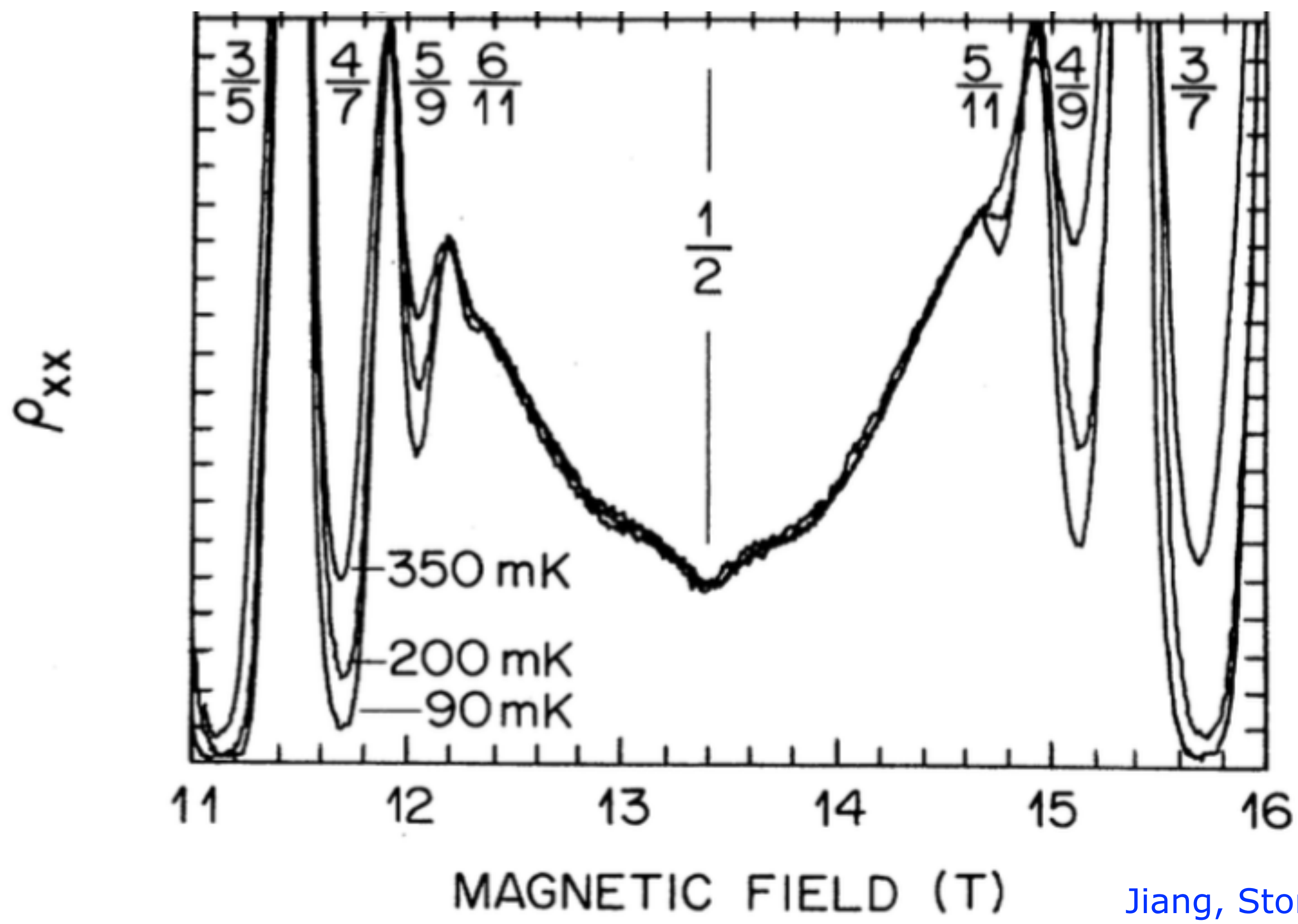
Fluctuations and magnetoresistance oscillations near a half-filled Landau level



Jiang, Stormer, Tsui, Pfeiffer, & West (1989)

Michael Mulligan
UC Riverside

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Scaling Theory of Two-Dimensional Metal-Insulator Transitions

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(Received 10 April 1997)

