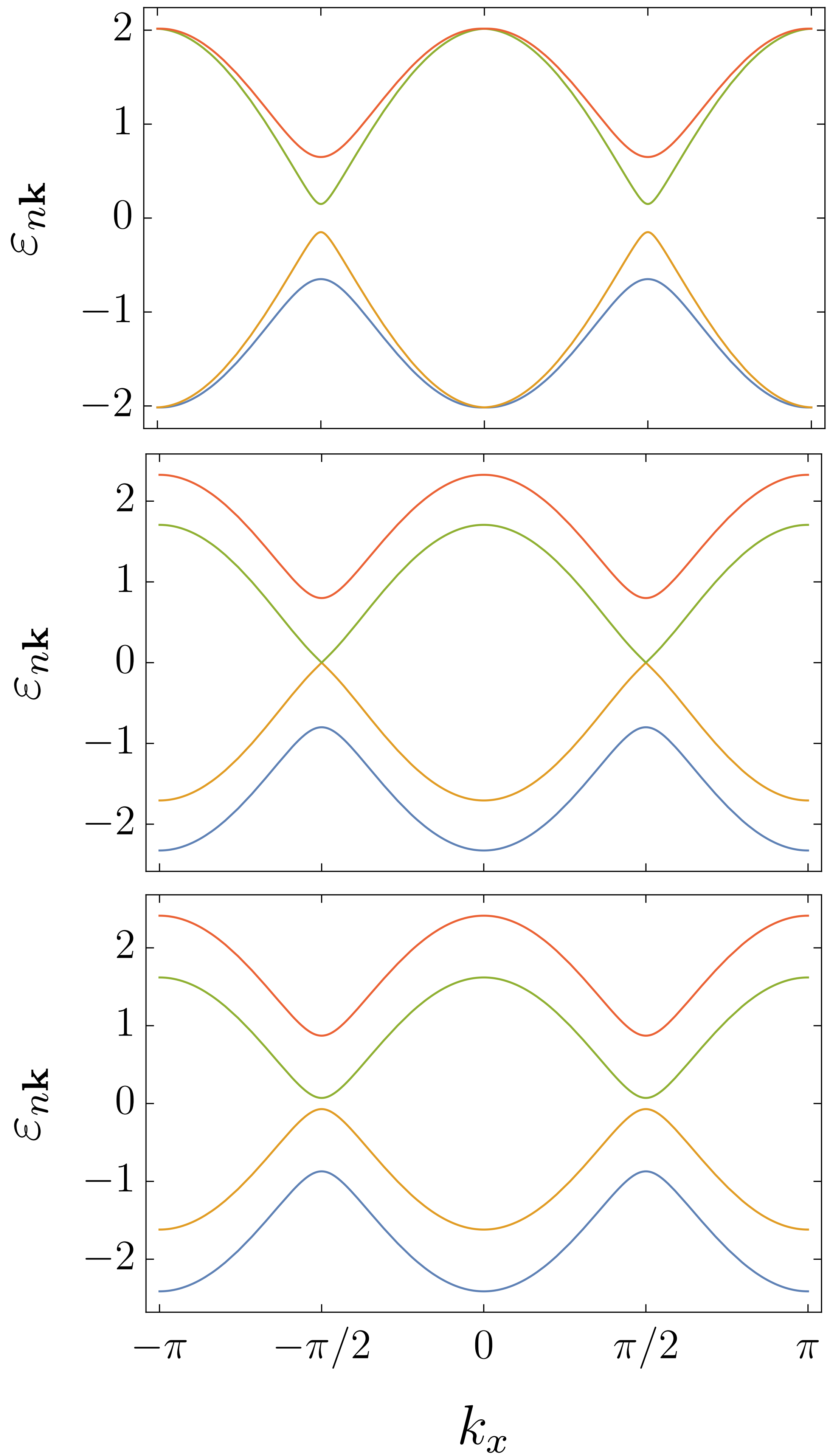
M. Scheurer and S. Sachdev, PRB **98**, 235126 (2018)

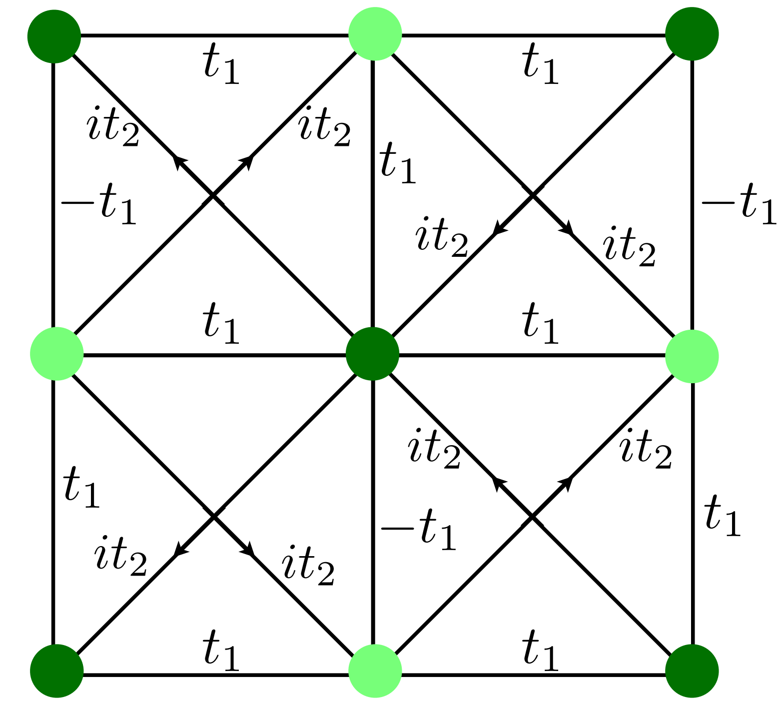




We focus on two possible mass terms for the Dirac fermions: m_1, induced by Néel order, and m_2 induced by chiral topological order. There is a line in the m_1, m_2 plane where the occupied bands switch from Chern numbers $\{1, -1\}$ (the Néel state) to $\{1, 1\}$ (Néel order co-existing with semion topological order).

