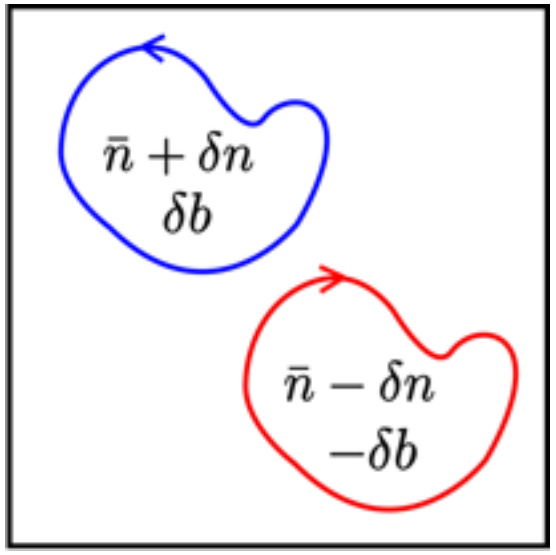
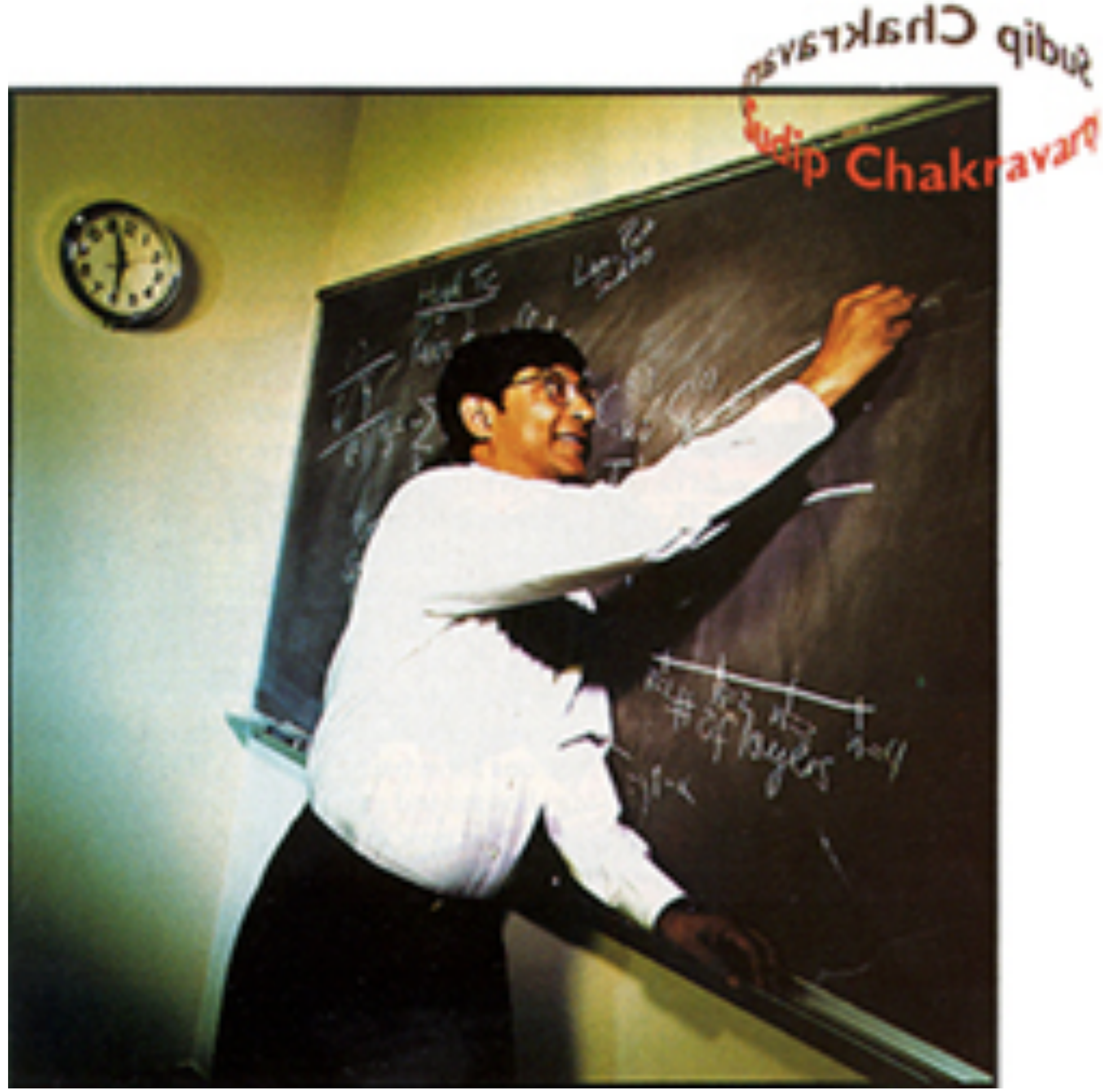
Half-filled Landau level with quenched disorder

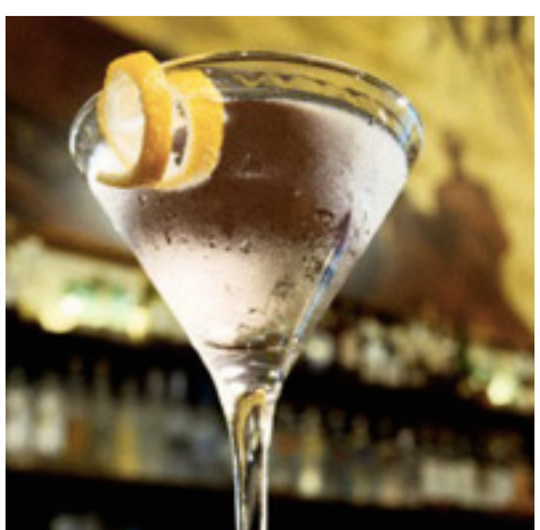
Srinivas Raghu (Stanford)
Prashant Kumar (Stanford)
Michael Mulligan (UCR)



arXiv:1903.06297
1805.06462
1803.07767

Sudip Chakravarty





happy birthday, Sudip!

Two-dimensional quantum Heisenberg antiferromagnet at low temperatures

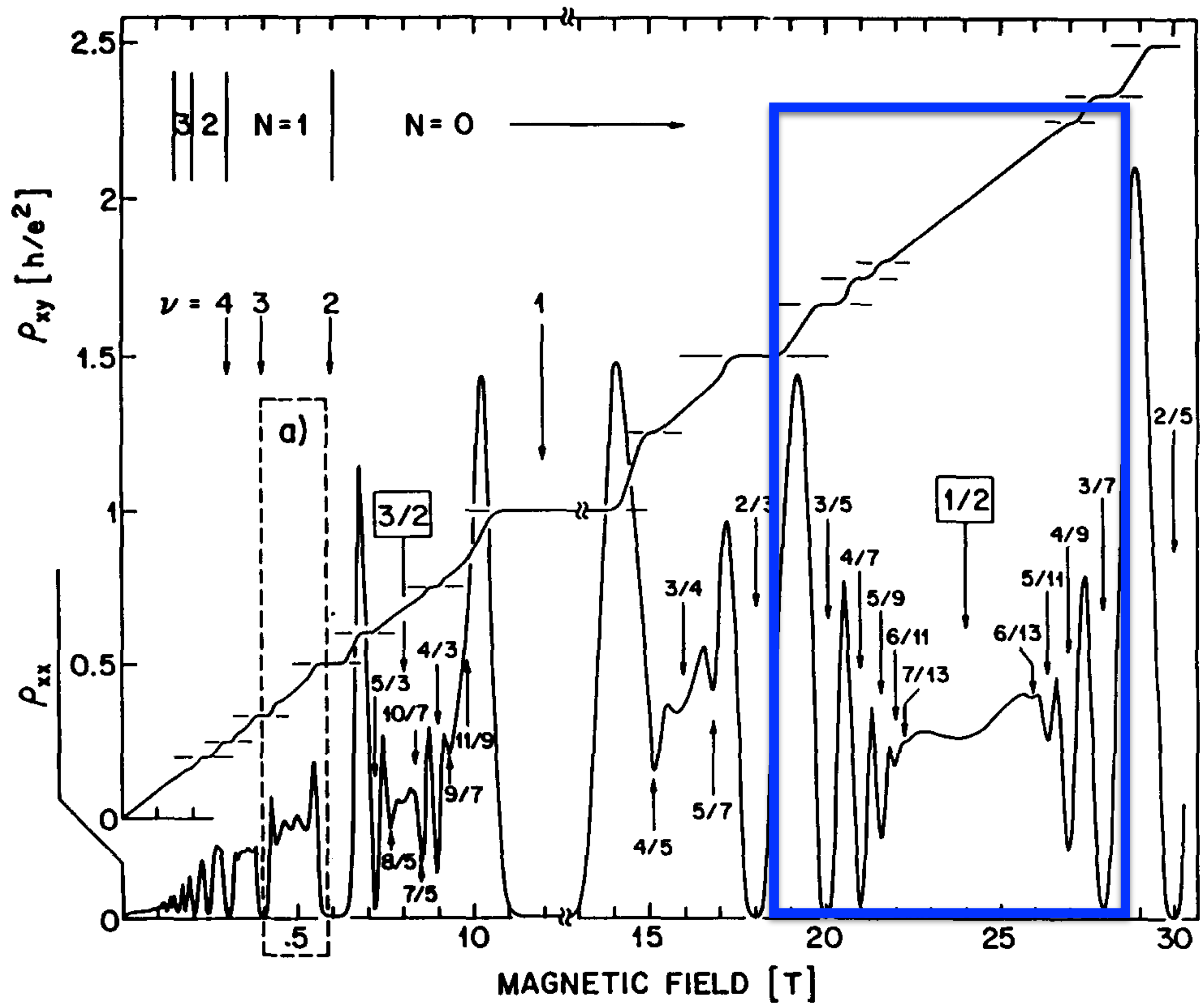
Sudip Chakravarty*
Department of Physics, State University of New York at Stony Brook, Stony Brook, New York 11794

Bertrand I. Halperin and David R. Nelson
Lyman Laboratory of Physics, Harvard University, Cambridge, Massachusetts 02138
(Received 18 August 1988)

Interactions and scaling in a disordered two-dimensional metal

Sudip Chakravarty and Lan Yin
Department of Physics and Astronomy, University of California Los Angeles, Los Angeles, California 90095-1547

Elihu Abrahams
Serin Physics Laboratory, Rutgers University, Piscataway, New Jersey 08855-0849
(Received 19 December 1997)



Quantum of resistance:

$$\frac{h}{e^2} = 2\pi$$

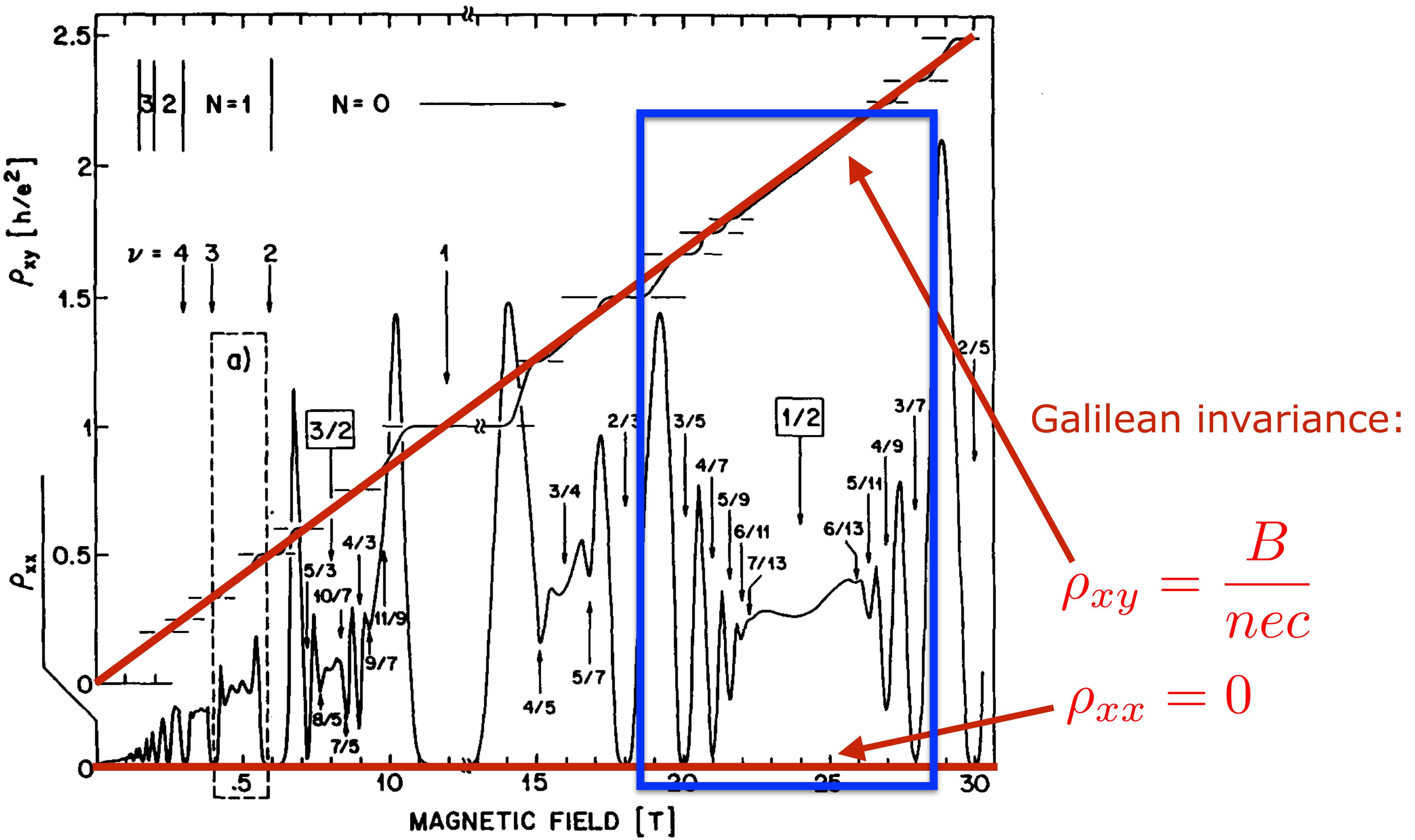
Quantum of flux:

$$\frac{h}{e} = 2\pi$$

Filling fraction:

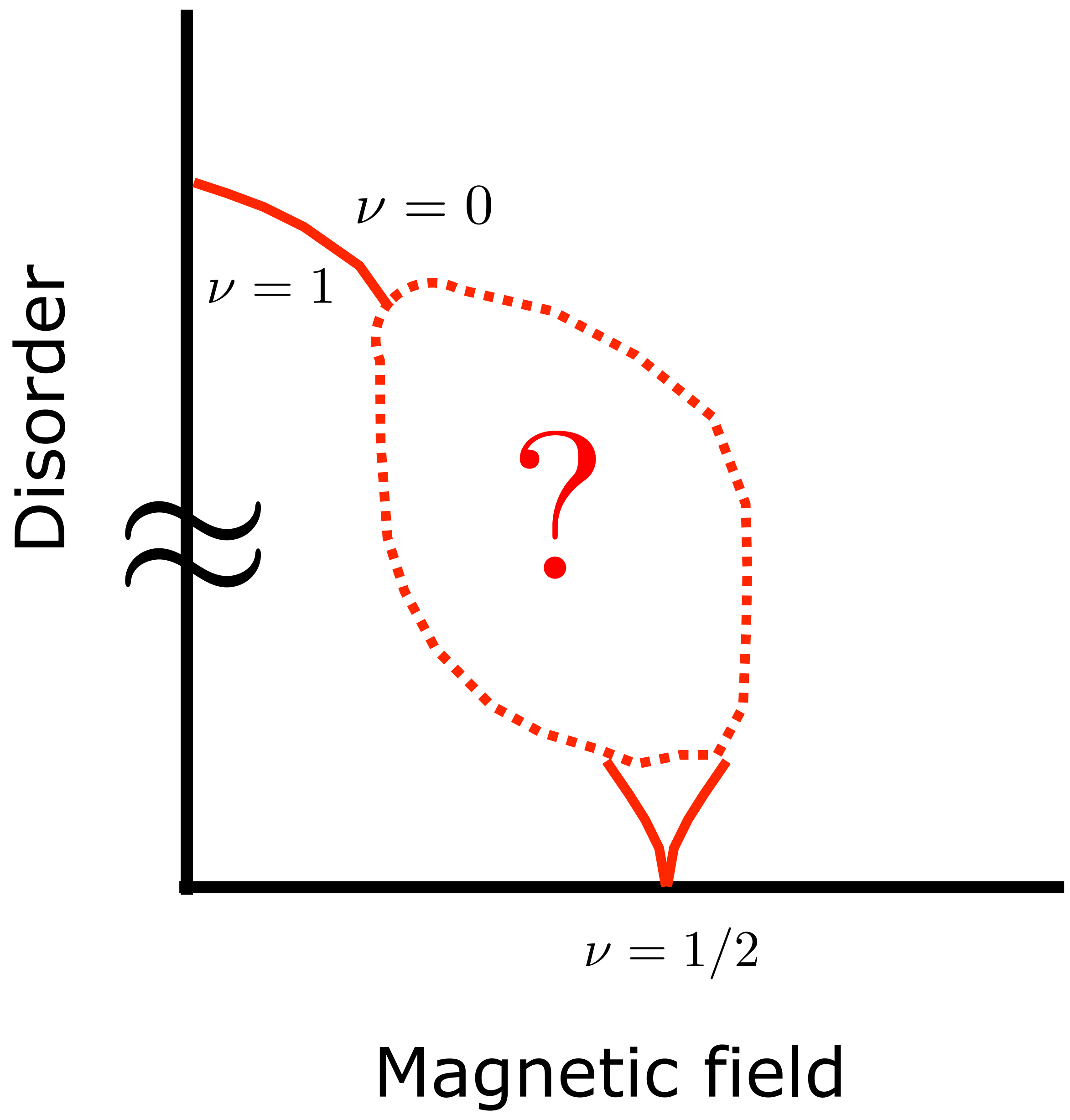
$$\nu = 2\pi\frac{n}{B}$$

D. Tsui, Phys. B (1990) 59.

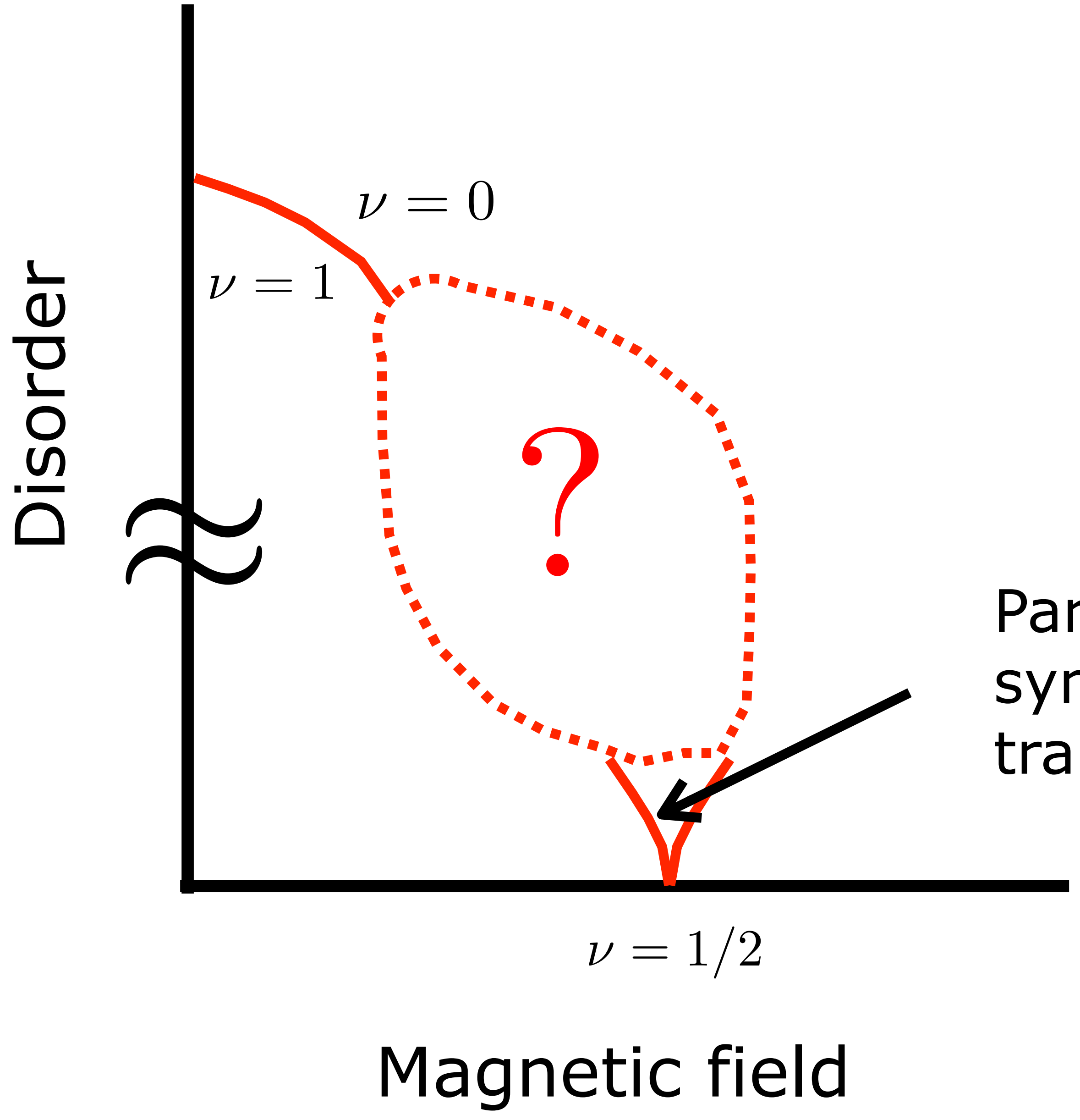


D. Tsui, Phys. B (1990) 59.

Plan for the talk

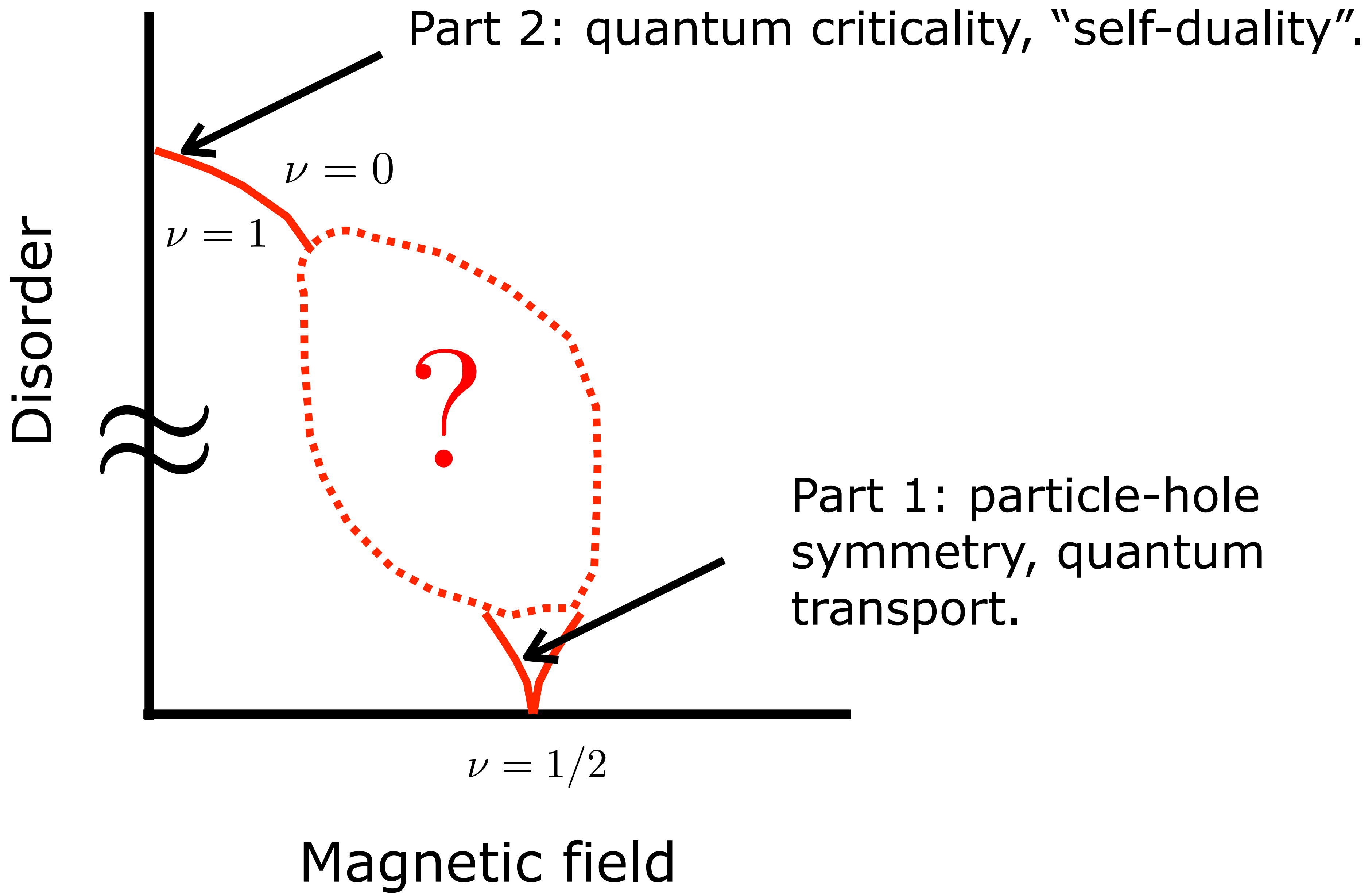


Plan for the talk

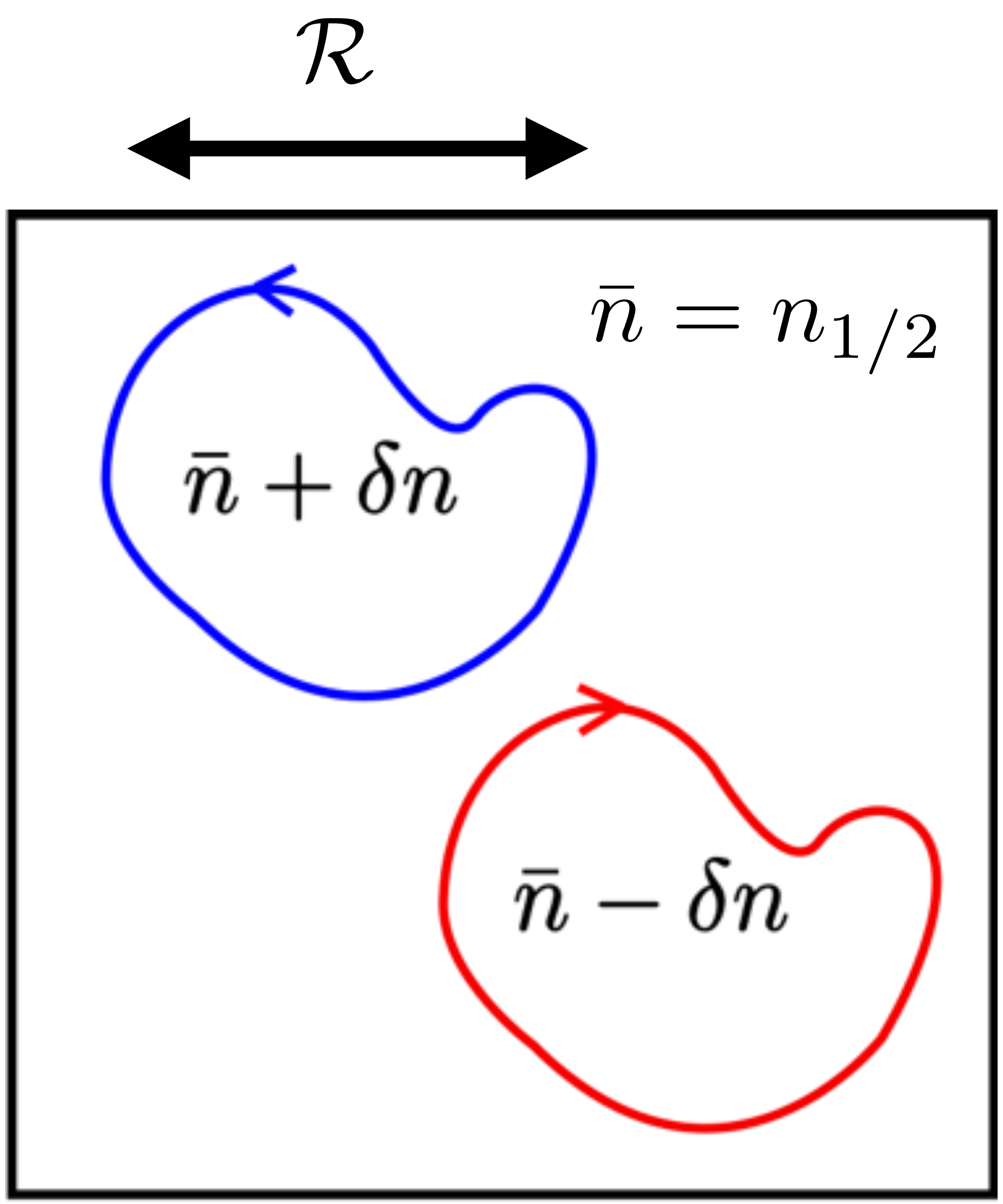


Part 1: particle-hole symmetry, quantum transport.

Plan for the talk



Disorder of interest



$$\overline{V(r)} = 0$$

$$\overline{V(r)V(r')} = g e^{-(\mathbf{x}-\mathbf{x}')^2/\mathcal{R}^2}$$

Long-wavelength disorder:

$$\mathcal{R} \gg \ell_B$$

Composite fermions

Lopez, Fradkin; Jain; Halperin,Lee,Read; Kalmeyer, Zhang.