We discuss the recently discovered two-dimensional metal-insulator transition in zero magnetic field in the light of the scaling theory of localization. We demonstrate that the observed symmetry relating conductivity and resistivity follows directly from the quantum critical behavior associated with such a transition. In addition, we show that very general scaling considerations imply that any disordered two-dimensional metal is a *perfect metal*, but most likely *not* a Fermi liquid. [S0031-9007(97)03673-9]

beautiful paper

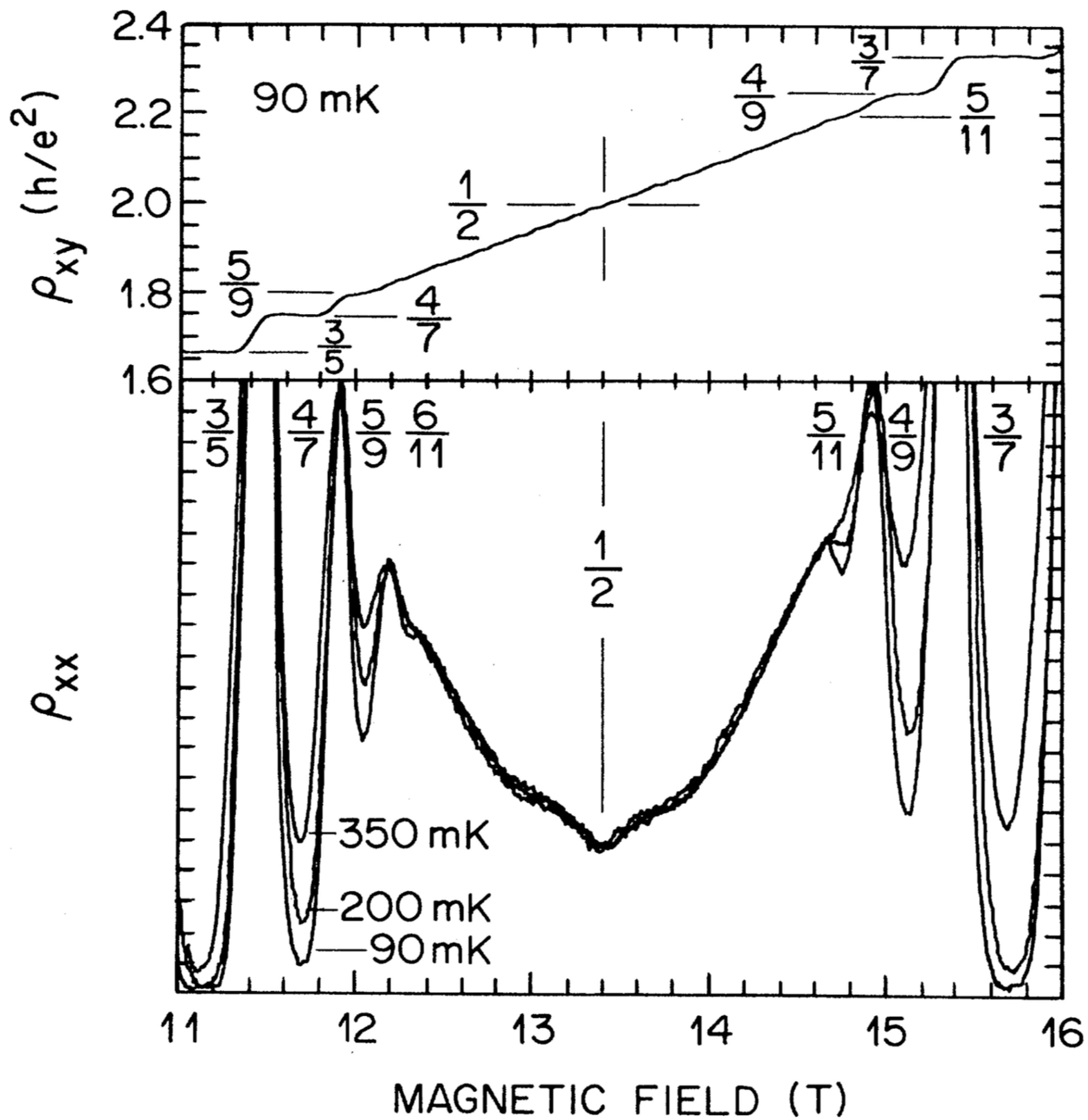
directly influenced the work with Plamadeala and Nayak finding a particular realization of a perfect metal in (1+1)d arXiv:1611.08910