S. Chatterjee, Haoyu Guo, S. Sachdev, R. Samajdar, M. S. Scheurer, N. Seiberg, and Cenke Xu, arXiv:1903.01992



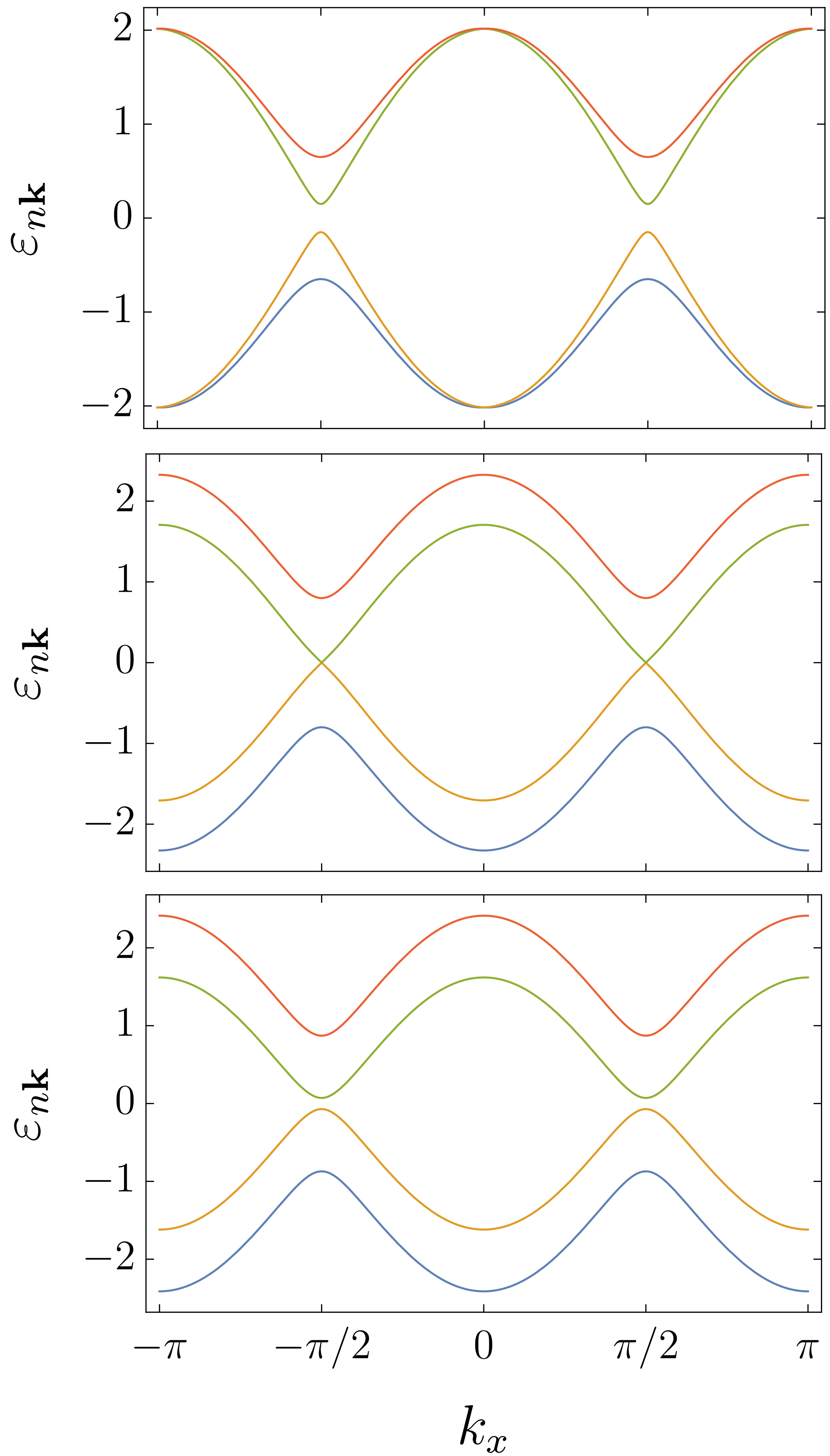


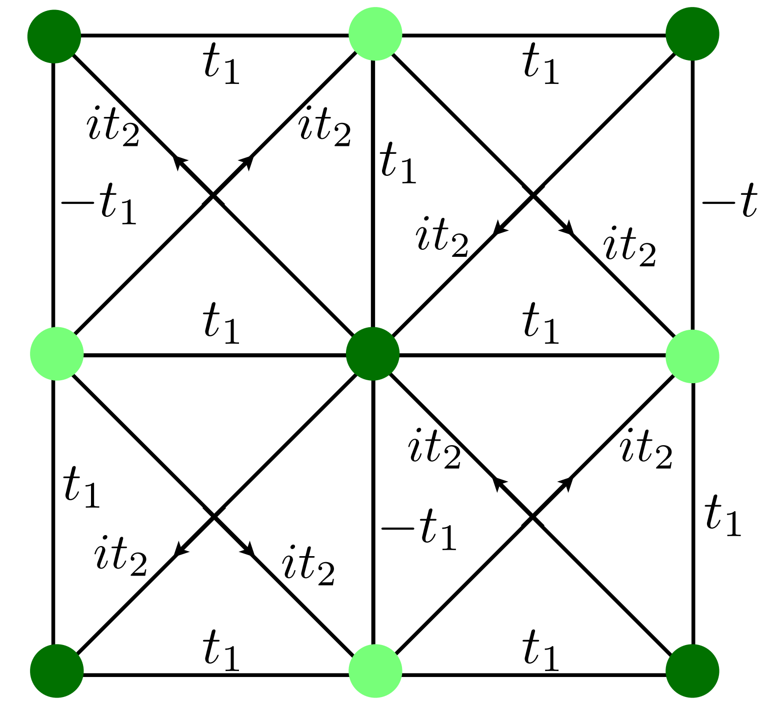
The vicinity of the critical point is described by $N_f = 1$ Dirac fermion coupled to a SU(2) gauge field A_μ at level $-1/2$

$$\mathcal{L}_f = \overline{f}\gamma^\mu(\partial_\mu - iA_\mu)f + m\overline{f}f - \frac{1}{2}\mathrm{CS}[A_\mu]$$

The transition is tuned by the change in sign of m.