Electrons

$$\mathcal{L}_{el} = \bar{\psi}\left(\hat{K}_A + \mu\right)\psi + \cdots$$

$$\hat{K}_A = iD_A^t + \frac{1}{2m}\vec{D}_A^2$$

$$D_A^\mu = \partial_\mu - iA_\mu, \quad \mu \in \{t, x, y\}$$

$$B = \partial_x A_y - \partial_y A_x$$

Composite fermions

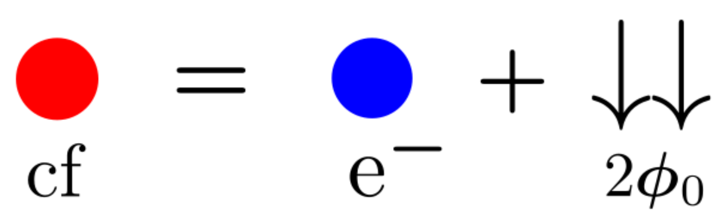
$$\mathcal{L}_{cf} = \bar{f}\left(\hat{K}_{A+a} + \mu\right) f + \frac{1}{2}\frac{1}{4\pi} ada + \cdots$$

$$ada = \epsilon_{\mu\nu\lambda} a_\mu \partial_\nu a_\lambda$$

Chern-Simons term

$$a_t \text{ eq. of motion :} \quad \langle \bar{f} f \rangle + \frac{1}{4\pi} b = 0$$

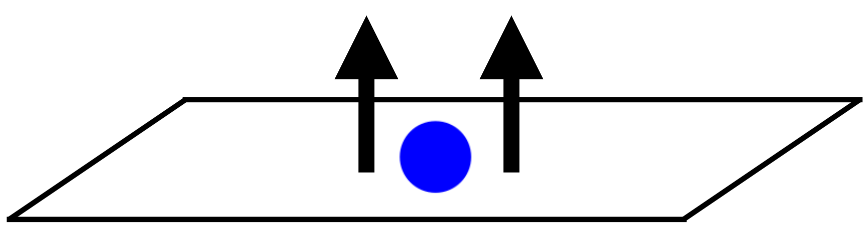
$$b = \partial_x a_y - \partial_y a_x$$

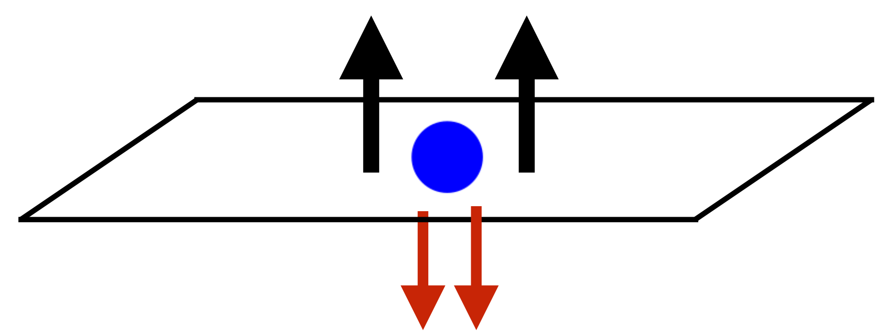


Flux-attachment

Composite fermions and the half-filled LL

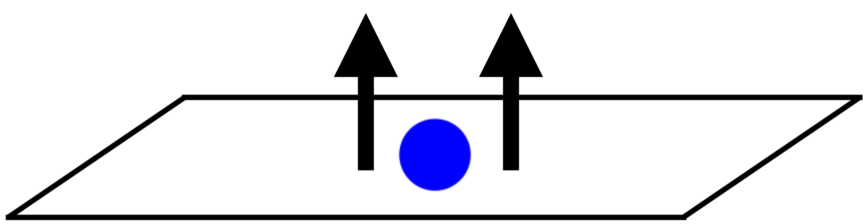
$\nu = 1/2 \quad \Rightarrow \quad \nu_{cf} = \infty$

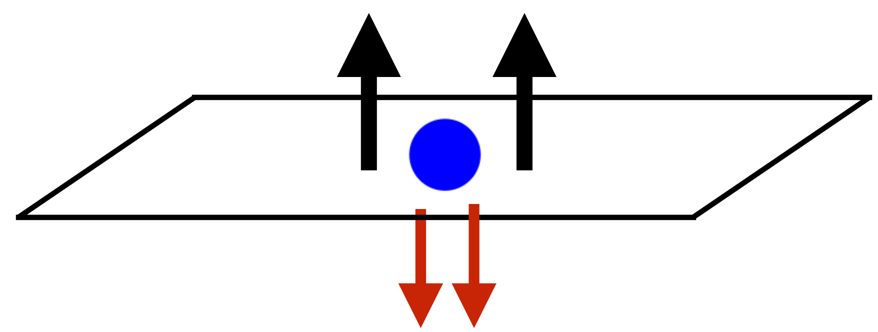




Composite fermions and the half-filled LL

$\nu = 1/2 \quad \Rightarrow \quad \nu_{cf} = \infty$





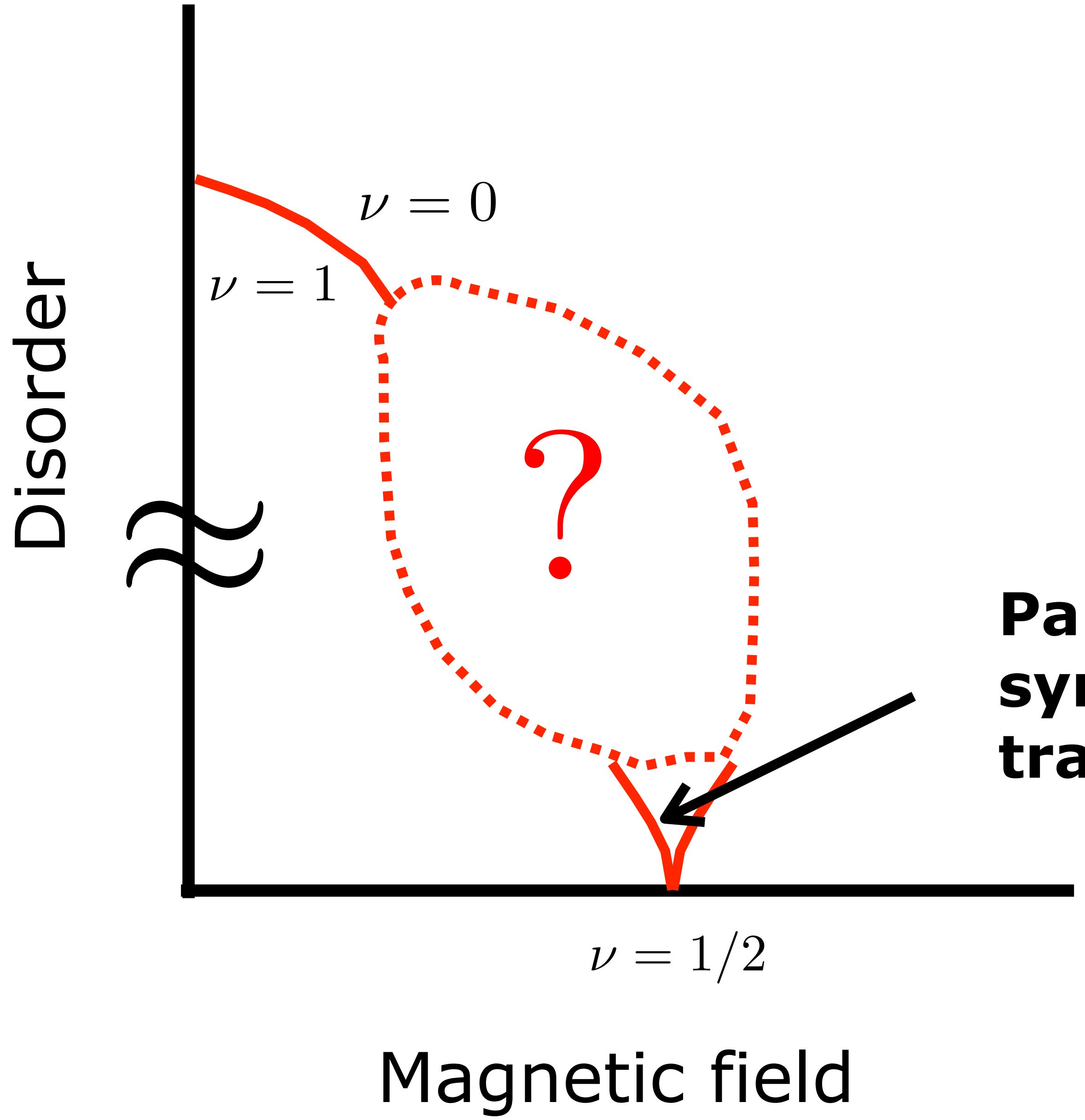
$$\mathcal{L}_{cf} = \bar{f}\left(\hat{K}_{A+a} + \mu\right) f + \frac{1}{2}\frac{1}{4\pi} ada + \cdots$$

EOM(a_t): $\langle \bar{f} f \rangle = -\frac{b}{4\pi}$

Half-filling: $\langle \bar{f} f \rangle = \frac{B}{4\pi}$

$\Big\}\quad b + B = 0$

Net zero field at half filling: Fermi sea of CFs.



Part 1: particle-hole symmetry, quantum transport.

C. Wang, N. Cooper, B. Halperin, A. Stern, PRX **7**, 031029 (2017).

P. Kumar, M. Mulligan, SR, PRB 2018.

Particle-hole symmetry at half-filling

ph symmetry constraint at half-filling: $\sigma_{xy} = \frac{1}{4\pi}$

This constraint holds even with disorder V(r) if

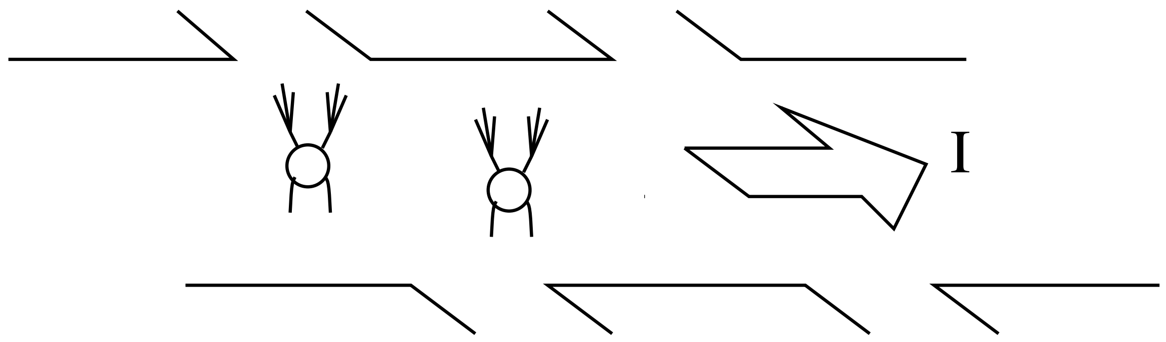
$$\overline{V^n(r)} = 0, \ n = odd$$

In this case, each disorder realization breaks ph, but disorder averaged quantities are ph symmetric.

Particle-hole symmetry at half-filling

S. Kivelson, D.-H. Lee, Y. Krotov, J. Gan, PRB 1997.

ph constraint at half-filling: $\sigma_{xy} = \frac{1}{4\pi}$



$$\rho^{cf}_{ab} = \rho_{ab} + 4\pi\epsilon_{ab}$$

$$\epsilon_{ab} = \begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix}$$

ph constraint at half-filling: $\sigma^{cf}_{xy} = -\frac{1}{4\pi}$

Disorder is crucial here.

How to get this from CFs in zero net field?

CFs with disorder

$$\mathcal{L}_{cf} = \bar{f}\left(\hat{K}_{A+a} + \mu\right) f + \frac{1}{2}\frac{1}{4\pi} a da + \cdots$$

CFs with disorder

$$\mathcal{L}_{cf} = \bar{f}\left(\hat{K}_{A+a} + \mu\right) f + \frac{1}{2}\frac{1}{4\pi} a da + \cdots$$

Disorder at half-filling: $\mu(r) = \mu_{1/2} + V(r)$

CFs with disorder

$$\mathcal{L}_{cf} = \bar{f}\left(\hat{K}_{A+a} + \mu\right) f + \frac{1}{2}\frac{1}{4\pi} ada + \cdots$$

Disorder at half-filling: $\mu(r) = \mu_{1/2} + V(r)$

$n_{1/2} = \frac{B}{4\pi} \quad \chi = \frac{m}{2\pi}$

Linear response: $n = n_{1/2} + \chi V$

CFs with disorder

$$\mathcal{L}_{cf} = \bar{f}\left(\hat{K}_{A+a} + \mu\right) f + \frac{1}{2}\frac{1}{4\pi} ada + \cdots$$

Disorder at half-filling: $\mu(r) = \mu_{1/2} + V(r)$

$n_{1/2} = \frac{B}{4\pi} \quad \chi = \frac{m}{2\pi}$

Linear response: $n = n_{1/2} + \chi V$

a_t eq. of motion : $n = -\frac{b}{4\pi}$ $\quad \Rightarrow V(r) = -\frac{b(r) + B}{2m}$

CFs with disorder

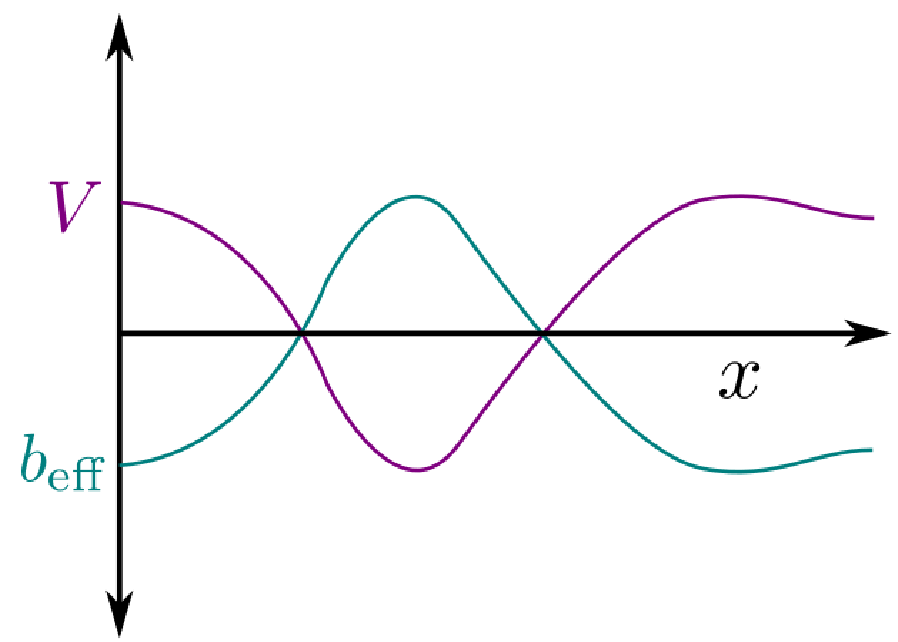
$$\mathcal{L}_{cf} = \bar{f}\left(\hat{K}_{A+a} + \mu\right) f + \frac{1}{2}\frac{1}{4\pi} a da + \cdots$$

Disorder at half-filling: $\mu(r) = \mu_{1/2} + V(r)$

$n_{1/2} = \dfrac{B}{4\pi} \qquad \chi = \dfrac{m}{2\pi}$

Linear response: $n = n_{1/2} + \chi V$

a_t eq. of motion : $n = -\dfrac{b}{4\pi}$ $\quad \Rightarrow V(r) = -\dfrac{b(r) + B}{2m}$



CFs with disorder

$$\mathcal{L}_{cf} = \bar{f}\left(\hat{K}_{A+a} + \mu\right) f + \frac{1}{2}\frac{1}{4\pi} ada + \cdots$$

Disorder at half-filling: $\mu(r) = \mu_{1/2} + V(r)$

$n_{1/2} = \frac{B}{4\pi} \quad \chi = \frac{m}{2\pi}$

Linear response: $n = n_{1/2} + \chi V$

a_t eq. of motion : $n = -\frac{b}{4\pi} \quad \Rightarrow V(r) = -\frac{b(r) + B}{2m}$

$$\mathcal{L}_{cf} = \bar{f}\left(\hat{K}_{A+a} + \mu_{1/2} - \frac{b(r) + B}{2m}\right) f + \frac{1}{2}\frac{1}{4\pi} ada + \cdots$$

$$\text{shift} : a \to a - A$$

CFs with disorder

$$\mathcal{L}_{cf} = \bar{f}\left(\hat{K}_{A+a} + \mu\right) f + \frac{1}{2}\frac{1}{4\pi} ada + \cdots$$