arXiv:1901.08070

thanks to my collaborators

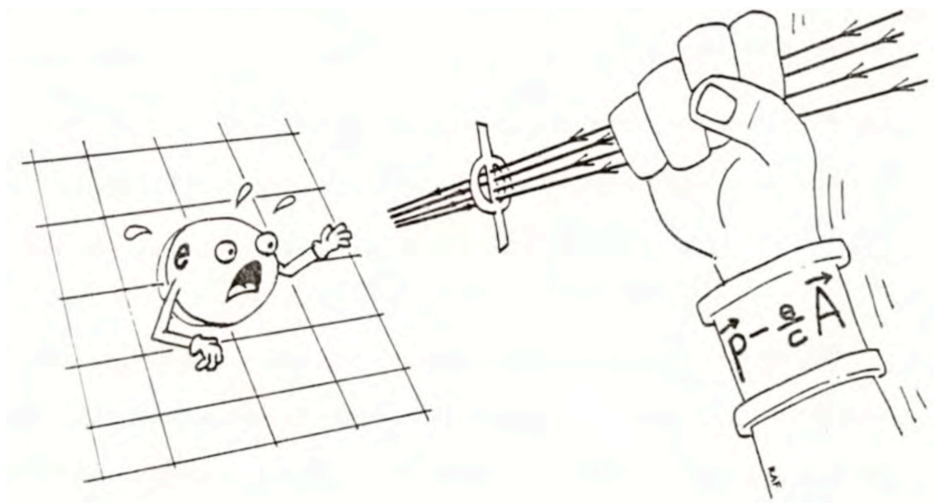
Alfred Cheung (Stanford), Amartya Mitra (UCR),
Sri Raghu (Stanford)

Objective: To argue that emergent gauge field fluctuations improve the comparison between composite fermion theories of the half-filled lowest Landau level and geometric resonance experiments.



Jiang, Stormer, Tsui, Pfeiffer, & West (1989)

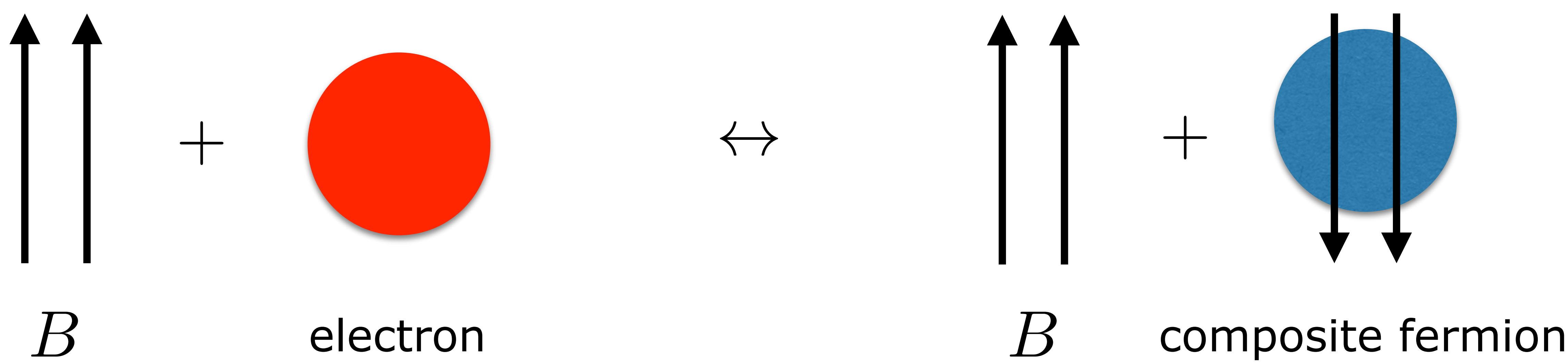
composite fermions



from D. Arovas' Ph.D thesis

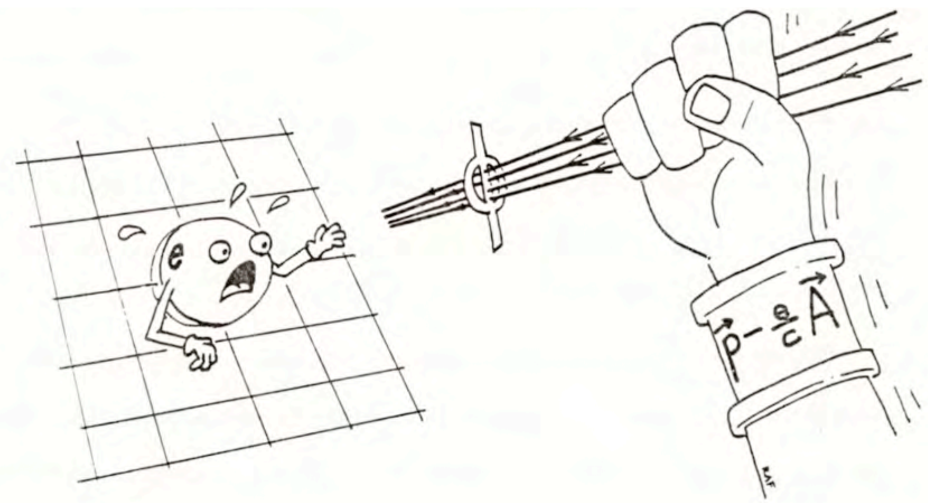
Girvin & MacDonald (1987); Read (1989); Jain(1989);
Hansson, Kivelson, & Zhang (1989); Lopez & Fradkin (1991);
Halperin, Lee, & Read (1993); Kalmeyer & Zhang (1992)