S. Chatterjee, Haoyu Guo, S. Sachdev, R. Samajdar, M. S. Scheurer, N. Seiberg, and Cenke Xu, arXiv:1903.01992

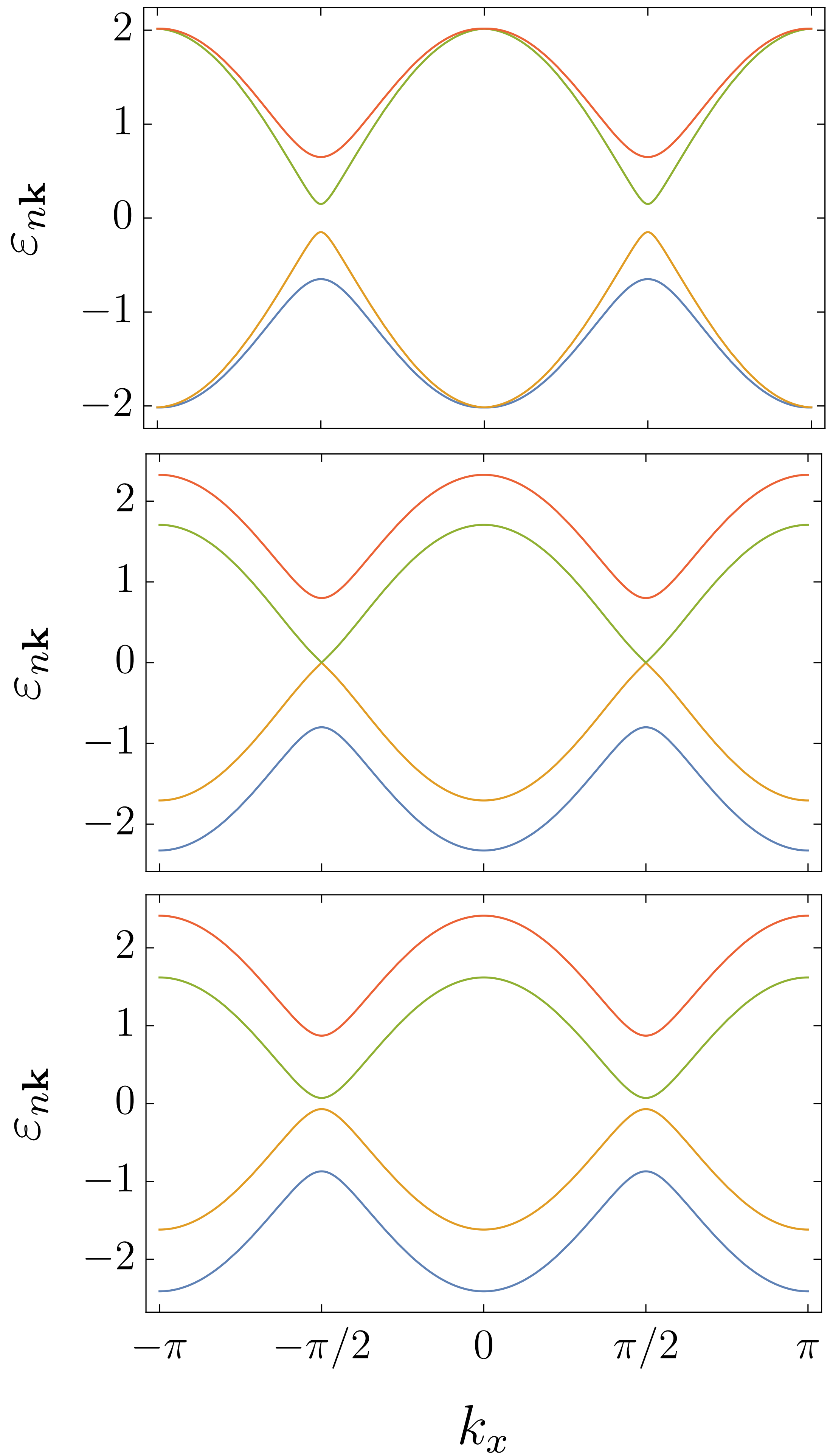


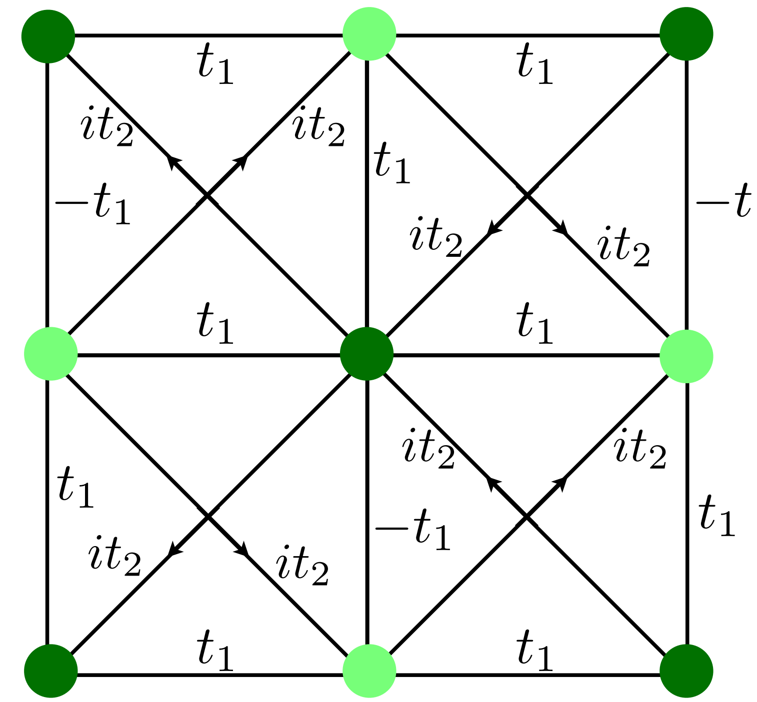


$$\mathcal{L}_f = \overline{f}\gamma^\mu(\partial_\mu - iA_\mu)f + m\overline{f}f - \frac{1}{2}\text{CS}[A_\mu]$$

When $m > 0$, we can integrate out f and there is no net CS term. The SU(2) gauge theory confines, and we obtain Néel order.

S. Chatterjee, Haoyu Guo, S. Sachdev, R. Samajdar, M. S. Scheurer, N. Seiberg, and Cenke Xu, arXiv:1903.01992





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When $m < 0$, we can integrate out f and we obtain a net CS term at level -1. The SU(2) gauge theory at level -1 describes the semion topological phase.

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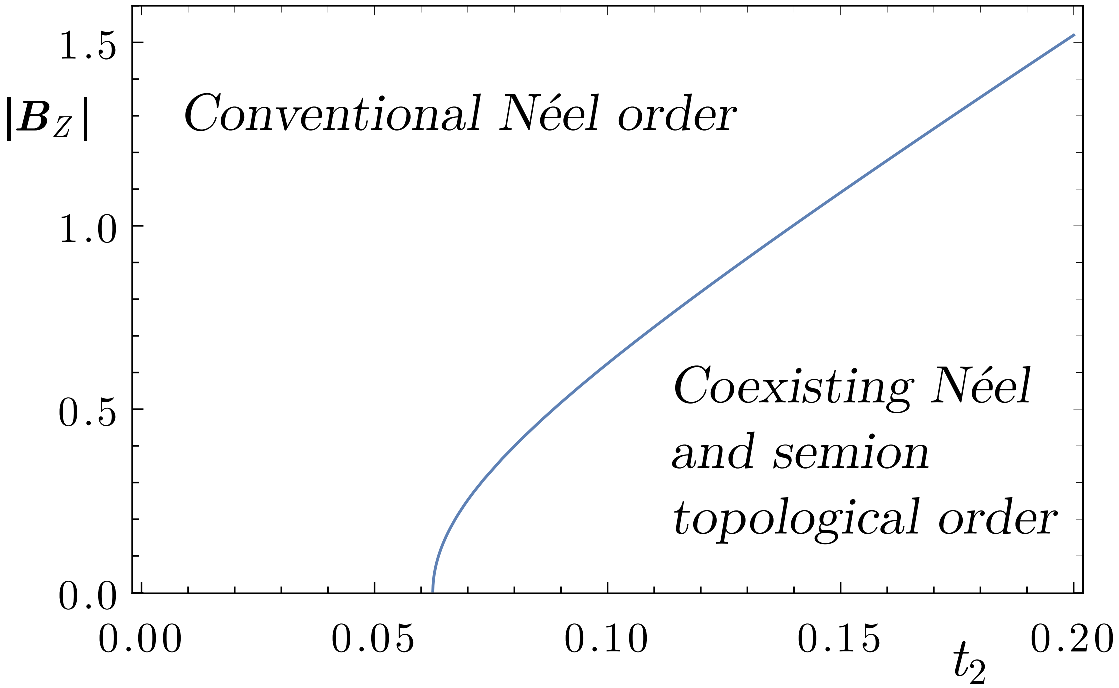