Disorder at half-filling: $\mu(r) = \mu_{1/2} + V(r)$

$$n_{1/2} = \frac{B}{4\pi} \qquad \chi = \frac{m}{2\pi}$$

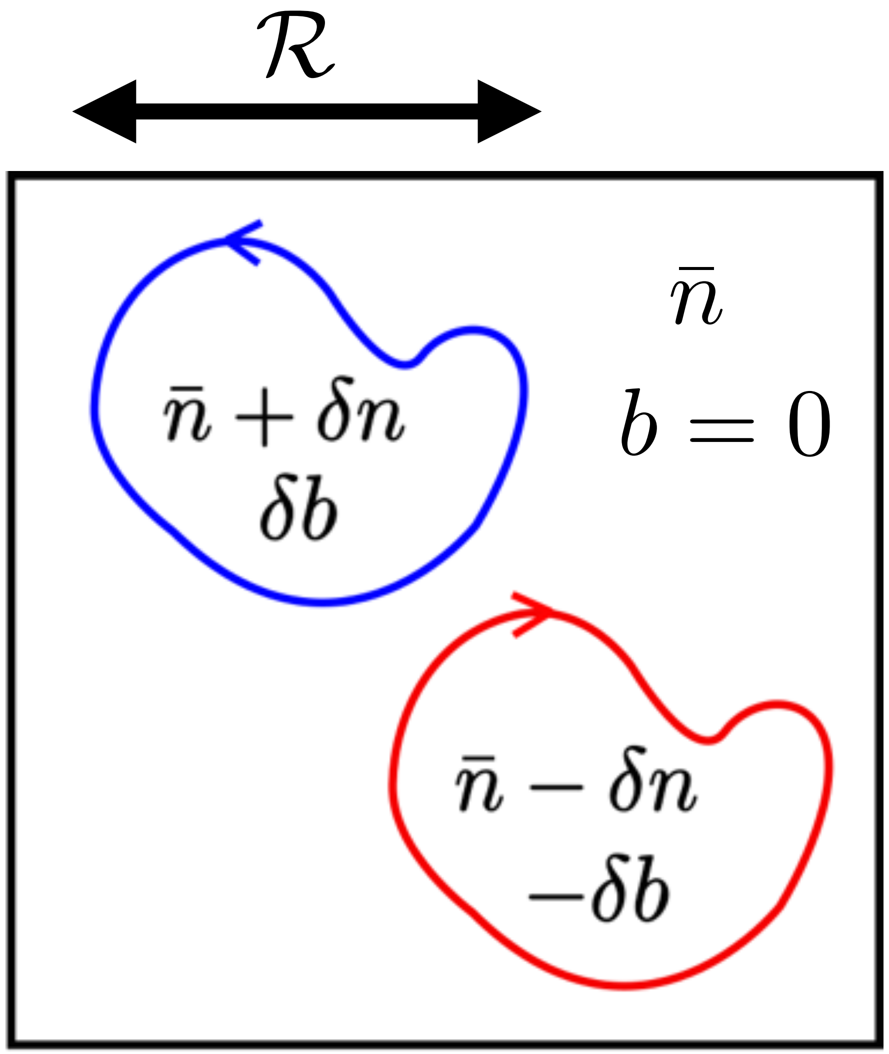
Linear response: $n = n_{1/2} + \chi V$

a_t eq. of motion : $n = -\frac{b}{4\pi}$ $\quad \Rightarrow V(r) = -\frac{b(r) + B}{2m}$

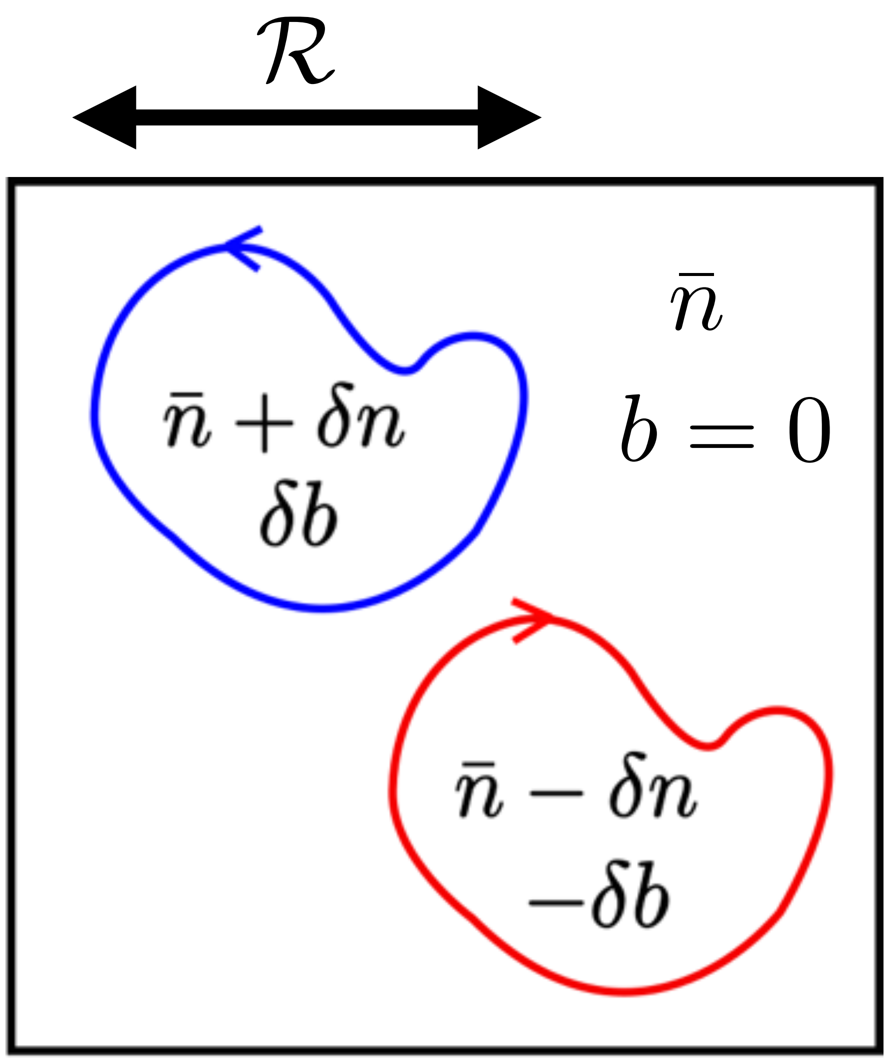
$$\mathcal{L}_{cf} = \bar{f}\left(\hat{K}_a + \mu_{1/2} - \frac{b(r)}{2m}\right) f + \frac{1}{2}\frac{1}{4\pi}(a - A)d(a - A) + \cdots$$

Disorder problem: random potential slaved to random flux.

Intuitive argument

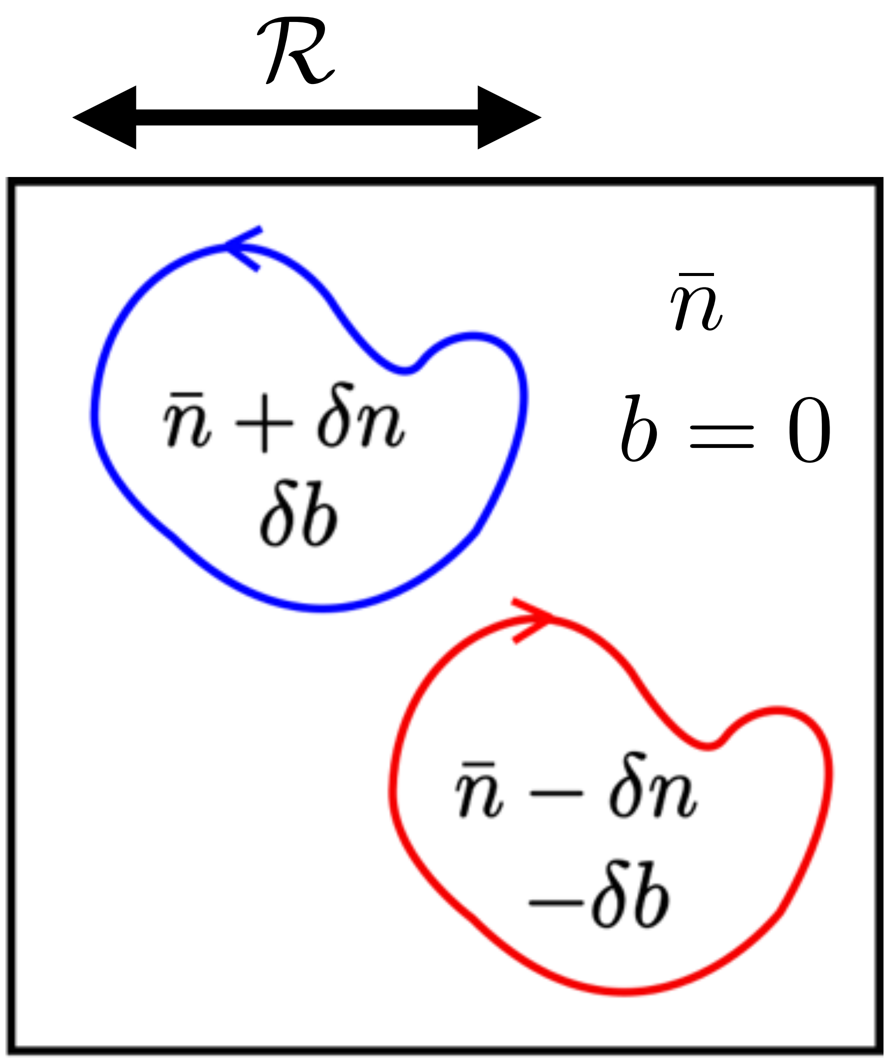


Intuitive argument



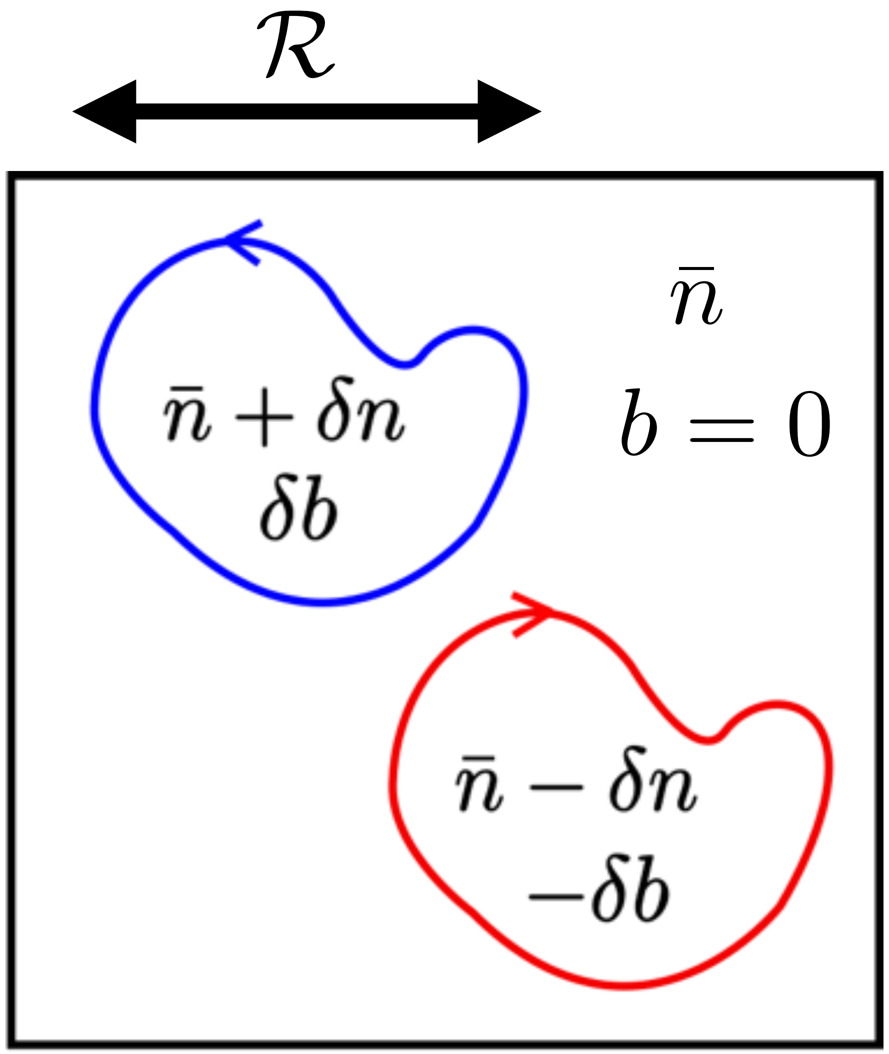
$$\nu_{eff} = 2\pi \frac{(\bar{n} + \delta n) - (\bar{n} - \delta n)}{2\delta b} = 2\pi \frac{\delta n}{\delta b}$$

Intuitive argument



$$\nu_{eff} = 2\pi \frac{(\bar{n} + \delta n) - (\bar{n} - \delta n)}{2\delta b} = 2\pi \underbrace{\frac{\delta n}{\delta b}}_{-\frac{1}{4\pi}}$$

Intuitive argument



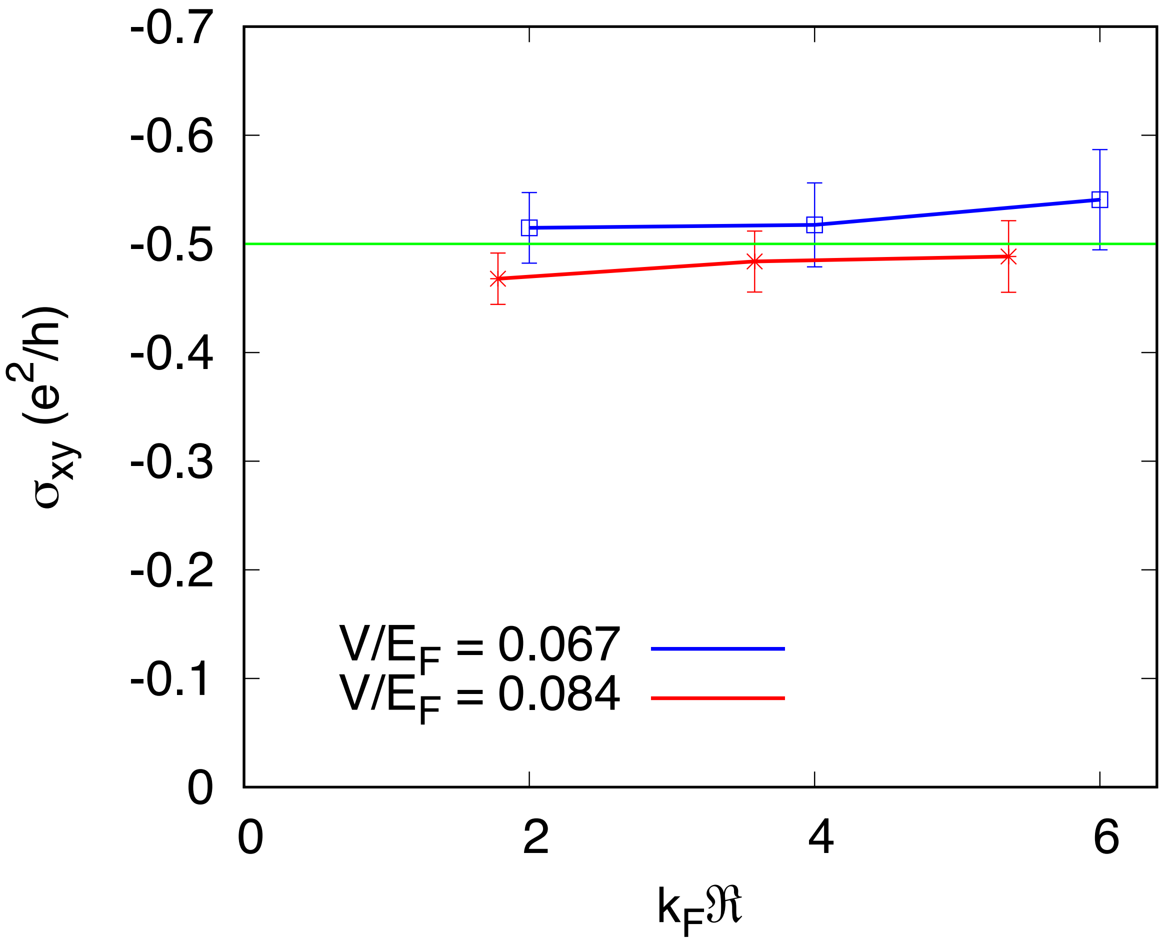
$$\nu_{eff} = 2\pi \frac{(\bar{n} + \delta n) - (\bar{n} - \delta n)}{2\delta b} = 2\pi \underbrace{\frac{\delta n}{\delta b}}_{-\frac{1}{4\pi}}$$