heuristic picture:
electrons at half-filling = composite fermions in zero effective field



$$b = B - 4\pi n_e$$

composite fermions



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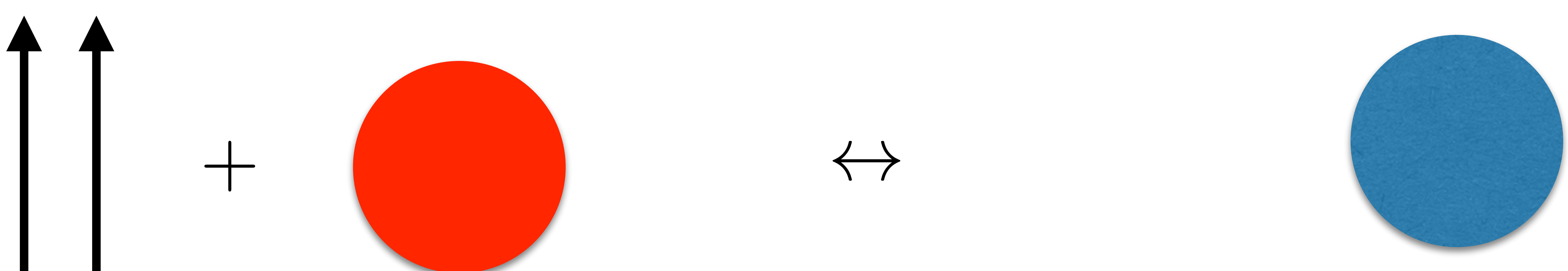


$$b = B - 4\pi n_e$$

HLR composite Fermi liquid

Halperin, Lee, & Read (1993); Kalmeyer & Zhang (1992)

$$\mathcal{L}_{\text{electron}} = \psi_e^\dagger \Big(i\partial_t + A_t + \frac{1}{2m_e}(\partial_j - iA_j)^2 \Big) \psi_e + \mathcal{L}_{\text{int}}[\psi_e^\dagger \psi_e]$$

$$\updownarrow$$

$$\mathcal{L}_{\text{CFL}} = \mathcal{L}_f + \mathcal{L}_{\text{gauge}} + \mathcal{L}_{\text{int}}[f^\dagger f],$$

$$\mathcal{L}_f = f^\dagger \Big(i\partial_t + (a_t + A_t) + \frac{1}{2m_f}(\partial_j - i(a_j + A_j))^2 \Big) f$$

$$\mathcal{L}_{\text{gauge}} = \frac{1}{2\pi}\epsilon^{\mu\nu\rho} a_\mu \partial_\nu \tilde{a}_\rho - \frac{2}{4\pi}\epsilon^{\mu\nu\rho} \tilde{a}_\mu \partial_\nu \tilde{a}_\rho$$

composite fermions provide a more “amenable” mean-field vacuum:
an emergent Fermi liquid-like state in zero effective field

“flux attachment”

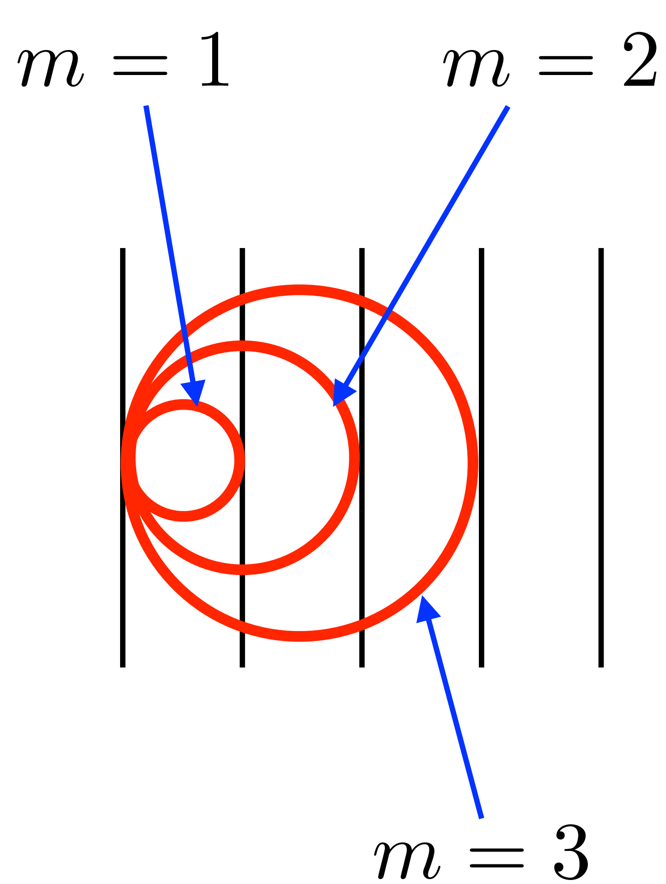
$$n_e = \langle f^\dagger f \rangle = -\frac{1}{4\pi}\langle \partial_x a_y - \partial_y a_x \rangle$$