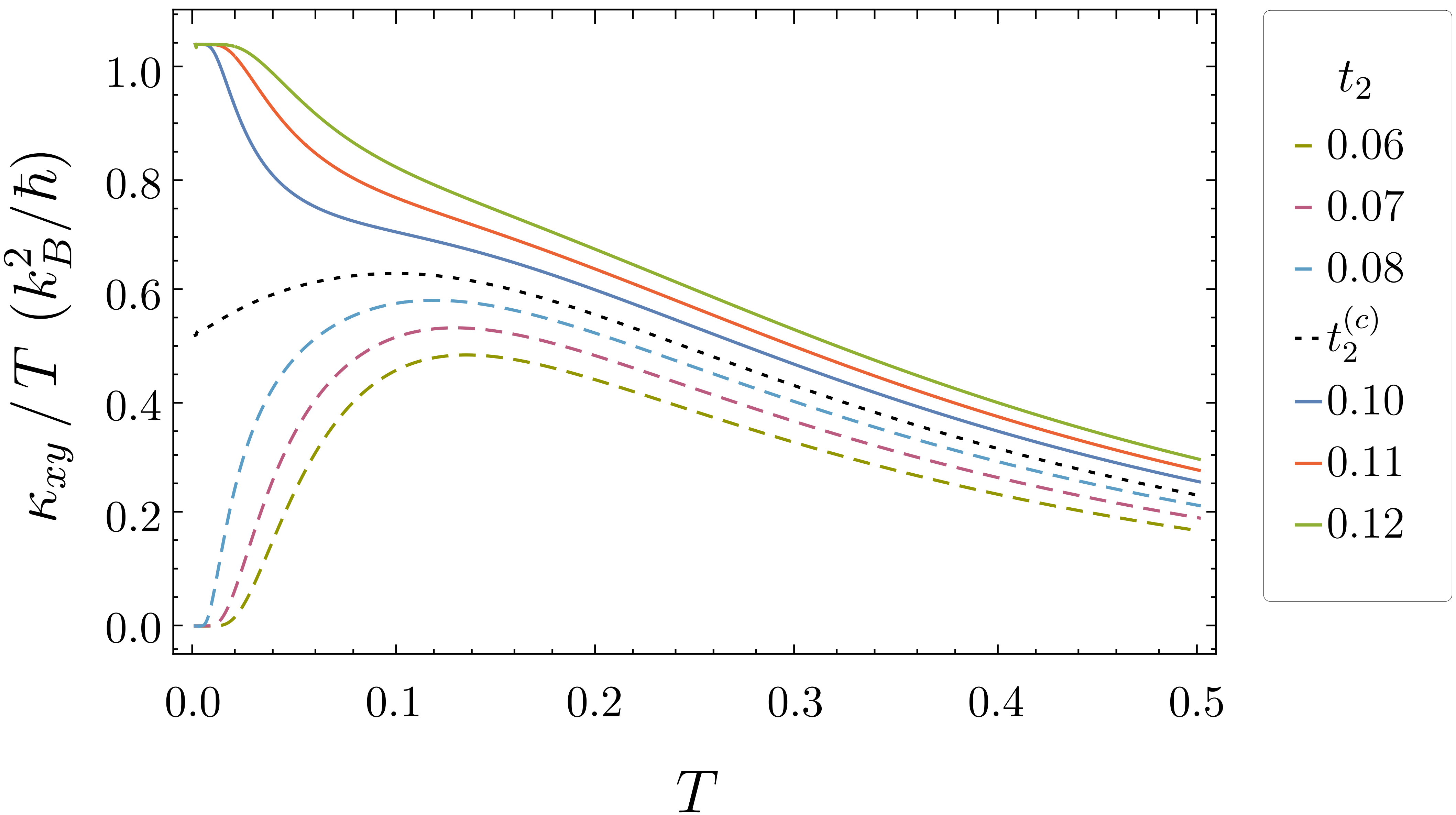

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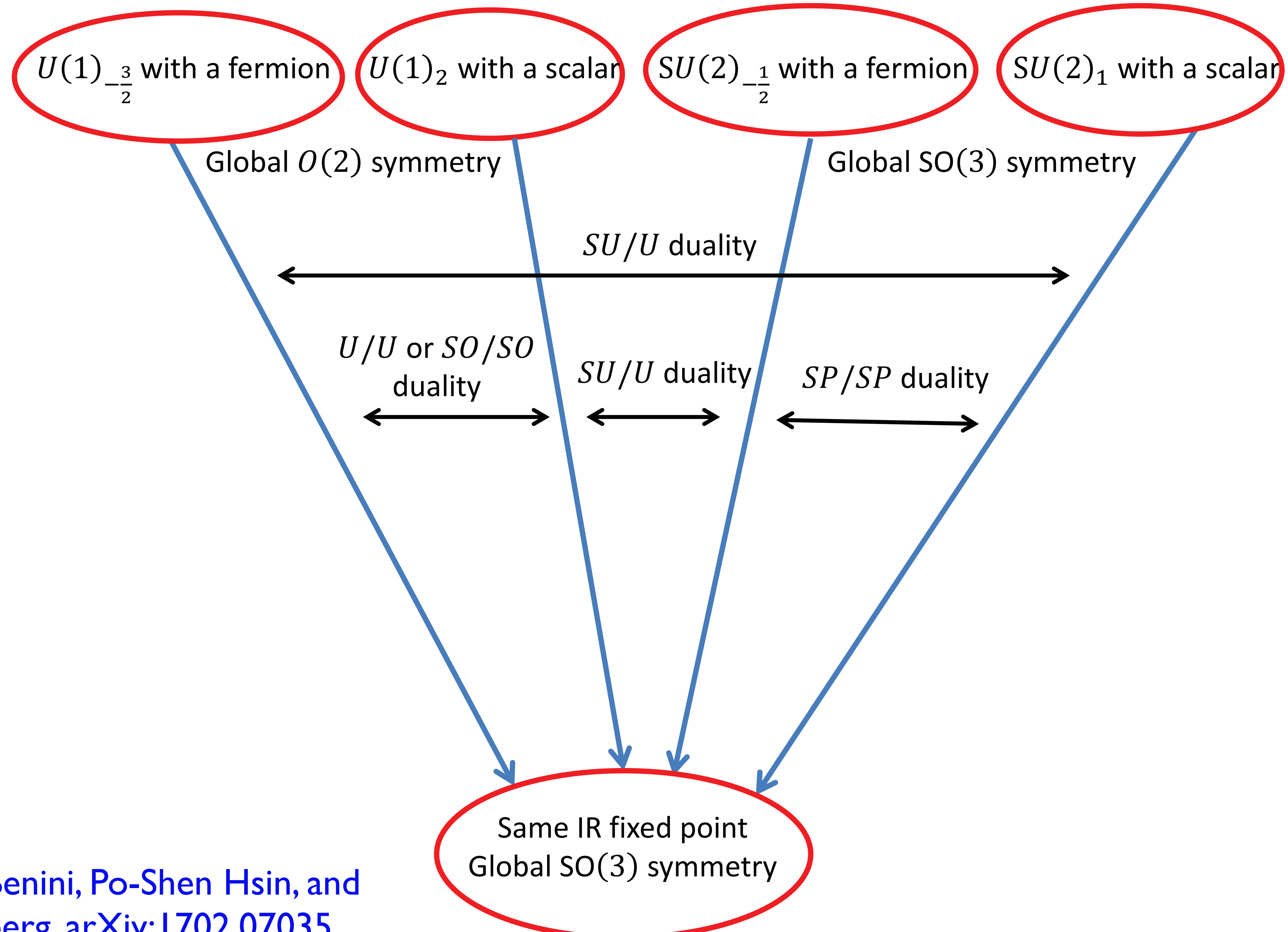
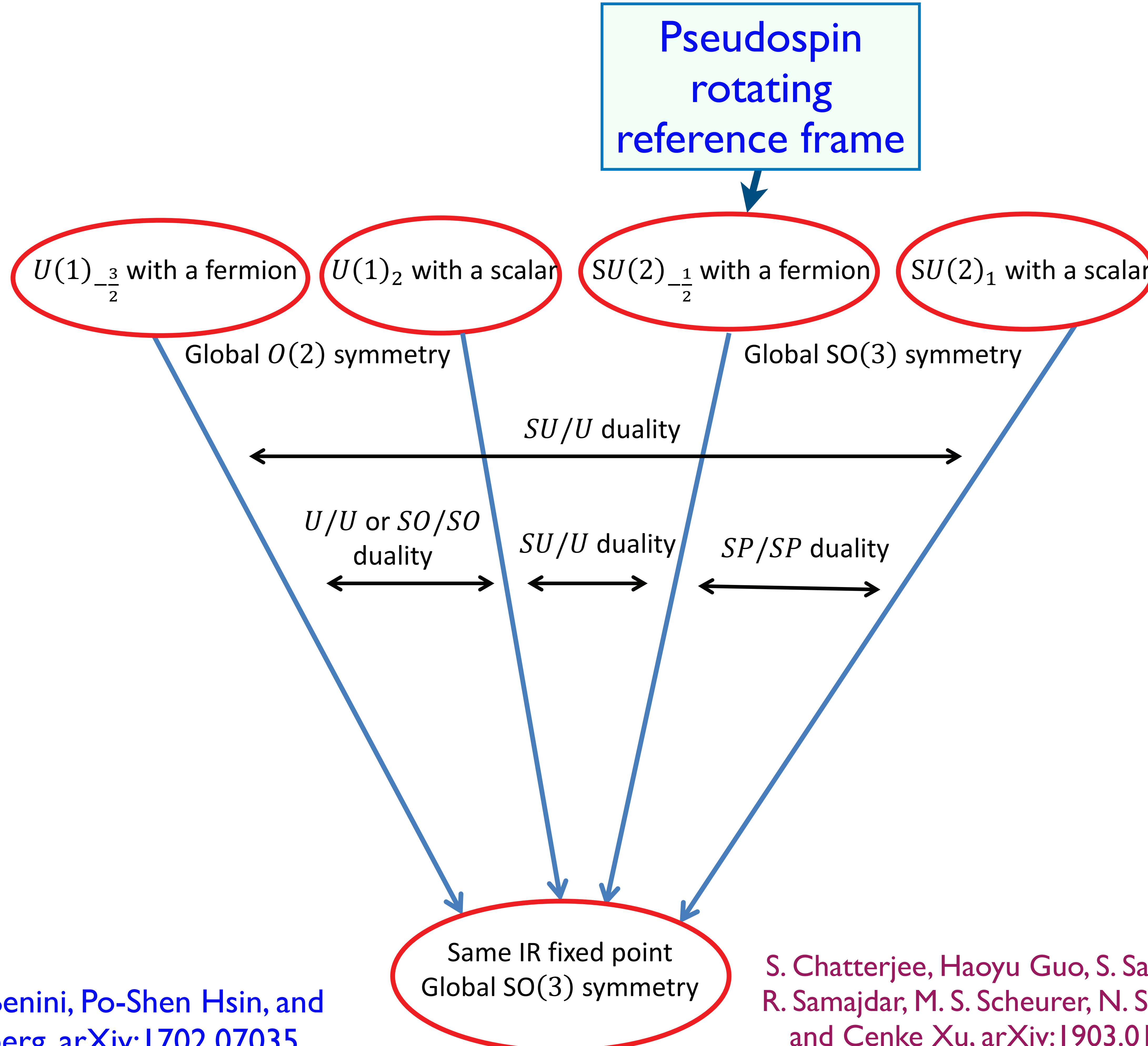