$$\nu_{eff} = -\frac{1}{2}$$

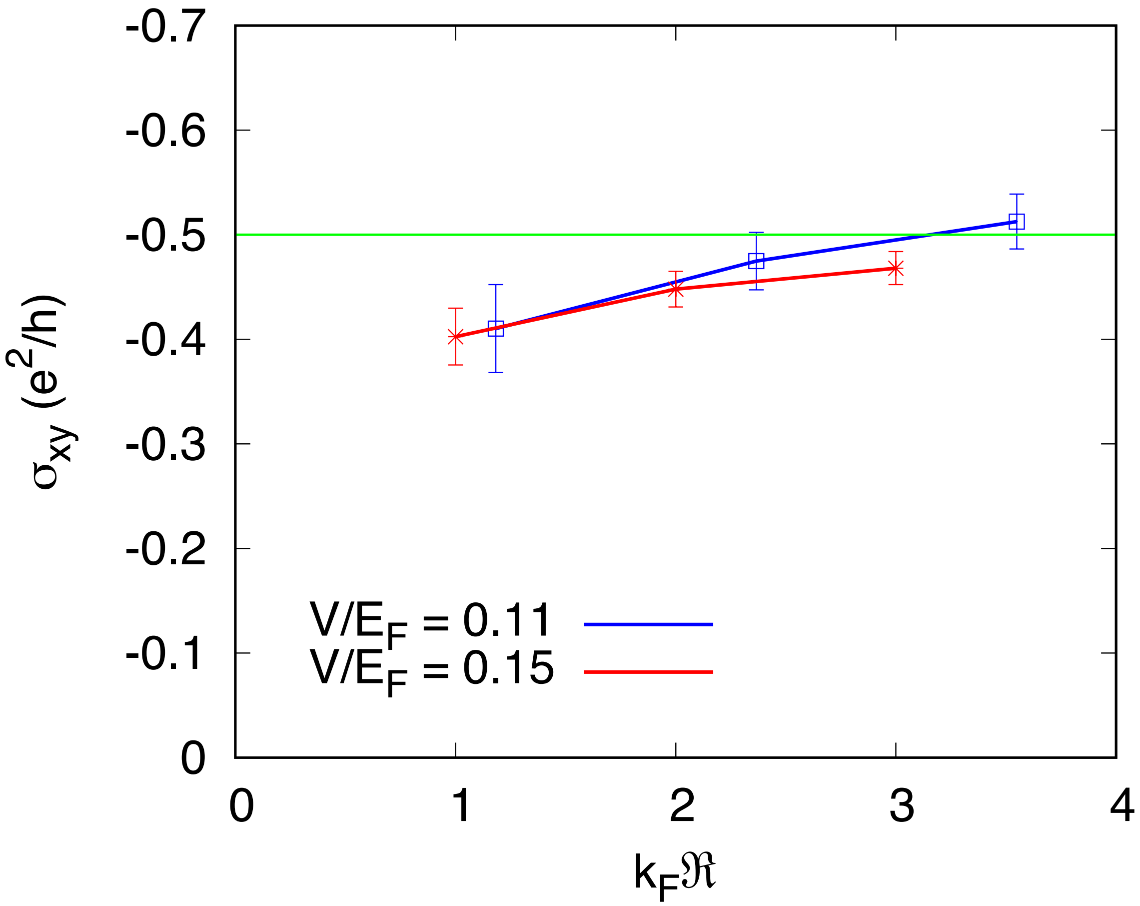
$$\sigma_{xy}^{cf} = -\frac{1}{4\pi}$$

Numerical calculation

momentum space calc.



real space lattice calc.



Particle-hole symmetry occurs in the long wavelength limit.

Analytic theory

We can treat the disorder problem non-perturbatively.

$$\mathcal{H}_{cf} = \frac{1}{2m}\left[(\boldsymbol{p}+\boldsymbol{a})^2 - b\right], \quad b = \nabla \times a$$

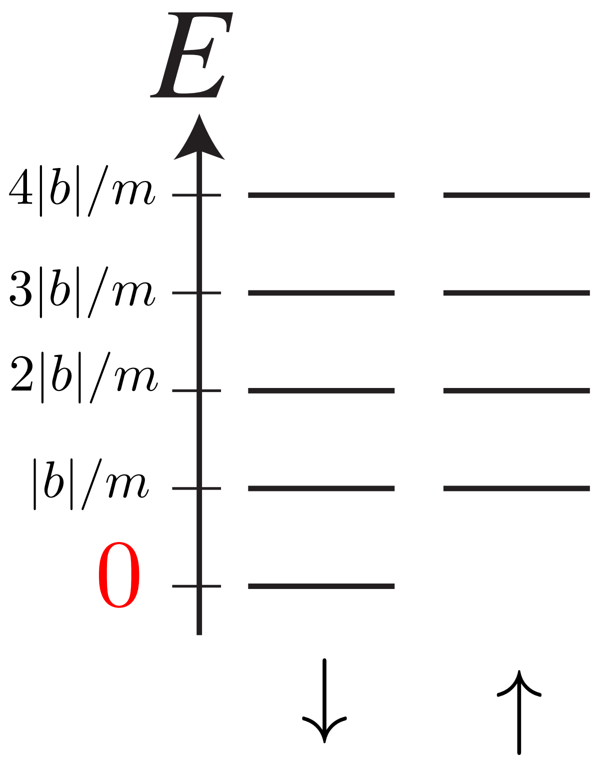
Analytic theory

We can treat the disorder problem non-perturbatively.

$$\mathcal{H}_{cf} = \frac{1}{2m}\left[(\boldsymbol{p}+\boldsymbol{a})^2 - b\right], \quad b = \nabla \times a$$

Analogy: spin-1/2 system in a magnetic field with g=2:

$$\mathcal{H} = \frac{1}{2m}\left[(\boldsymbol{p}+\boldsymbol{a})^2\,\hat{1} + \frac{g}{2} b\sigma^z\right]$$



Analytic theory

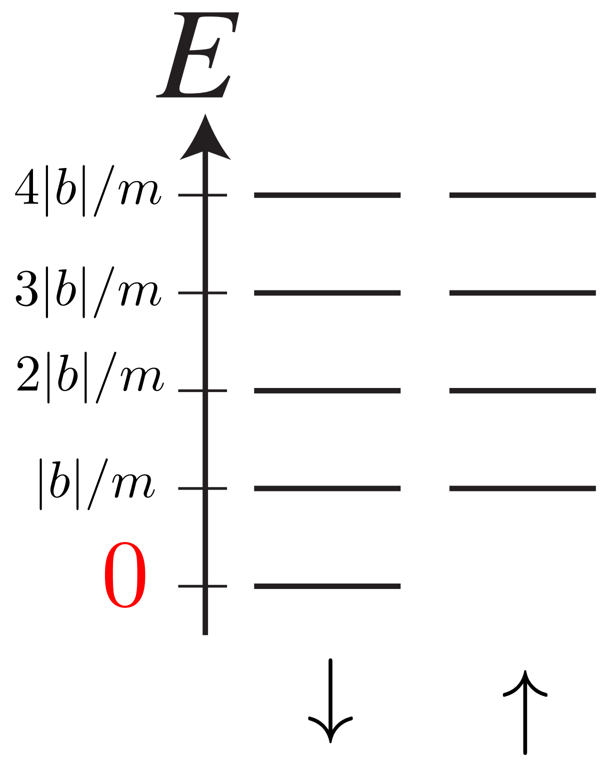
We can treat the disorder problem non-perturbatively.

$$\mathcal{H}_{cf} = \frac{1}{2m}\left[(\boldsymbol{p}+\boldsymbol{a})^2 - b\right], \quad b = \nabla \times a$$

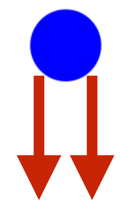
Analogy: spin-1/2 system in a magnetic field with g=2:

$$\mathcal{H} = \frac{1}{2m}\left[(\boldsymbol{p}+\boldsymbol{a})^2\,\hat{1} + \frac{g}{2} b\sigma^z\right]$$

unpaired zero energy mode occurs for arbitrary disorder strength provided g=2.



Why g=2: 1) Flux attachment



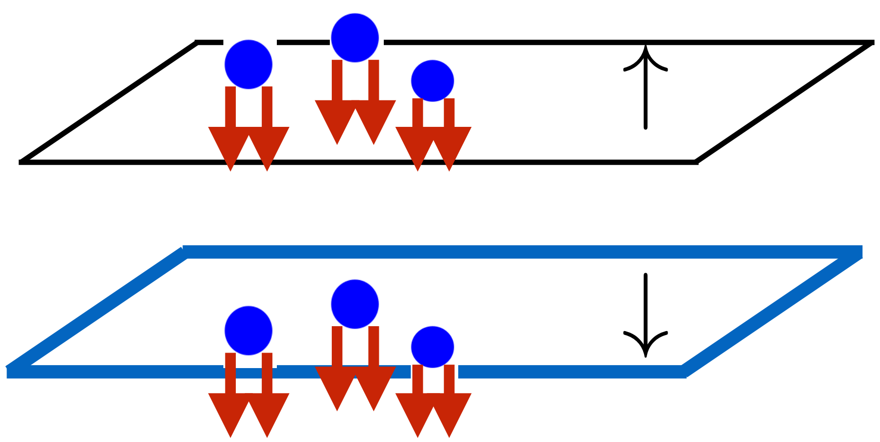
2) Mean-field/ linear response:

$$\chi = \frac{m}{2\pi}$$

$$\delta n = \chi \delta \mu$$

g=2 realizes SUSY quantum mechanics

$$\Pi_\mu = p_\mu + a_\mu$$



$$\mathcal{H}_\uparrow = \frac{1}{2m}\left[\Pi^2 + b(r)\right] = QQ^\dagger$$

$$\mathcal{H}_\downarrow = \mathcal{H}_{cf} = \frac{1}{2m}\left[\Pi^2 - b(r)\right] = Q^\dagger Q$$

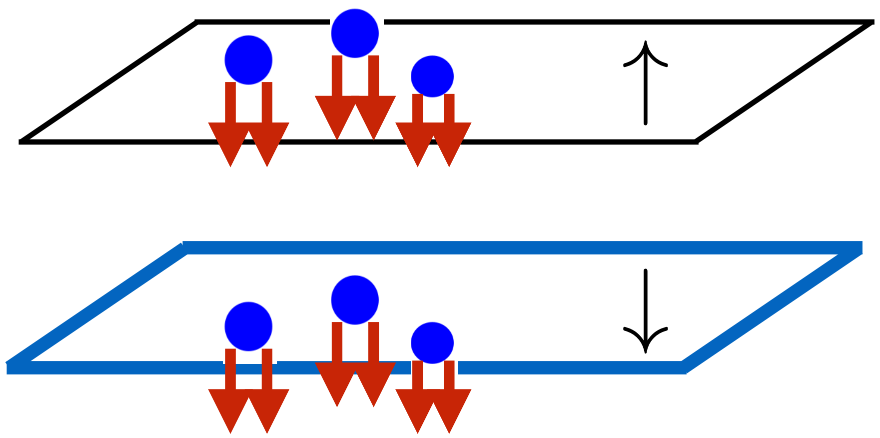
$$Q = \frac{\Pi_x + i\Pi_y}{\sqrt{2m}}$$

Idea: use the doubled system to compute σ_{xy}^{cf}.

$\Pi_\mu = p_\mu + a_\mu$

$$\mathcal{H}_\uparrow = \frac{1}{2m}\left[\Pi^2 + b(r)\right]$$

$$\mathcal{H}_\downarrow = \frac{1}{2m}\left[\Pi^2 - b(r)\right]$$



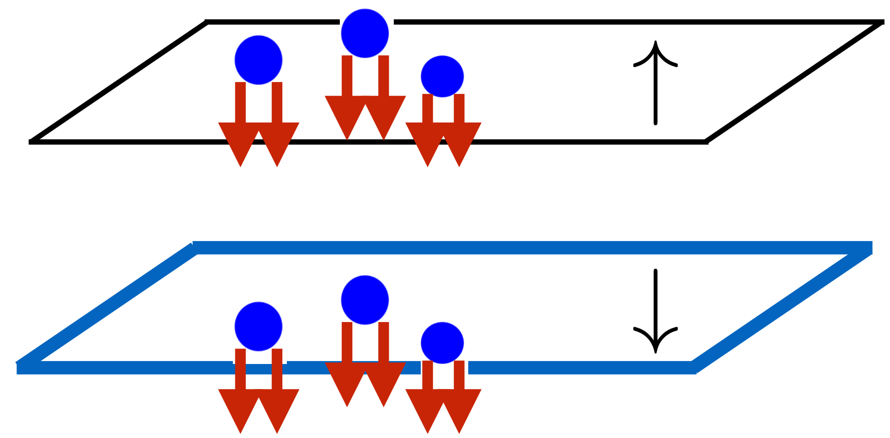
$\bar{\sigma}^{\uparrow}_{xy}$

$\bar{\sigma}^{\downarrow}_{xy} = \sigma^{cf}_{xy}$

$\Pi_\mu = p_\mu + a_\mu$

$$\mathcal{H}_\uparrow = \frac{1}{2m}\left[\Pi^2 + b(r)\right]$$

$$\mathcal{H}_\downarrow = \frac{1}{2m}\left[\Pi^2 - b(r)\right]$$



$$\bar{\sigma}^{\uparrow}_{xy}$$

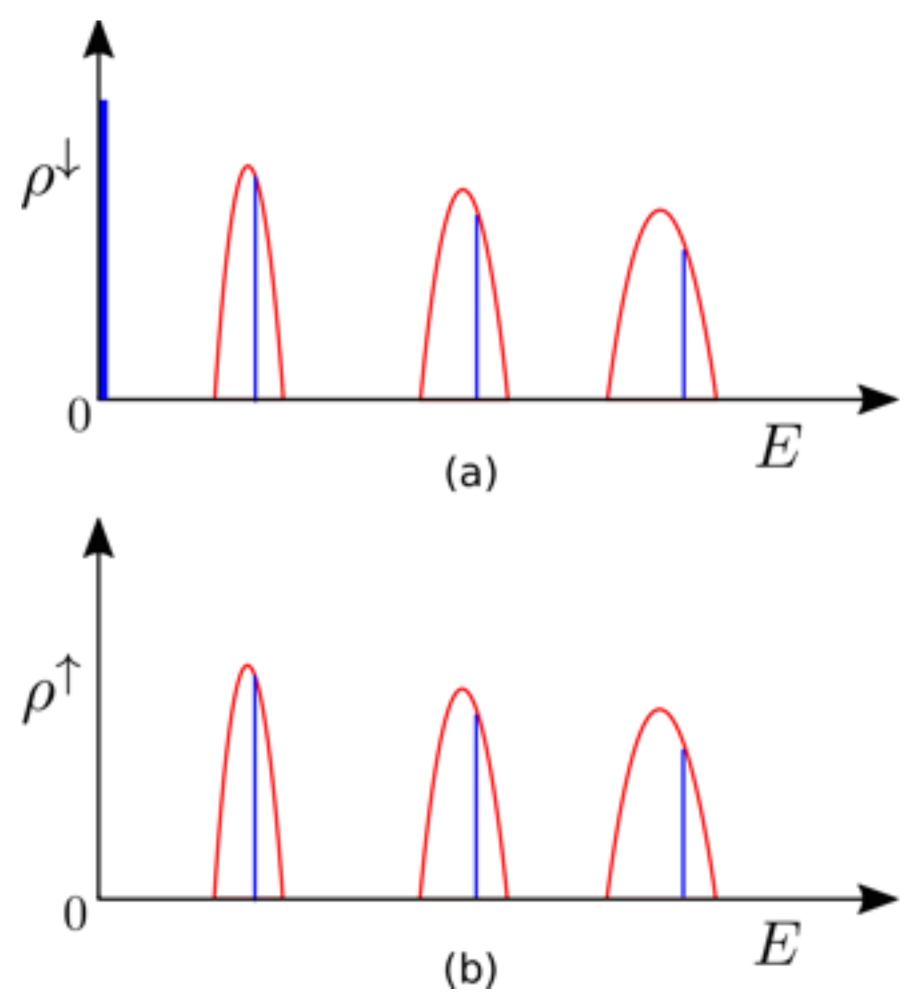
$$\bar{\sigma}^{\downarrow}_{xy} = \sigma^{cf}_{xy}$$

For <u>any</u> disorder strength at g=2:

$$\bar{\sigma}^{\uparrow}_{xy} + \bar{\sigma}^{\downarrow}_{xy} = 0$$

$$\bar{\sigma}^{\downarrow}_{xy} - \bar{\sigma}^{\uparrow}_{xy} = -\frac{1}{2\pi}$$

$$\Rightarrow \sigma^{\downarrow}_{xy} = \sigma^{cf}_{xy} = -\frac{1}{4\pi}$$



Particle-hole symmetric transport.

Note - since CFs see average zero field: anomalous Hall affect.