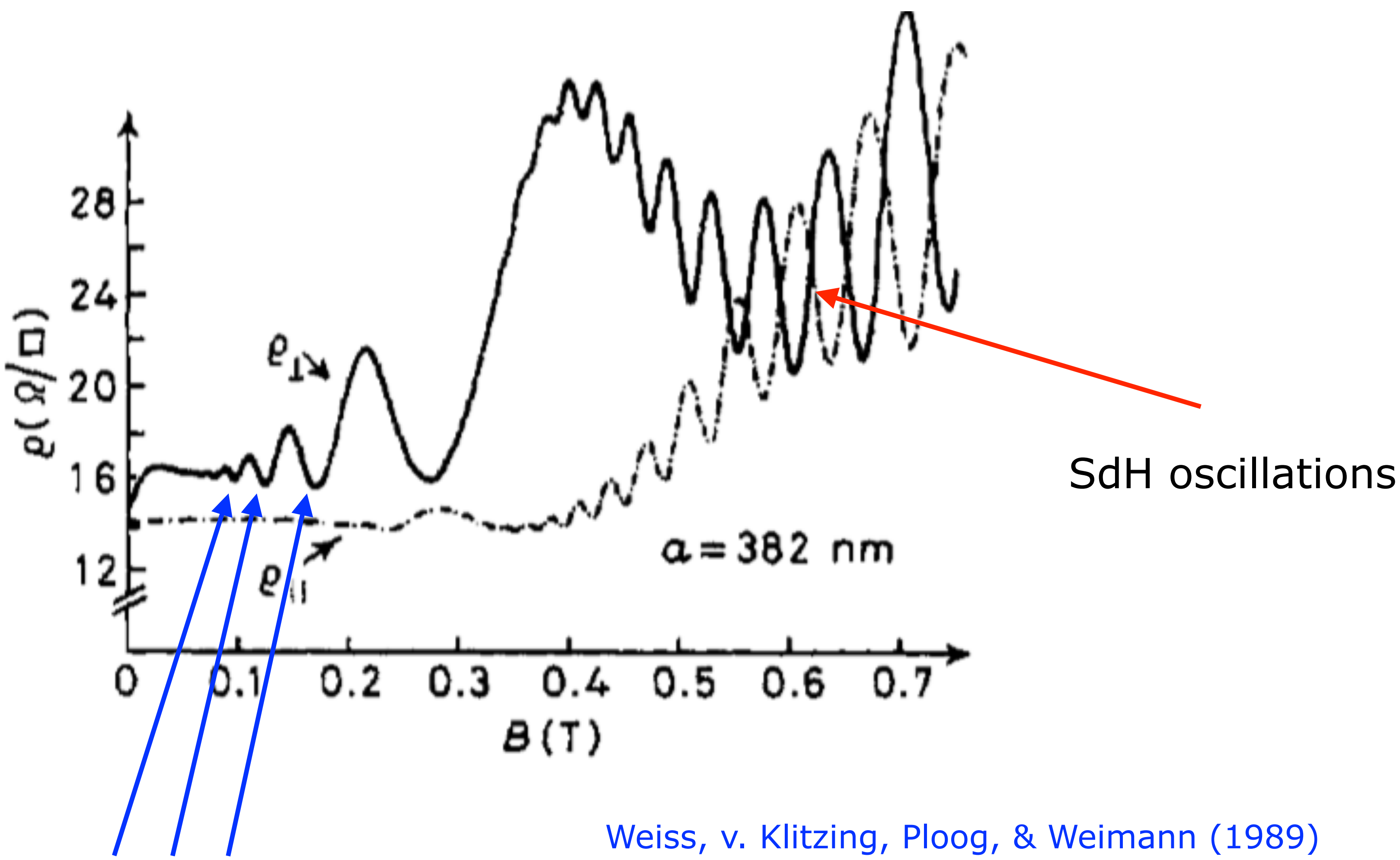
$$\langle \partial_x a_y - \partial_y a_x \rangle = -\Big(\partial_x A_y - \partial_y A_x \Big) = -B < 0$$

geometric resonances

quantum oscillations that occur **because** of the presence of a, e.g. 1D, periodic scalar/vector ($\phi = \mp\frac{1}{4}$) potential of wavelength a

$$\frac{c\hbar}{e|B_m|} = \frac{a}{2k_F}(m+\phi),\ \ m = 1, 2, 3$$





Weiss, v. Klitzing, Ploog, & Weimann (1989)

geometric resonances (m = 3, 2, 1)