Francesco Benini, Po-Shen Hsin, and Nathan Seiberg, arXiv:1702.07035



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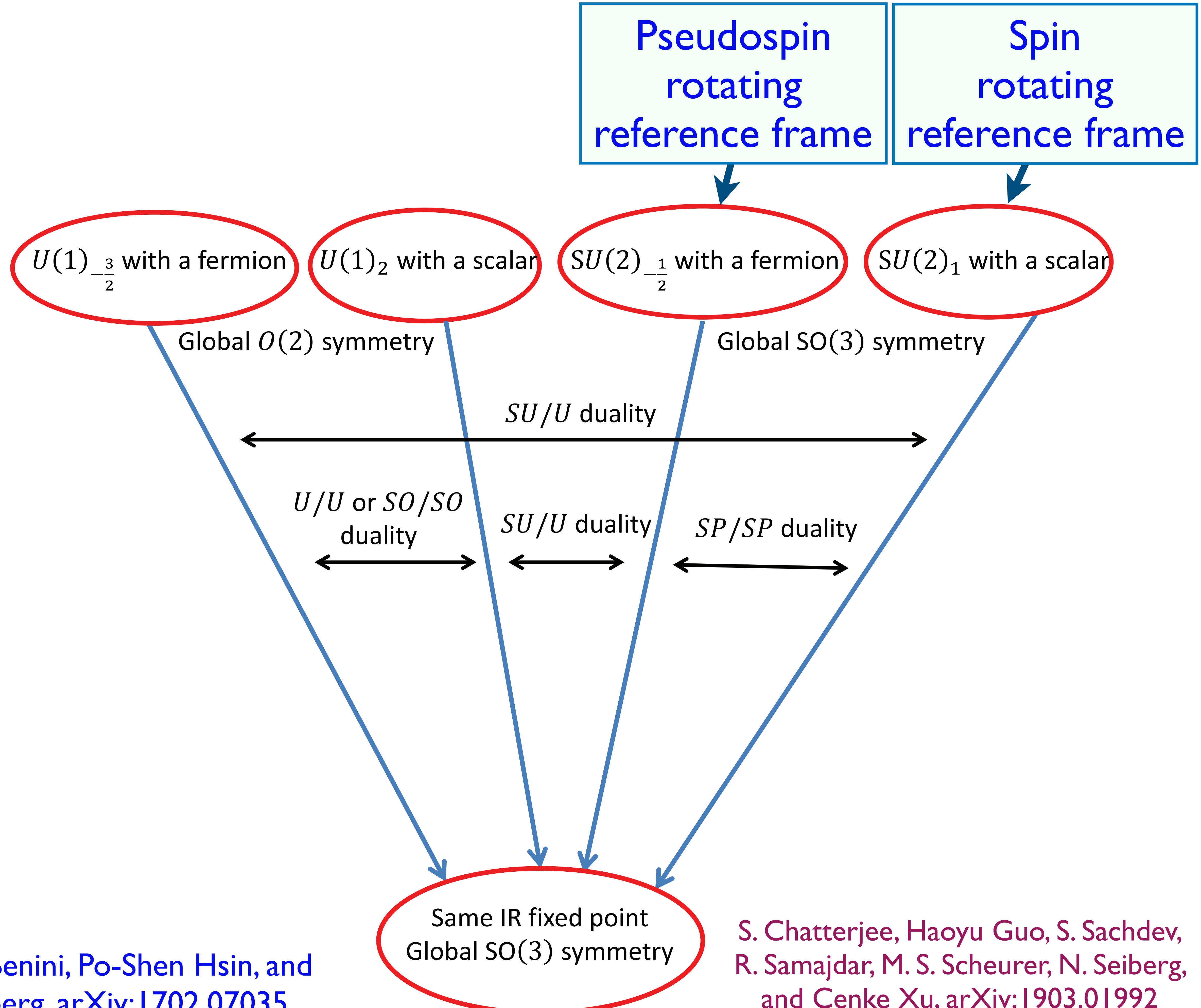

Francesco Benini, Po-Shen Hsin, and Nathan Seiberg, arXiv:1702.07035

S. Chatterjee, Haoyu Guo, S. Sachdev, R. Samajdar, M. S. Scheurer, N. Seiberg, and Cenke Xu, arXiv:1903.01992

SU(2) gauge theory of rotating reference frame in pseudospin space (similar to Schwinger fermions):
Write the lattice electron operator $c_{i\alpha}$ as

$$C_i = \begin{pmatrix} c_{i\uparrow} & -c_{i\downarrow}^\dagger \\ c_{i\downarrow} & c_{i\uparrow}^\dagger \end{pmatrix} \quad , \quad C_i = F_i R_{ci}$$

$$F_i = \begin{pmatrix} f_{i\uparrow} & -f_{i\downarrow}^\dagger \\ f_{i\downarrow} & f_{i\uparrow}^\dagger \end{pmatrix} \quad , \quad R_{ci} = \begin{pmatrix} b_{i1} & b_{i2} \\ -b_{i2}^* & b_{i1}^* \end{pmatrix}$$

F are fermionic spinons, R_c is a SU(2) rotation. Pseudospin rotations are *right* multiplication of R_c, while *left* multiplication is an emergent SU(2) gauge symmetry:

$$F \to FU, \quad R_c \to U^\dagger R_c \,.$$

M. Scheurer and S. Sachdev, PRB **98**, 235126 (2018)

SU(2) gauge theory of rotating reference frame in spin space (similar to Schwinger bosons):
Write the lattice electron operator $c_{i\alpha}$ as

$$C_i = \begin{pmatrix} c_{i\uparrow} & -c_{i\downarrow}^\dagger \\ c_{i\downarrow} & c_{i\uparrow}^\dagger \end{pmatrix} \quad , \quad C_i = R_{si} \Psi_i$$

$$\Psi_i = \begin{pmatrix} \psi_{i+} & -\psi_{i-}^\dagger \\ \psi_{i-} & \psi_{i+}^\dagger \end{pmatrix} \quad , \quad R_{si} = \begin{pmatrix} z_{i\uparrow} & -z_{i\downarrow}^* \\ z_{i\downarrow} & z_{i\uparrow}^* \end{pmatrix}$$

Ψ are fermionic 'chargons', R_s is a SU(2) rotation. Spin rotations are *left* multiplication of R_s, while *right* multiplication is an emergent SU(2) gauge symmetry:

$$\Psi \to U\Psi, \quad R_s \to R_s U^\dagger .$$

S. Sachdev, M.A. Metlitski, Y. Qi, and C. Xu, PRB **80**, 155129 (2009)

SU(2) gauge theory of rotating reference frame in spin space (similar to Schwinger bosons):