Berry Curvature on the Fermi Surface: Anomalous Hall Effect as a Topological Fermi-Liquid Property

F. D. M. Haldane

Department of Physics, Princeton University, Princeton New Jersey 08544-0708, USA

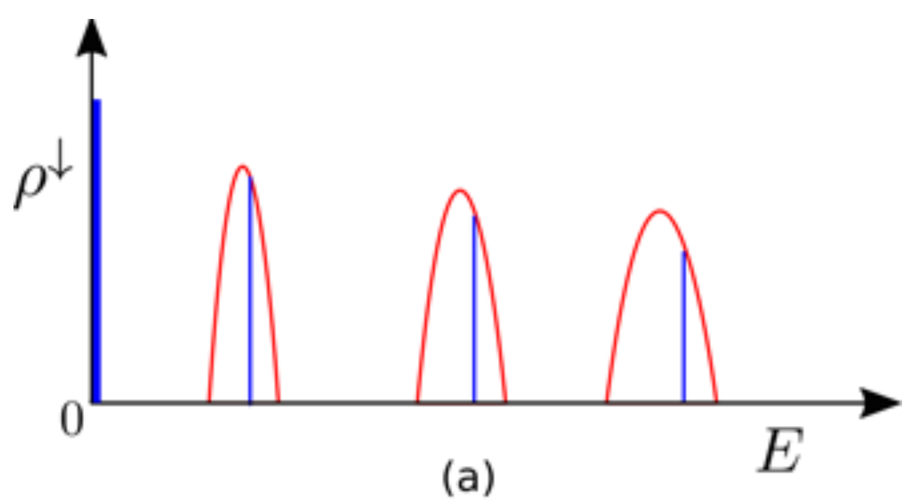
(Received 28 June 2004; revised manuscript received 20 October 2004; published 11 November 2004)

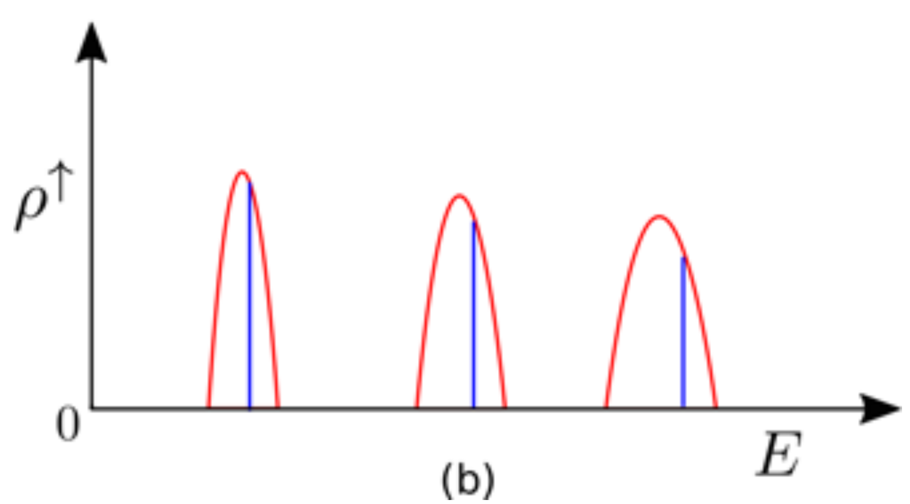
$$\sigma_{xy}^{cf} = -\frac{1}{4\pi} \quad \Longrightarrow \quad \pi \text{ Berry phase.}$$

Summary (so far)

CF mean-field theory with disorder: slaved potential and flux.

PH symmetric dc transport from composite fermions

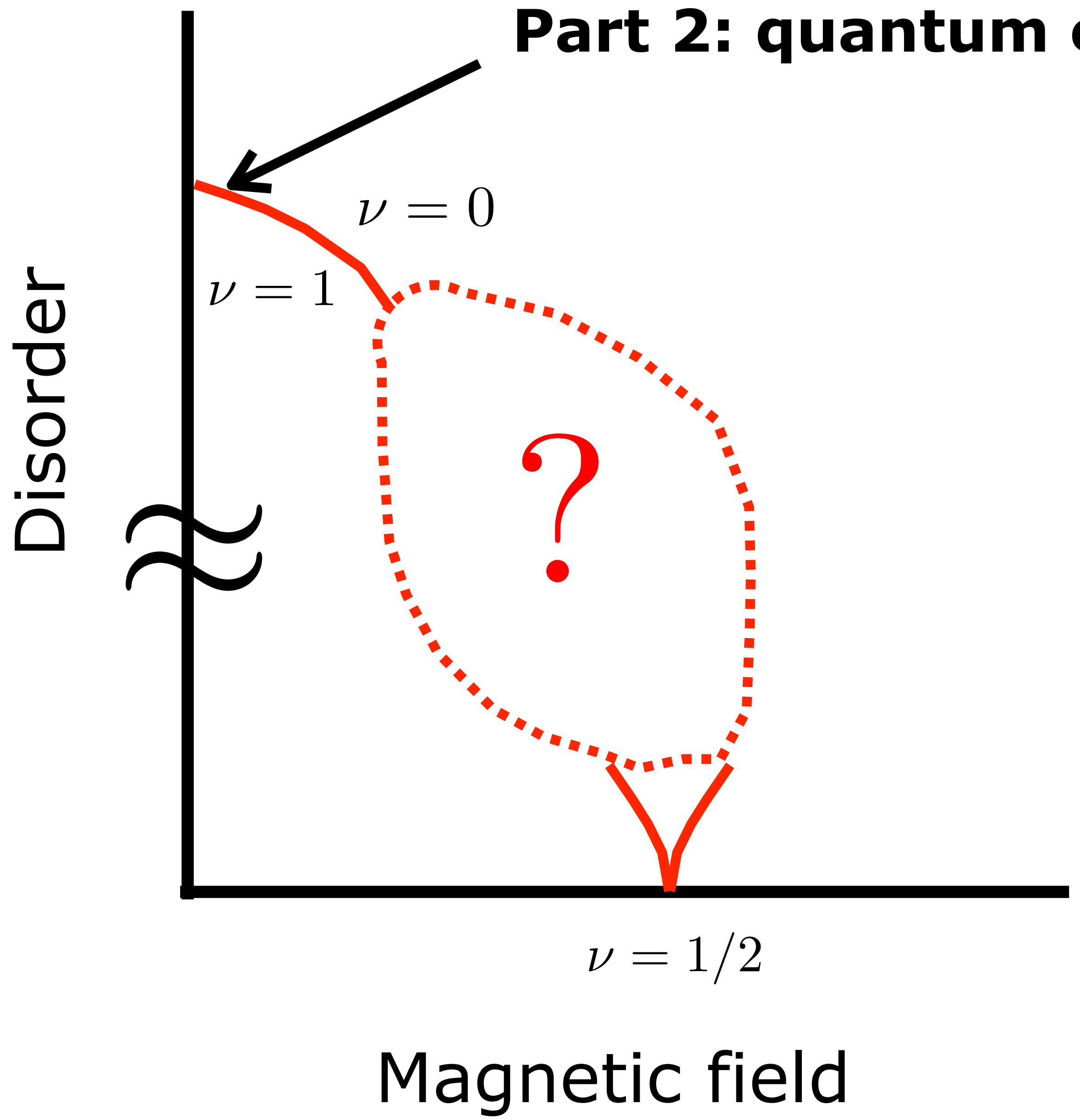




Zero modes of CF mean-field theory lead to proper dc transport.

Next: QCP at stronger disorder.

Part 2: quantum criticality, "self-duality".

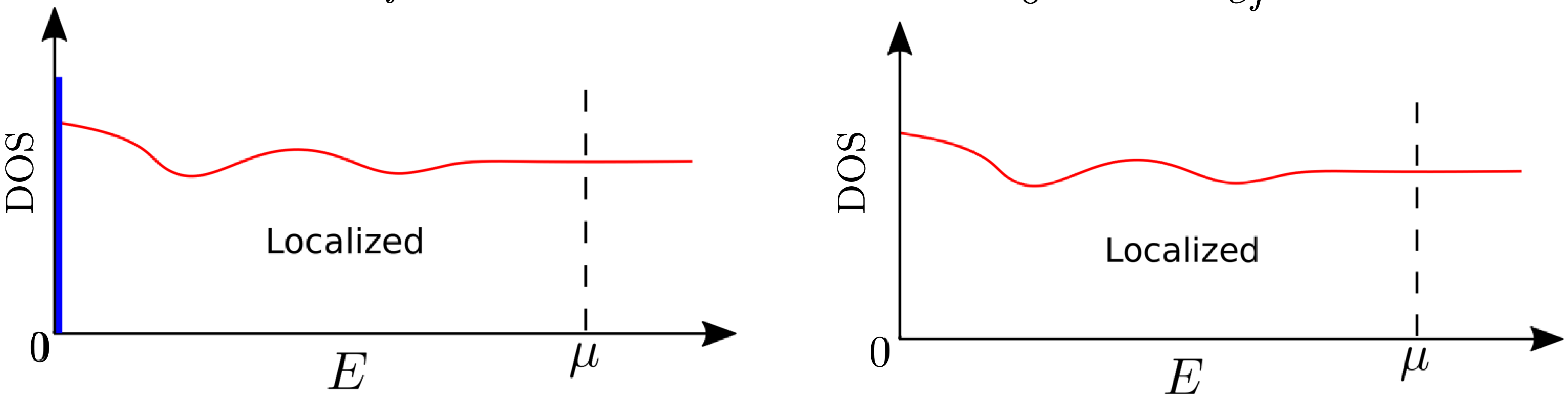


P. Kumar *et al.*, unpublished.

$\nu = 1/2$ as a quantum critical point

Let $b(r) = b_0 + \delta b(r)$ with $\overline{\delta b(r)} = 0$.

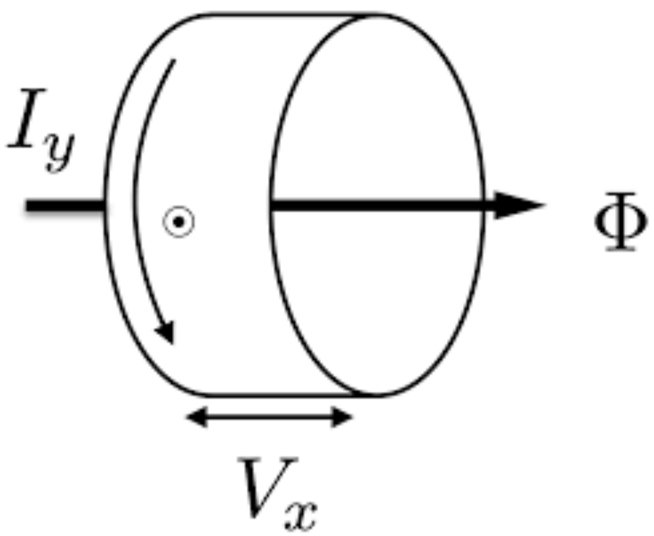
$$b_0 < 0 : \nu_{cf} = -1 \qquad\qquad b_0 > 0 : \nu_{cf} = 0$$



$b_0 = 0 : \nu = 1/2$ Integer QH transition of cfs.

At the qcp: $\sigma_{xy}^{cf} = -1/4\pi$.

$\nu = 1/2$ as a quantum critical point



Laughlin gauge argument:

States at E_F localized $\Longrightarrow$ $\sigma_{xy} = \frac{1}{2\pi} \times \text{integer}$

$\sigma_{xy}^{cf} = -1/4\pi$ $\Longrightarrow$ States at E_F must be extended.

Extended states at E_F: topological term of a non-linear sigma model.

Explicit approach

K. Efetov, A. Larkin, D. Khemlnitskii, JETP 1981.

We can derive this explicitly from our model. Here is the sketch:

$$\mathcal{L}_{cf} = \bar{f}\left(\hat{K}_a + \mu_{1/2} - \frac{g}{2}\frac{b(r)}{2m}\right) f$$

Explicit approach

K. Efetov, A. Larkin, D. Khemlnitskii, JETP 1981.

We can derive this explicitly from our model. Here is the sketch:

$$\mathcal{L}_{cf} = \bar{f}\left(\hat{K}_a + \mu_{1/2} - \frac{g}{2}\frac{b(r)}{2m}\right) f$$

Hubbard-Stratanovich transformation at g=2.

$$\mathcal{L} = \bar{\psi} \not{D}_a \psi + \psi^\dagger \mu \psi + \mathcal{O}(g-2)$$

Explicit approach

K. Efetov, A. Larkin, D. Khemlnitskii, JETP 1981.

We can derive this explicitly from our model. Here is the sketch:

$$\mathcal{L}_{cf} = \bar{f}\left(\hat{K}_a + \mu_{1/2} - \frac{g}{2}\frac{b(r)}{2m}\right) f$$

Hubbard-Stratanovich transformation at g=2.

$$\mathcal{L} = \bar{\psi} \not{D}_a \psi + \psi^\dagger \mu \psi + \mathcal{O}(g-2)$$

Gradient expansion of Goldstone modes about saddle point.

$$\mathcal{L}_{IR} = \frac{\sigma_{xx}}{8\pi}\mathrm{Tr}\left[(\partial_\mu \Omega)^2\right] - \frac{\sigma_{xy}}{8\pi}\epsilon_{\mu\nu}\mathrm{Tr}\left[\Omega\partial_\mu\Omega\partial_\nu\Omega\right]$$

$\mu, \nu = x, y$

$G/H = \frac{U(2n)}{U(n)\times U(n)}$

$n \to 0$

“Self-duality”

The theory of the transition in cf language:

$$\mathcal{L}_{IR} = \frac{\sigma_{xx}}{8\pi}\mathrm{Tr}\left[(\partial_\mu \Omega)^2\right] - \frac{\sigma_{xy}}{8\pi}\epsilon_{\mu\nu}\mathrm{Tr}\left[\Omega\partial_\mu\Omega\partial_\nu\Omega\right]$$

$$\mu, \nu = x, y$$

$$G/H = \frac{U(2n)}{U(n)\times U(n)}$$

$$n \to 0$$

“Self-duality”

The theory of the transition in cf language:

$$\mathcal{L}_{IR} = \frac{\sigma_{xx}}{8\pi}\mathrm{Tr}\left[(\partial_\mu \Omega)^2\right] - \frac{\sigma_{xy}}{8\pi}\epsilon_{\mu\nu}\mathrm{Tr}\left[\Omega\partial_\mu\Omega\partial_\nu\Omega\right]$$

$\mu, \nu = x, y$

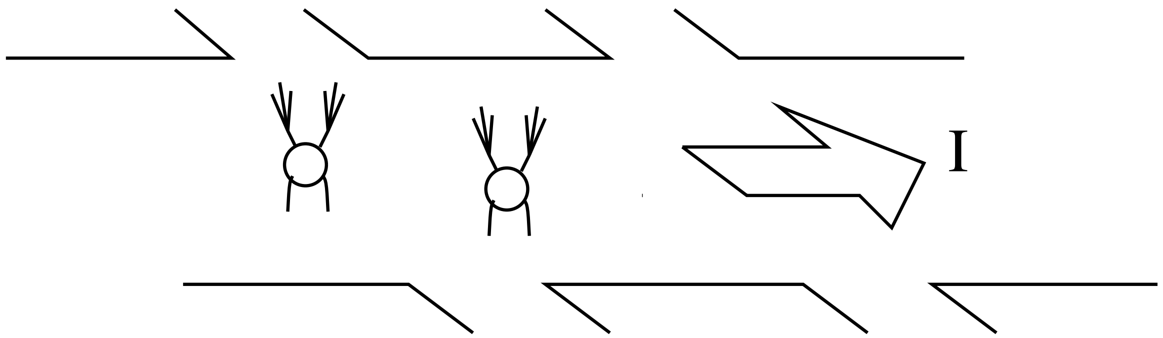
In electron coordinates (Pruisken, Dung-Hai Lee)

$G/H = \frac{U(2n)}{U(n)\times U(n)}$

$n \to 0$

$$\mathcal{L}_{IR} = \frac{\sigma_{xx}}{8\pi}\mathrm{Tr}\left[(\partial_\mu Q)^2\right] + \frac{\sigma_{xy}}{8\pi}\epsilon_{\mu\nu}\mathrm{Tr}\left[Q\partial_\mu Q\partial_\nu Q\right]$$

Same theory in 2 different languages: “self-duality”.