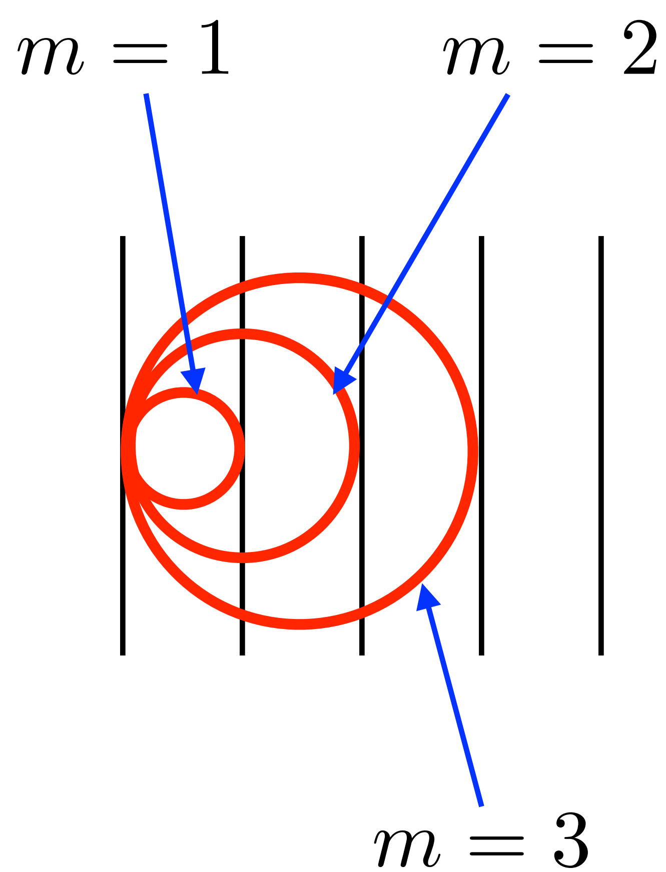
geometric resonances of composite fermions (pre 2014)

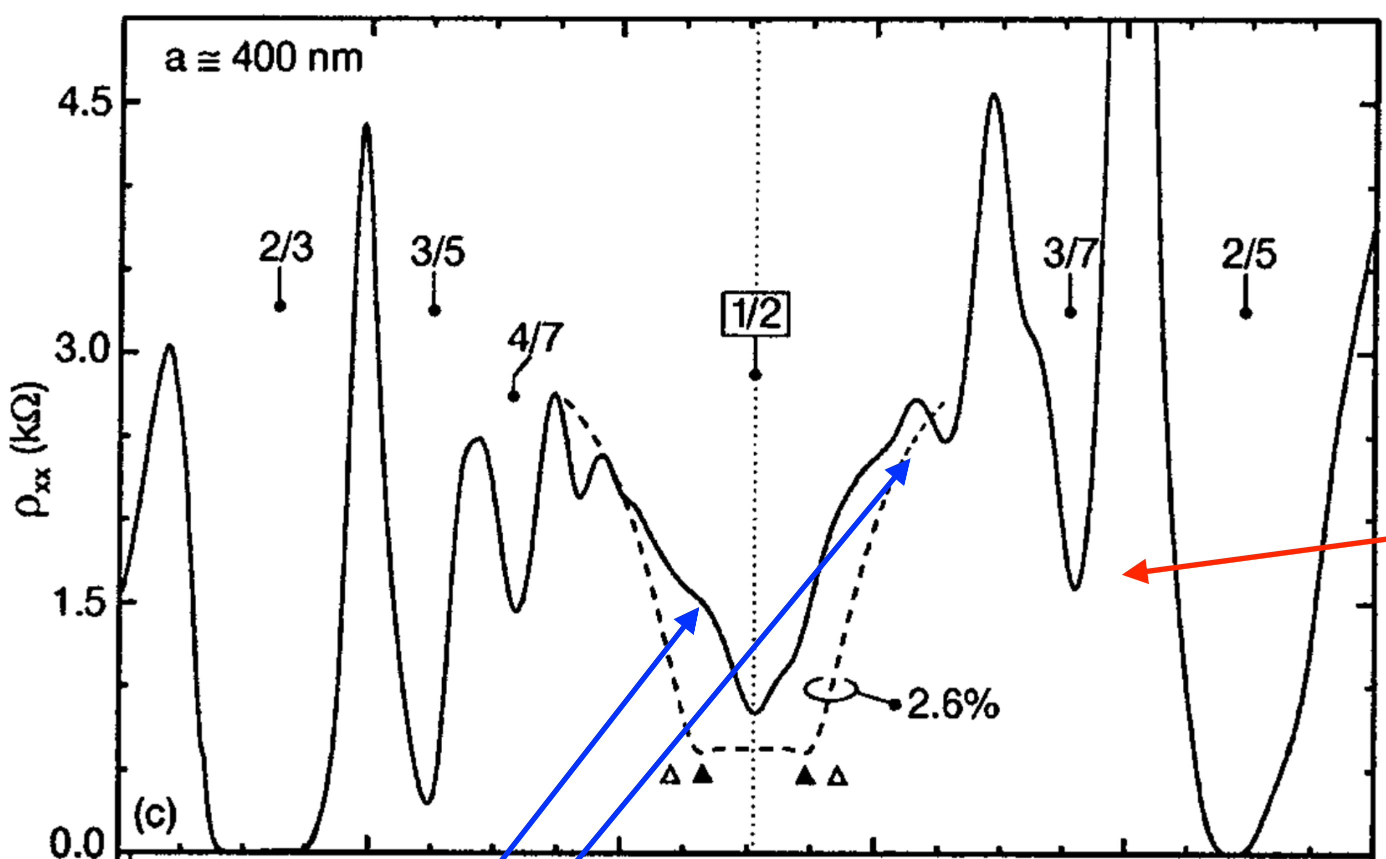
(i) composite fermions respond to an applied periodic electric modulation as a magnetic modulation