Write the lattice electron operator $c_{i\alpha}$ as

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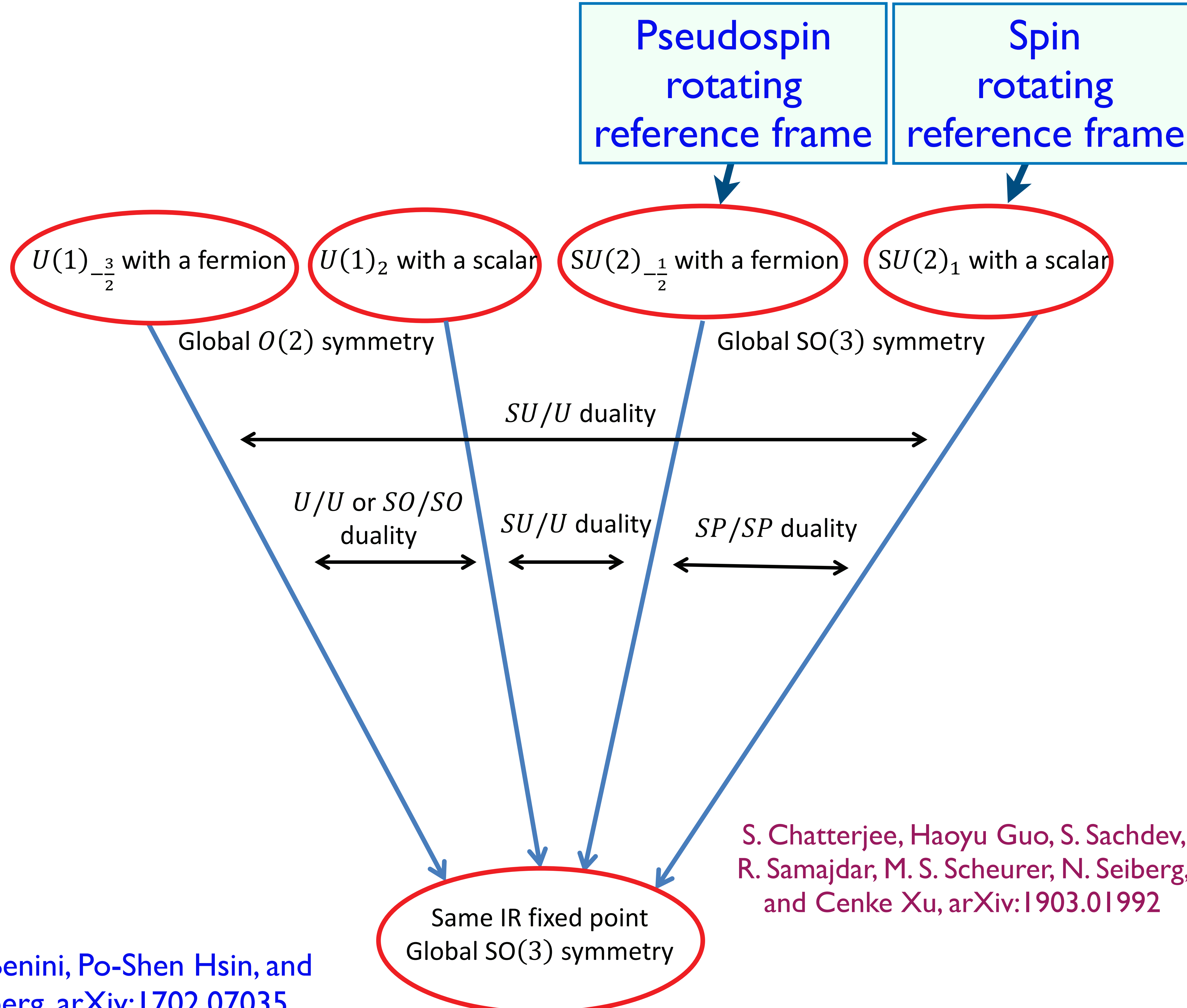
Ψ are fermionic 'chargons', R_s is a SU(2) rotation. Spin rotations are *left* multiplication of R_s, while *right* multiplication is an emergent SU(2) gauge symmetry:

$$\Psi \to U\Psi, \quad R_s \to R_s U^\dagger .$$

The fermionic chargons Ψ fully occupy a band with Chern number 1, and are gapped. When the R bosons are also gapped, we obtain the state with semion topological order. When the bosons condense, we obtain a trivial state. The critical theory is a SU(2) gauge theory at level 1 coupled to massless self-interacting scalar.

S. Chatterjee, Haoyu Guo, S. Sachdev, R. Samajdar, M. S. Scheurer, N. Seiberg, and Cenke Xu, arXiv:1903.01992



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Francesco Benini, Po-Shen Hsin, and Nathan Seiberg, arXiv:1702.07035

Another non-Abelian duality

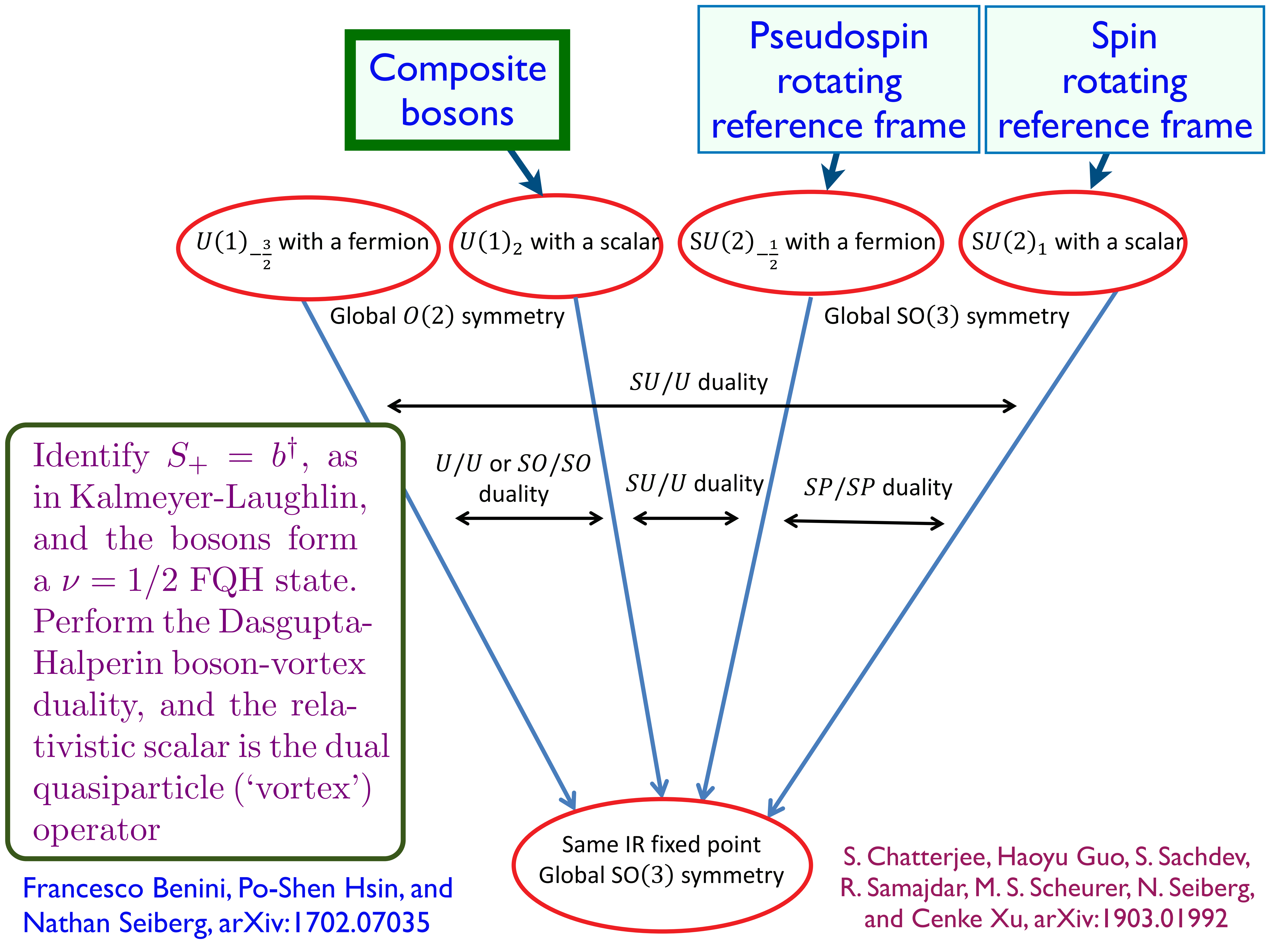
Critical SU(2) gauge theory of $N_1 = 2$ relativistic bosons
at CS level 1
is dual to
SU(2) gauge theory of $N_f = 1$ Dirac fermion
at CS level -1/2.