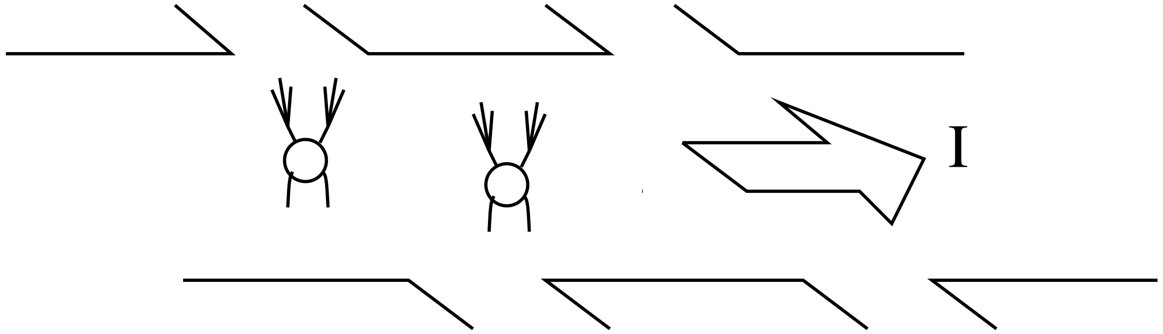
$$\rho^{cf}_{ab} = \rho_{ab} + 4\pi\epsilon_{ab}$$

self-duality: $\rho^{cf}_{ab} = \rho_{ba}$

using $\sigma^{cf}_{xy} = -\dfrac{1}{4\pi}$,

$$(\rho_{xx}, \rho_{xy}) = 2\pi\,(1, 1)$$

At the critical point.



$$\rho^{cf}_{ab} = \rho_{ab} + 4\pi\epsilon_{ab}$$

self-duality: $\rho^{cf}_{ab} = \rho_{ba}$

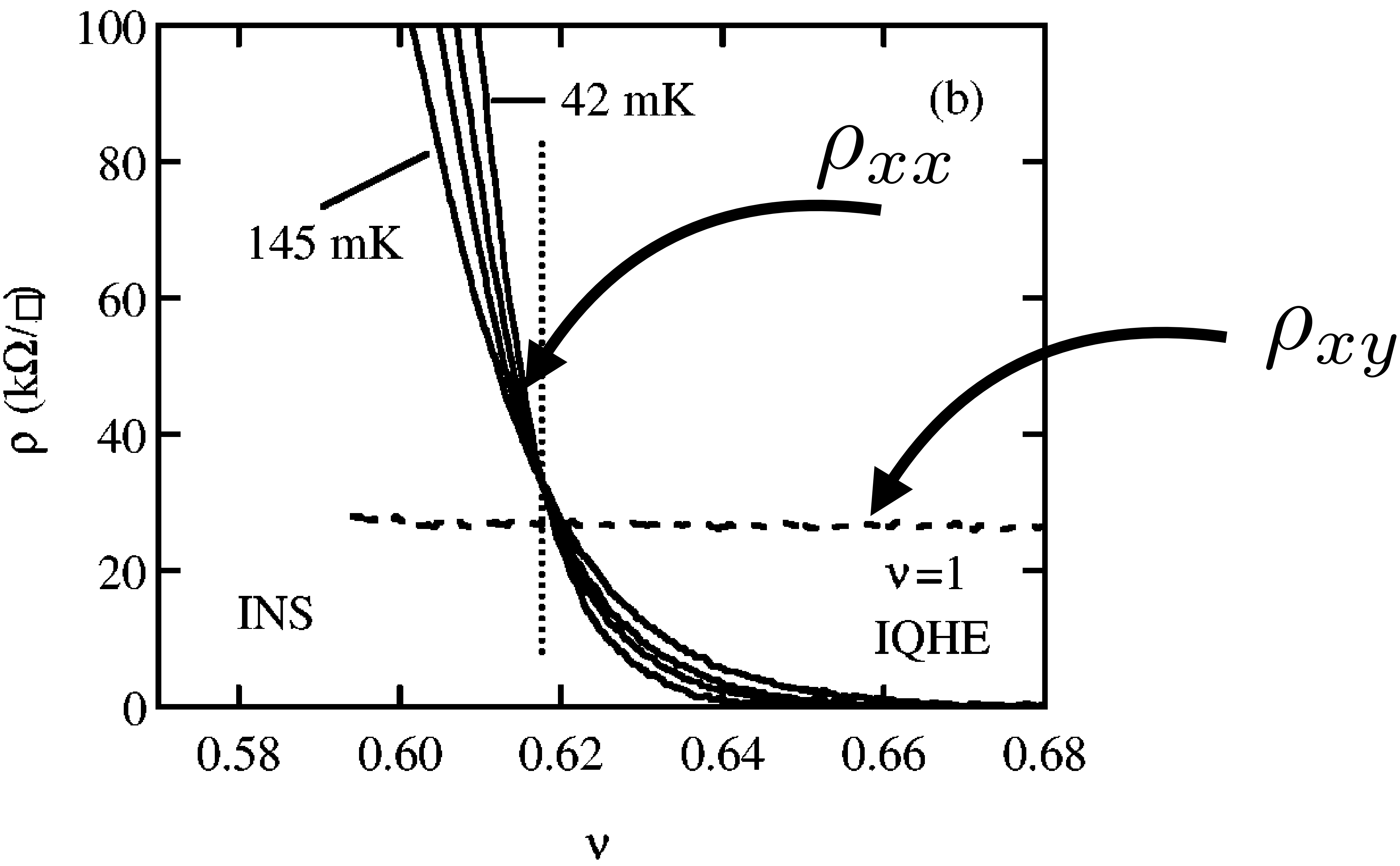
using $\sigma^{cf}_{xy} = -\dfrac{1}{4\pi}$,

$$(\rho_{xx}, \rho_{xy}) = 2\pi\,(1, 1)$$

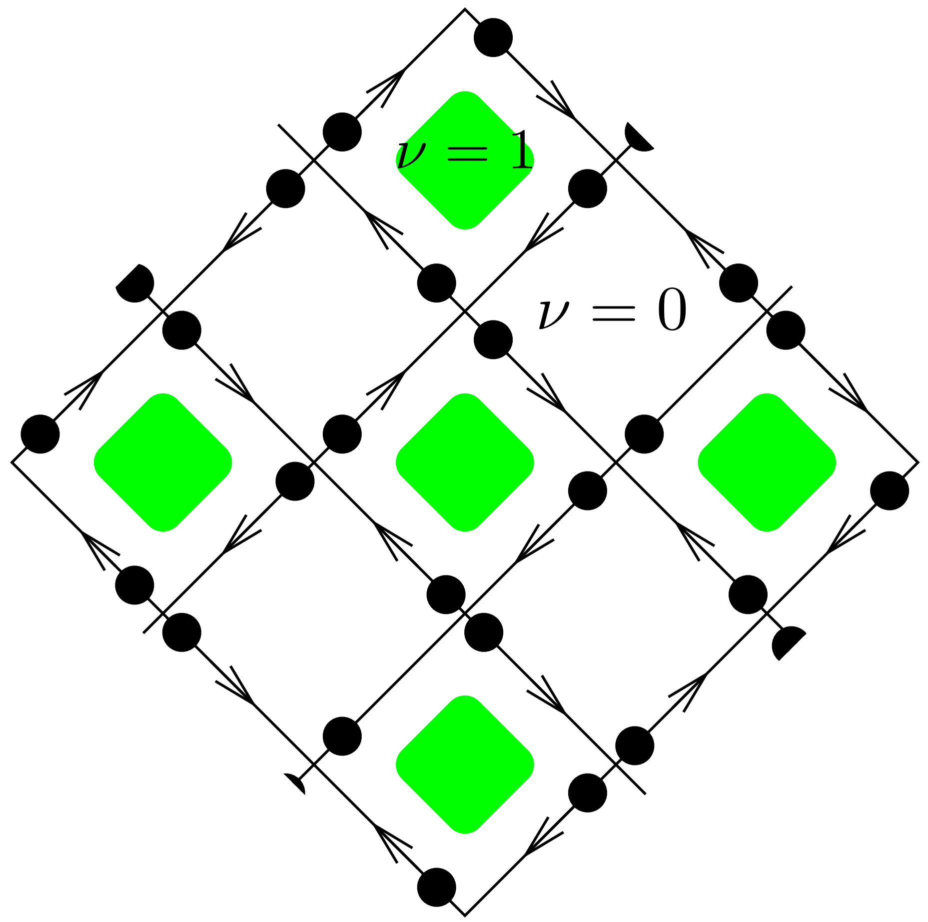
At the critical point.

$\Rightarrow \sigma^{cf}_{xx} = \dfrac{1}{4\pi}$ At the critical point.