(composite fermions are electrically-neutral dipoles)

Read (1995); Haldane & Pasquier (1997); Lee (1998); Stern, Halperin, von Oppen & Simon (1999); Murthy & Shankar (2003)

(ii) composite fermion Fermi momentum $k_F^{\mathrm{CF}} = \sqrt{4\pi n_e}$

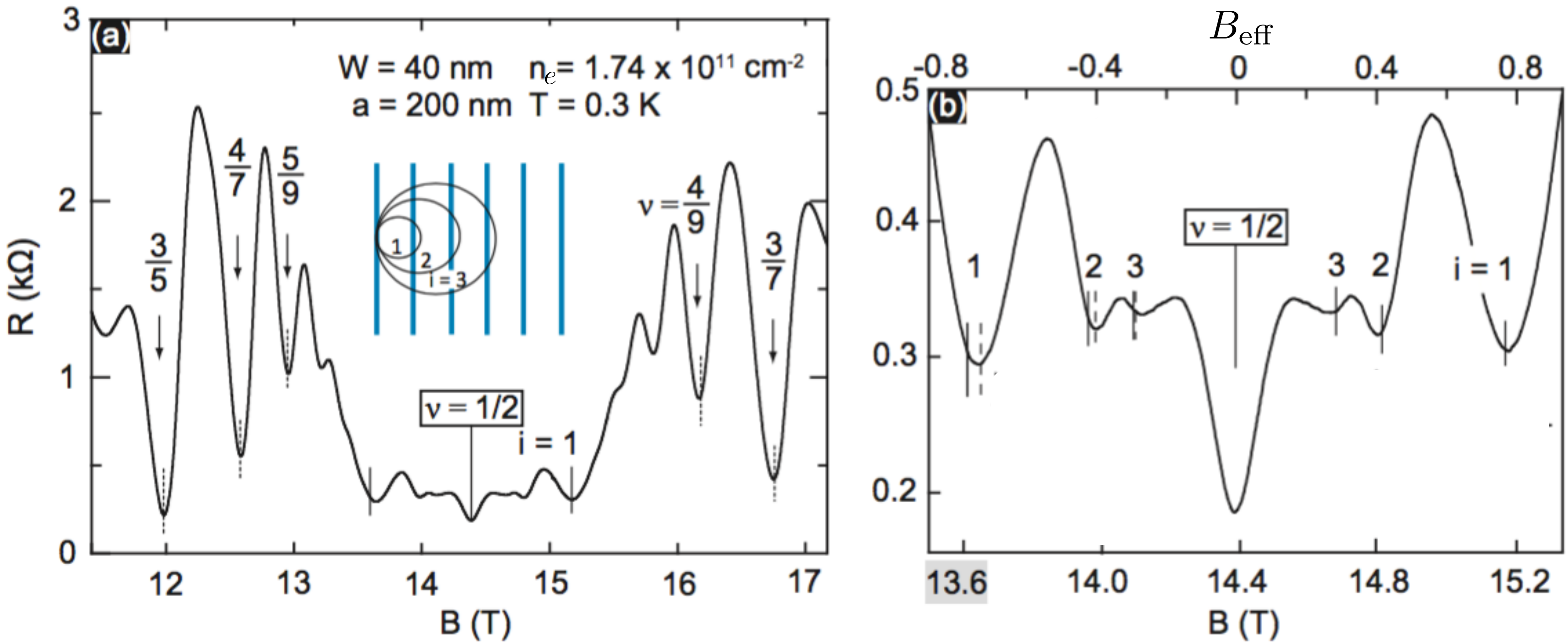




Smet, Jobst, v. Klitzing, Weiss, Wegscheider, & Umansky (1999)

geometric resonances (m = 2, 1)

geometric resonances of composite fermions (post 2014)