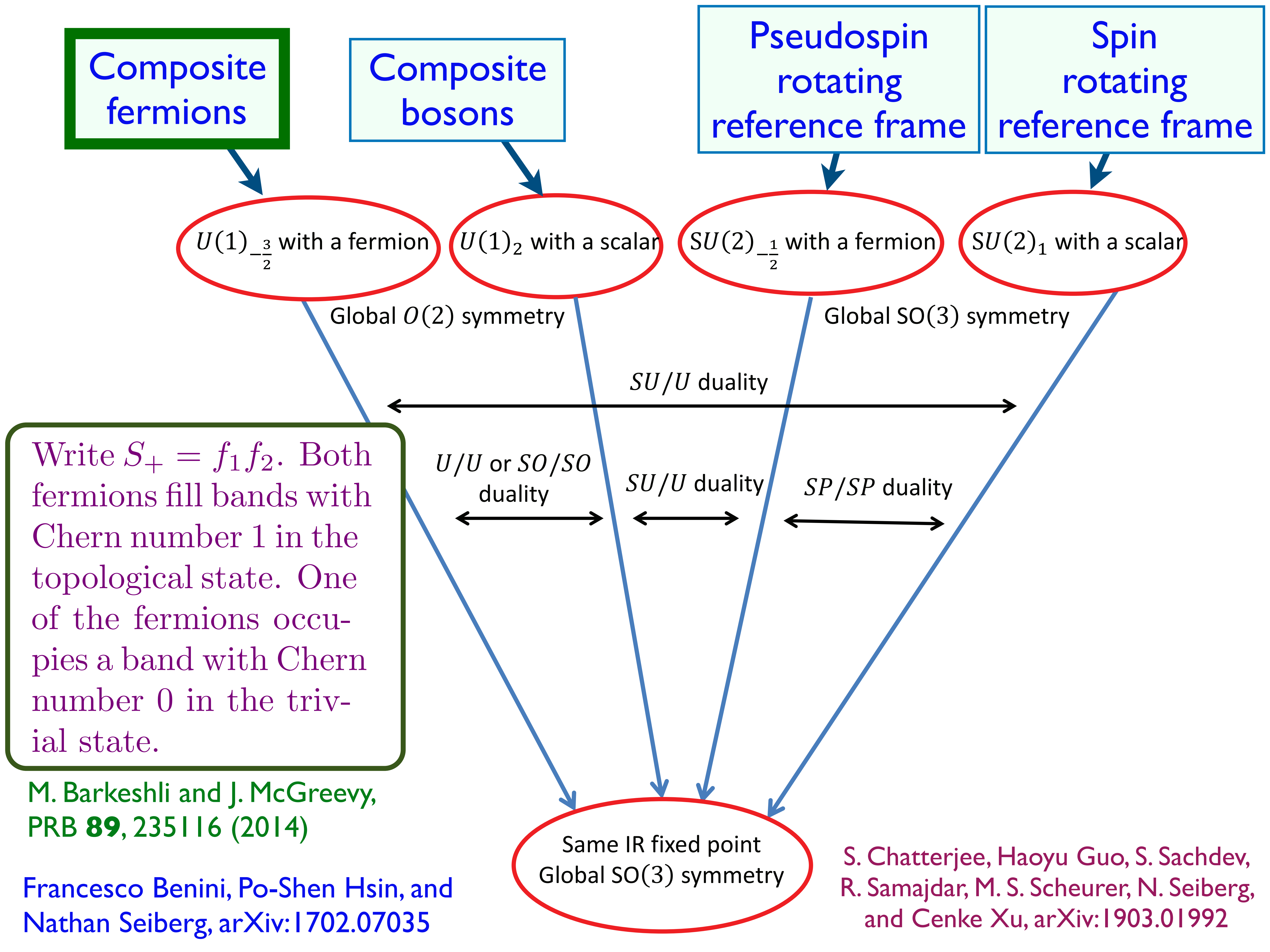
$$\mathcal{L}_z = |(\partial_\mu - iA_\mu)z|^2 + s|z|^2 + u(|z|^2)^2 + \mathrm{CS}[A_\mu]$$

$$\mathcal{L}_f = \overline{f}\gamma^\mu(\partial_\mu - iA_\mu)f + m\overline{f}f - \frac{1}{2}\mathrm{CS}[A_\mu]$$

Both theories have an emergent global SO(3) symmetry

S. Chatterjee, Haoyu Guo, S. Sachdev, R. Samajdar, M. S. Scheurer, N. Seiberg, and Cenke Xu, arXiv:1903.01992


Composite bosons
Pseudospin rotating reference frame
Spin rotating reference frame
$U(1)_{-\frac{3}{2}}$ with a fermion
$U(1)_2$ with a scalar
$SU(2)_{-\frac{1}{2}}$ with a fermion
$SU(2)_1$ with a scalar
Global $O(2)$ symmetry
Global SO(3) symmetry
SU/U duality
U/U or SO/SO duality
SU/U duality
SP/SP duality
Identify $S_+ = b^\dagger$, as in Kalmeyer-Laughlin, and the bosons form a $\nu = 1/2$ FQH state. Perform the Dasgupta-Halperin boson-vortex duality, and the relativistic scalar is the dual quasiparticle ('vortex') operator
Same IR fixed point
Global SO(3) symmetry
Francesco Benini, Po-Shen Hsin, and Nathan Seiberg, arXiv:1702.07035
S. Chatterjee, Haoyu Guo, S. Sachdev, R. Samajdar, M. S. Scheurer, N. Seiberg, and Cenke Xu, arXiv:1903.01992



Composite fermions
Composite bosons
Pseudospin rotating reference frame
Spin rotating reference frame
$U(1)_{-\frac{3}{2}}$ with a fermion
$U(1)_2$ with a scalar
$SU(2)_{-\frac{1}{2}}$ with a fermion
$SU(2)_1$ with a scalar
Global $O(2)$ symmetry
Global SO(3) symmetry
SU/U duality
U/U or SO/SO duality
SU/U duality
SP/SP duality
Write $S_+ = f_1 f_2$. Both fermions fill bands with Chern number 1 in the topological state. One of the fermions occupies a band with Chern number 0 in the trivial state.
Same IR fixed point
Global SO(3) symmetry
M. Barkeshli and J. McGreevy, PRB **89**, 235116 (2014)
Francesco Benini, Po-Shen Hsin, and Nathan Seiberg, arXiv:1702.07035
S. Chatterjee, Haoyu Guo, S. Sachdev, R. Samajdar, M. S. Scheurer, N. Seiberg, and Cenke Xu, arXiv:1903.01992

A quadrilarity

$$\mathcal{L}_z = |(\partial_\mu - iA_\mu)z|^2 + s|z|^2 + u(|z|^2)^2 + \mathrm{CS}[A_\mu]$$

$$\mathcal{L}_f = \overline{f}\gamma^\mu(\partial_\mu - iA_\mu)f + m\overline{f}f - \frac{1}{2}\mathrm{CS}[A_\mu]$$

$$\mathcal{L}_\phi = |(\partial_\mu - ia_\mu)\phi|^2 + s|\phi|^2 + u(|\phi|^2)^2 + 2\mathrm{CS}[a_\mu]$$

$$\mathcal{L}_g = \overline{g}\gamma^\mu(\partial_\mu - ia_\mu)g + m\overline{g}g - \frac{3}{2}\mathrm{CS}[a_\mu]$$

Francesco Benini, Po-Shen Hsin, and Nathan Seiberg, arXiv:1702.07035

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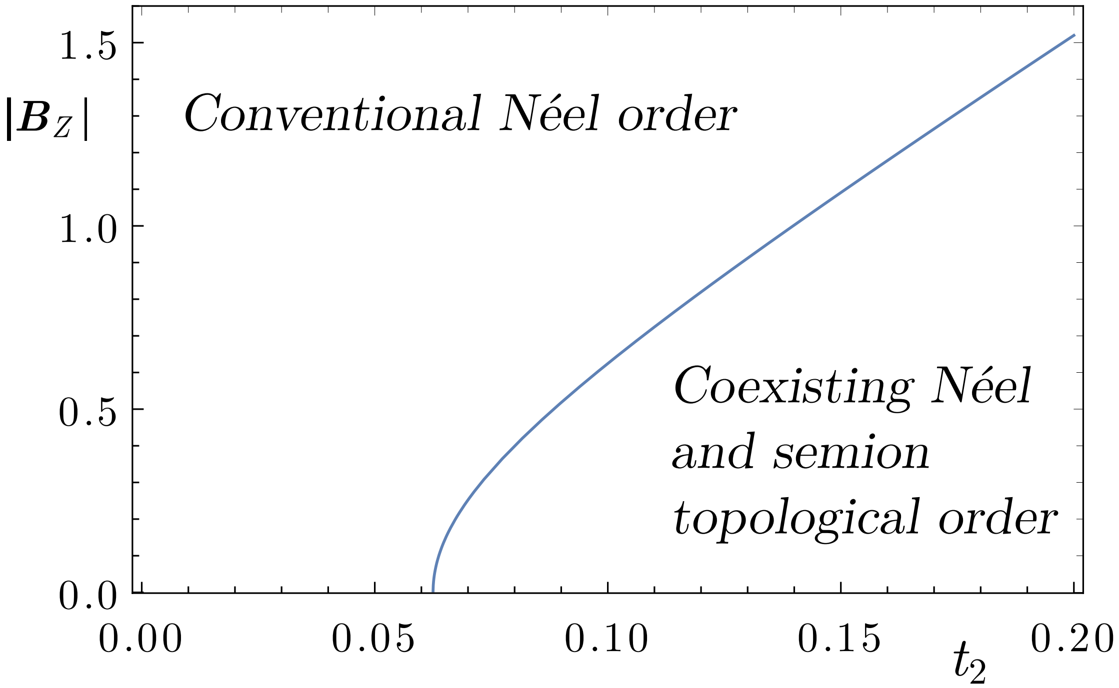