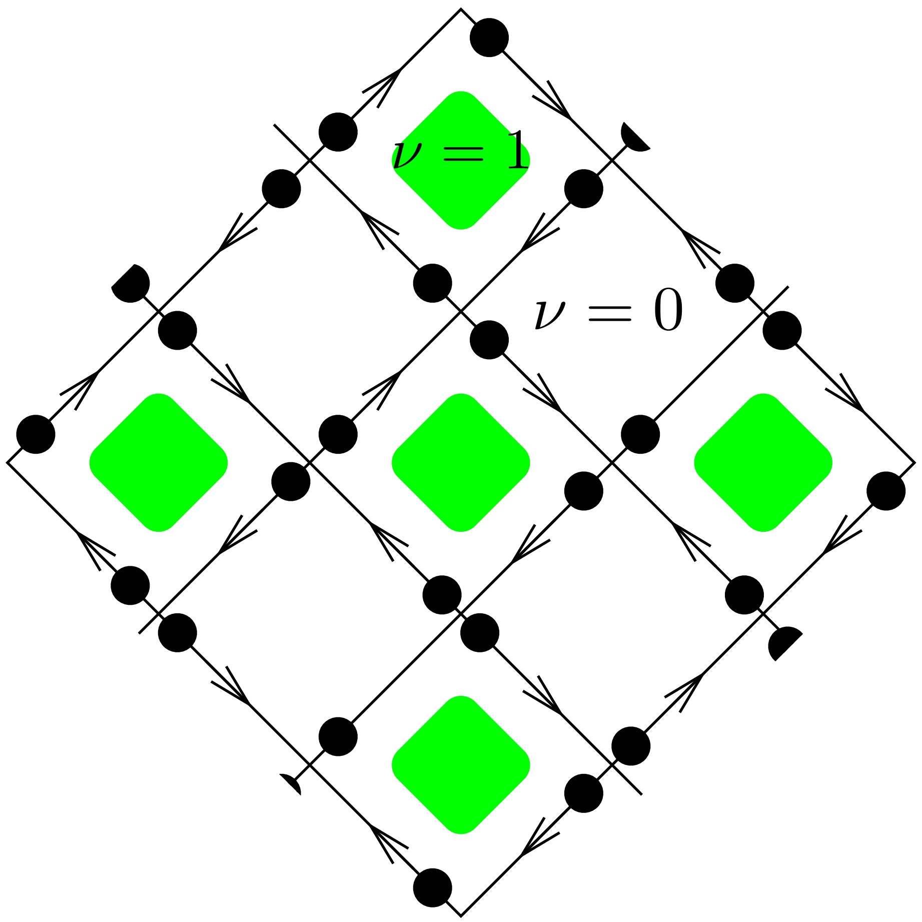
D. Shahar *et al.* PRL 1997

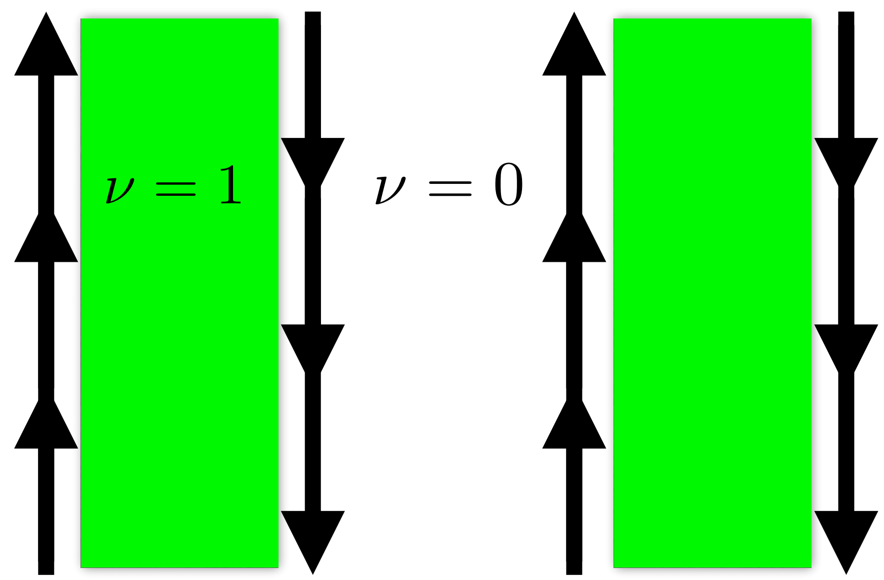


Heuristic: network models

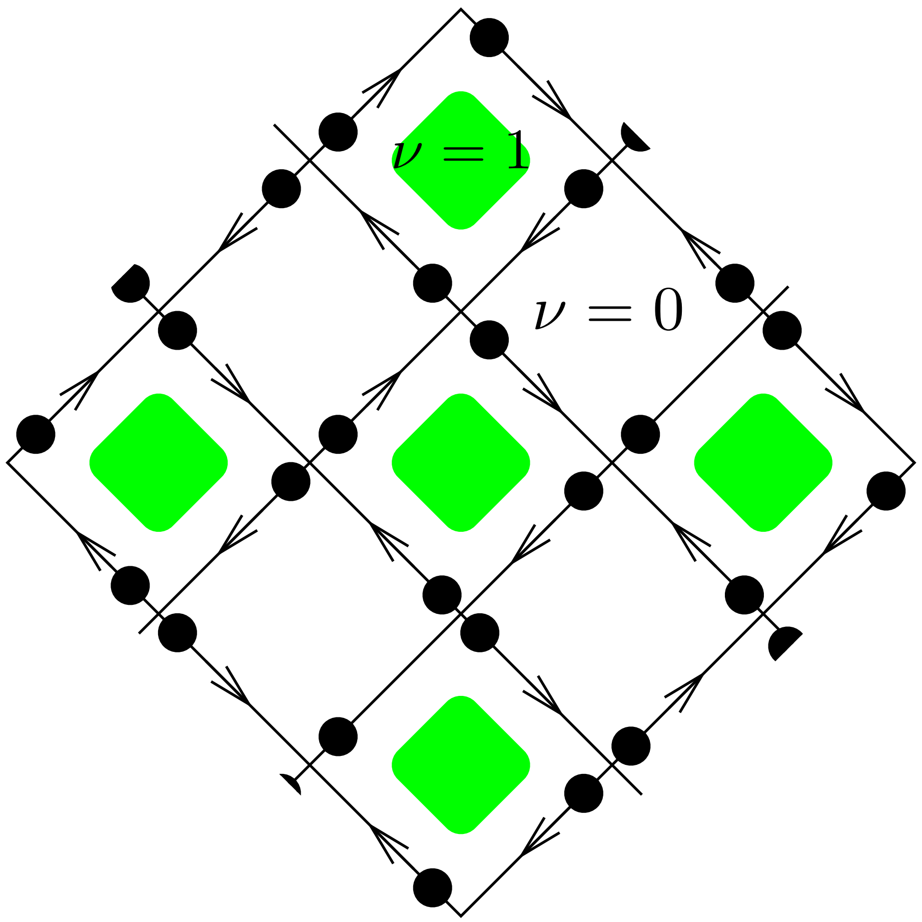


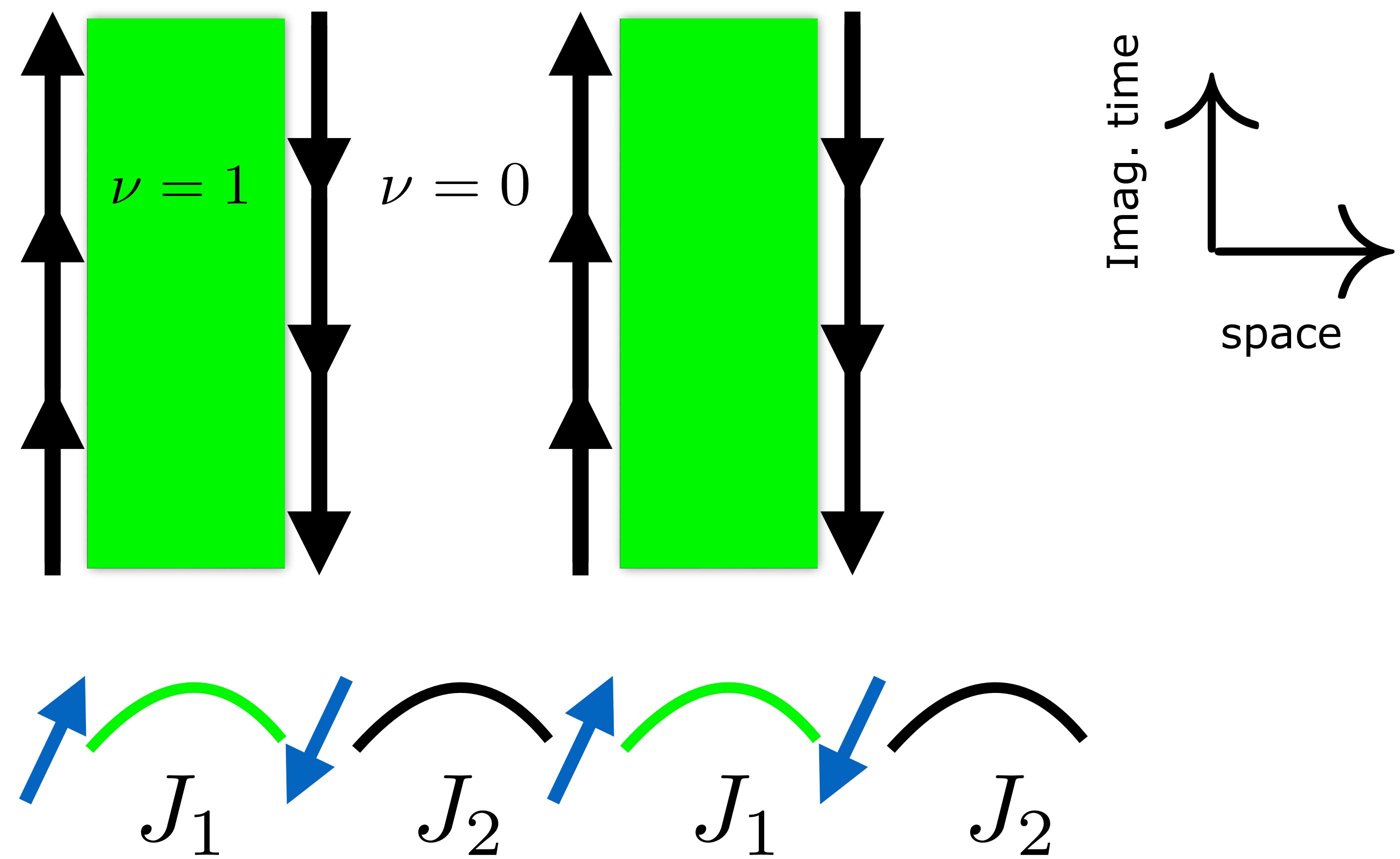
Heuristic: network models





Heuristic: network models





Network model = worldline of a SU(2n) spin chain.

J_1, J_2: Antiferromagnetic exchange couplings.

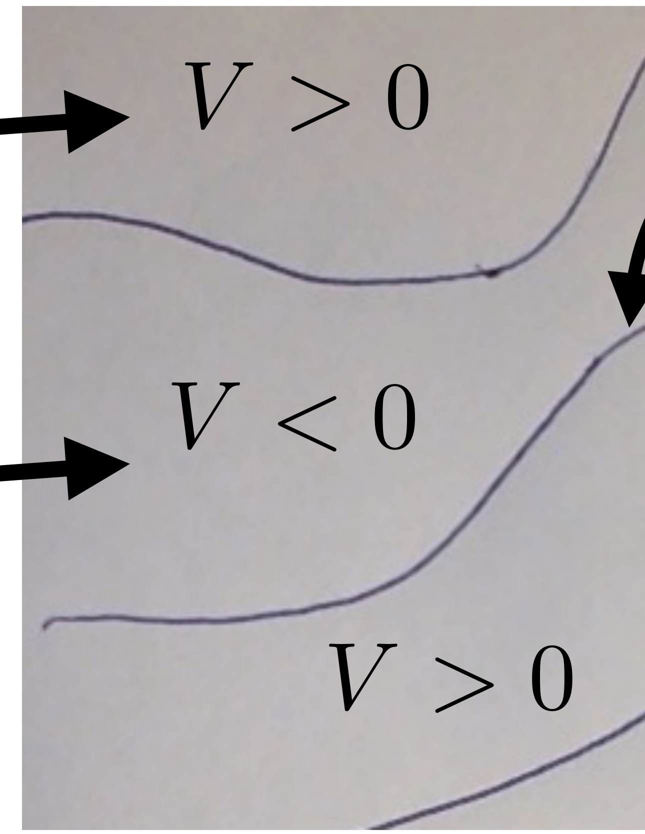
QCP = “spin Pierels” transition of spin chain.

Heuristic: network models

Network model for Composite fermions:

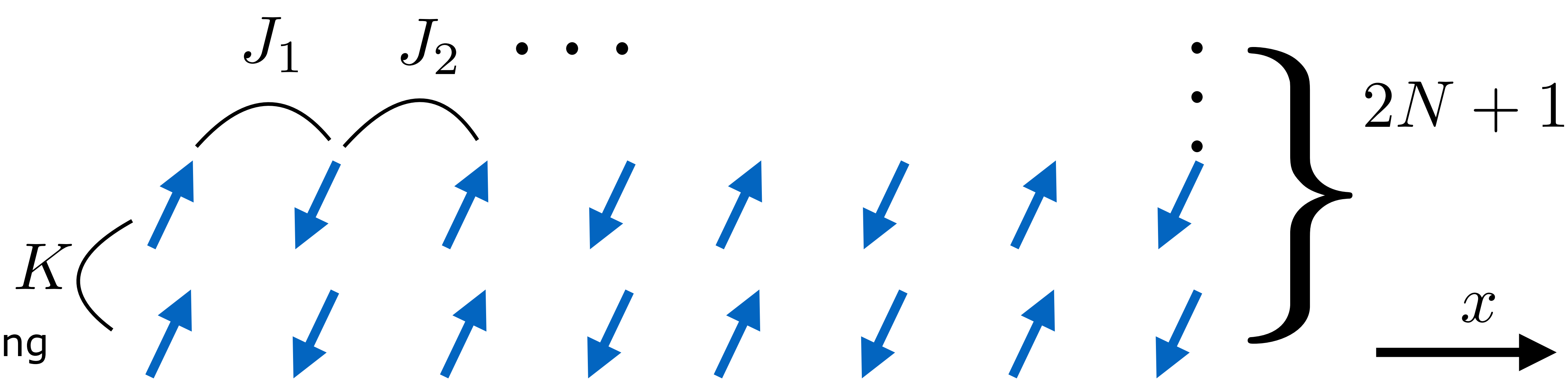
$$\mathcal{L}_{eff} = \frac{N}{4\pi} ada$$

$$\mathcal{L}_{eff} = -\frac{N+1}{4\pi} ada$$



2N+1 edge modes

$SU(2n)$ Spin ladder with 2N+1 legs



K: FM coupling

Summary

1) weak-disorder: particle-hole symmetric dc transport.

2) stronger disorder: qh-to-insulator qcp. System does not localize due to a topological term in the NLSM.

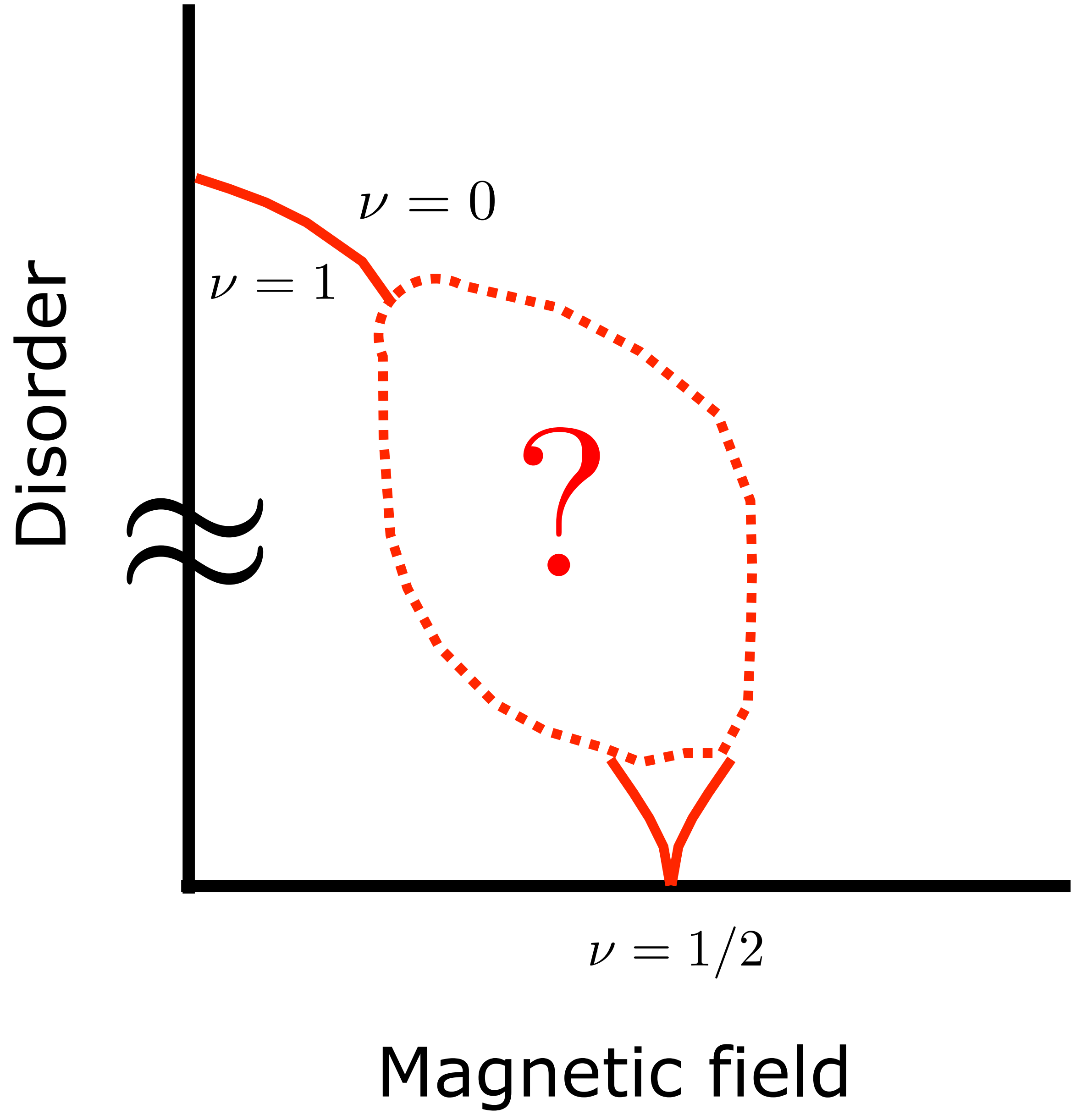
Summary

1) weak-disorder: particle-hole symmetric dc transport.

2) stronger disorder: qh-to-insulator qcp. System does not localize due to a topological term in the NLSM.

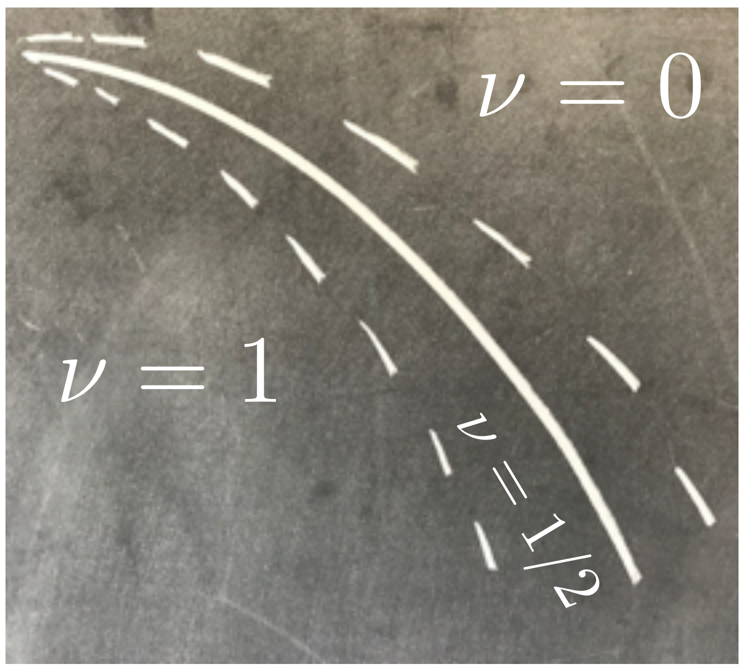
3) For the experts: Fermion zero modes of HLR mean-field theory suggest an equivalence with Dirac composite fermion theory (i.e. both theories flow to the same IR fixed pt).

Sudip's 80th Birthday: partial wish-list



Sudip's 80th Birthday: partial wish-list

Disorder

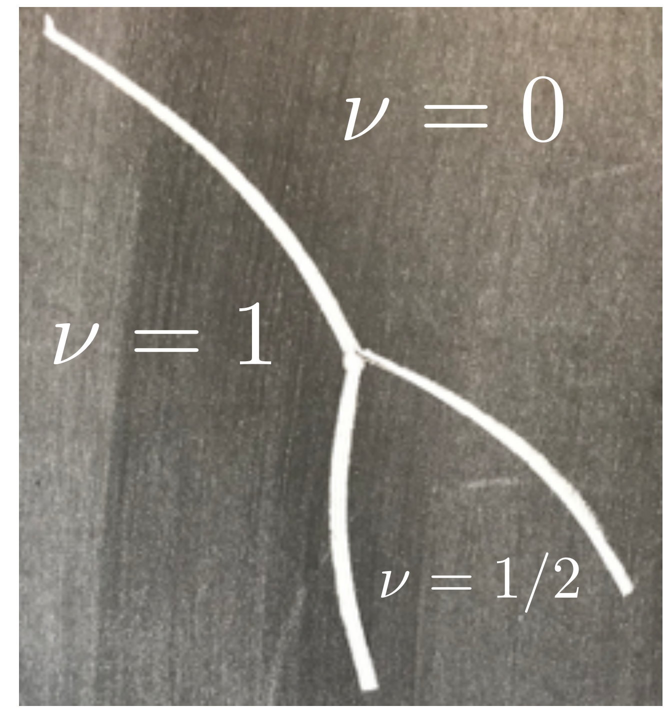


Magnetic field

nu=1/2: critical point
between two qh states.

vs

Disorder



Magnetic field

critical point and
nu=1/2 phase distinct.

Which is correct??

Two-dimensional quantum Heisenberg antiferromagnet at low temperatures

Sudip Chakravarty*
Department of Physics, State University of New York at Stony Brook, Stony Brook, New York 11794

Bertrand I. Halperin and David R. Nelson
Lyman Laboratory of Physics, Harvard University, Cambridge, Massachusetts 02138
(Received 18 August 1988)

Interactions and scaling in a disordered two-dimensional metal

Sudip Chakravarty and Lan Yin
Department of Physics and Astronomy, University of California Los Angeles, Los Angeles, California 90095-1547

Elihu Abrahams
Serin Physics Laboratory, Rutgers University, Piscataway, New Jersey 08855-0849
(Received 19 December 1997)

happy birthday, Sudip!