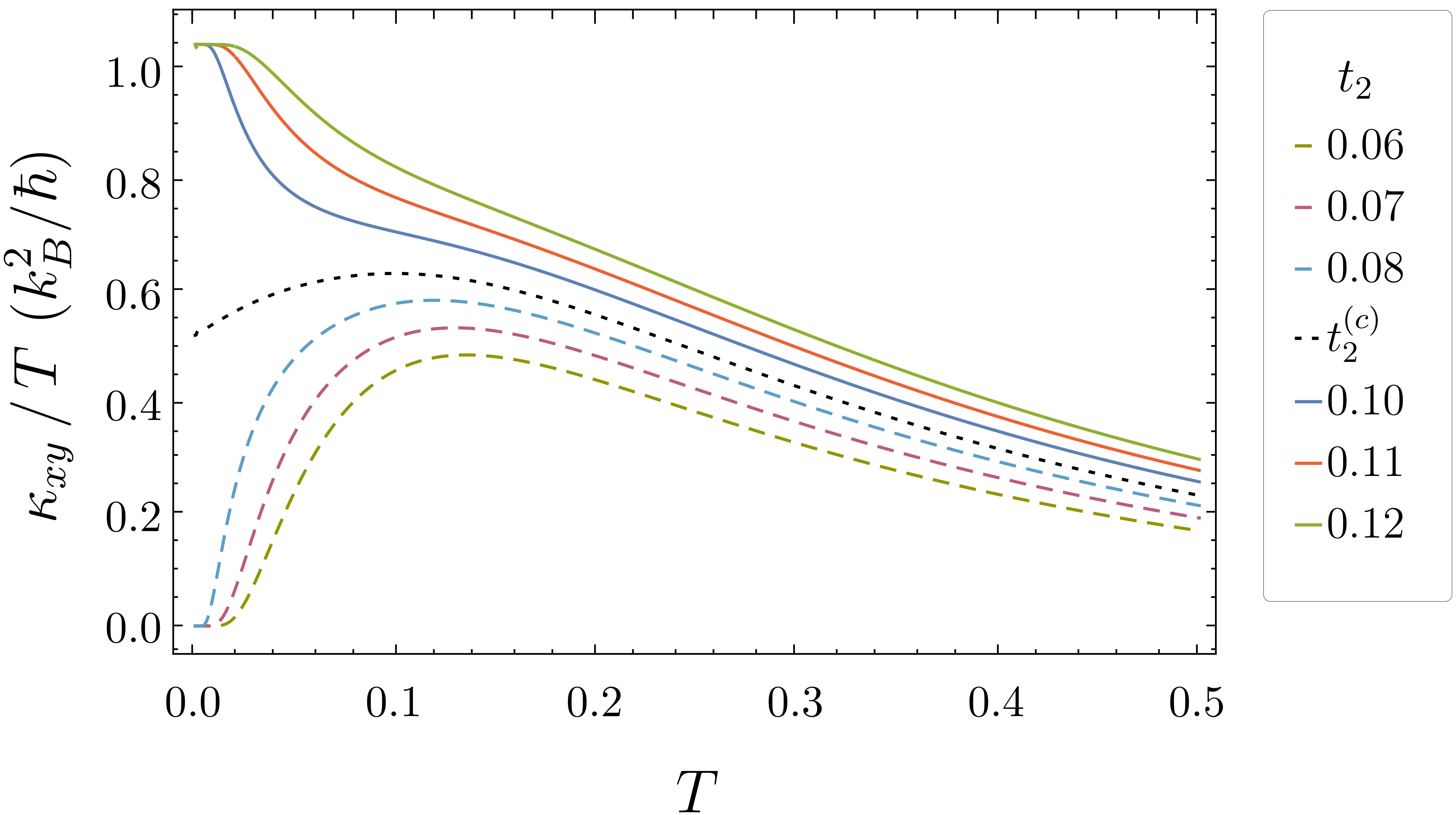

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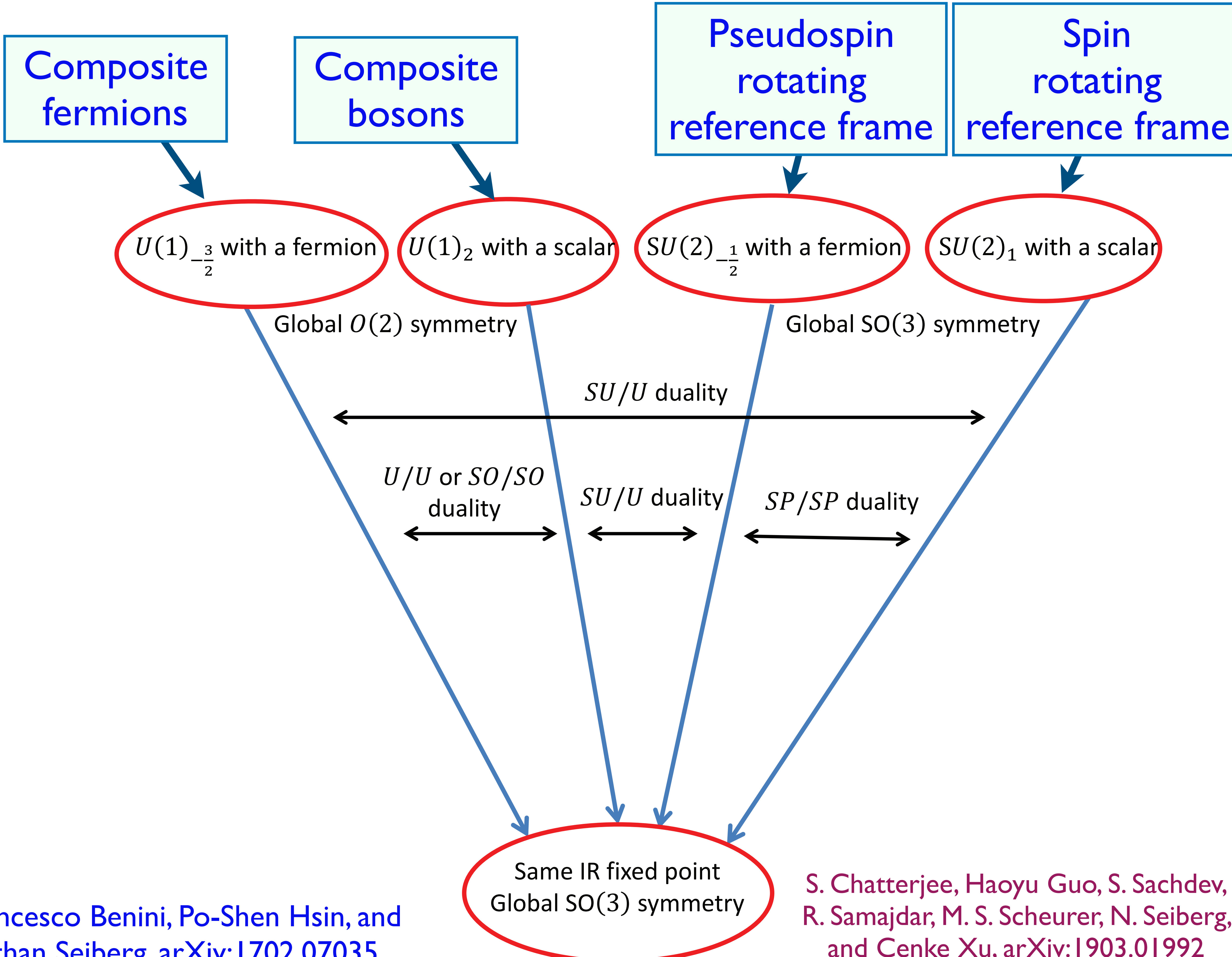

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