Kamburov, Liu, Mueed, Shayegan, Pfeiffer, West, & Baldwin (2014)

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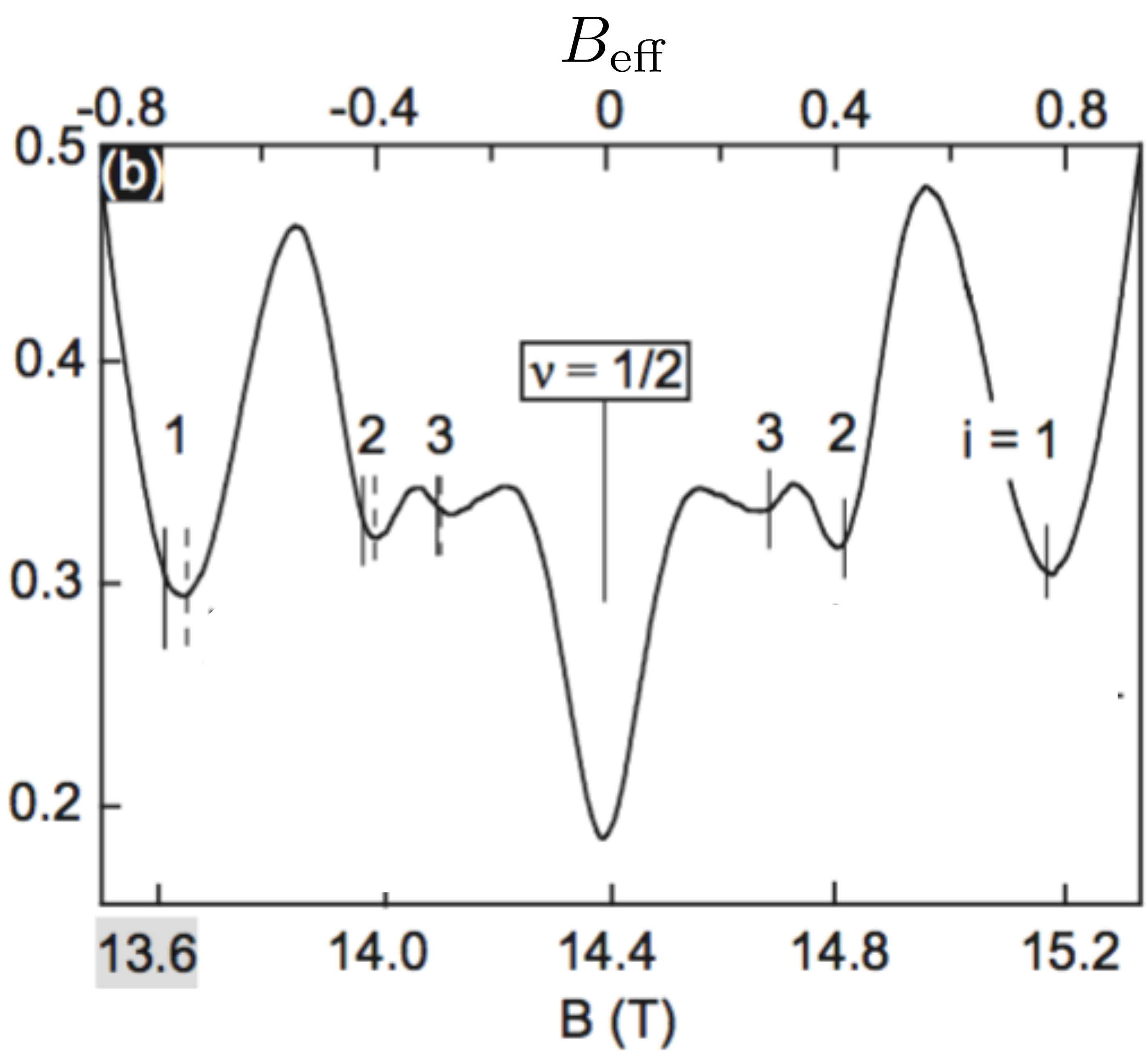

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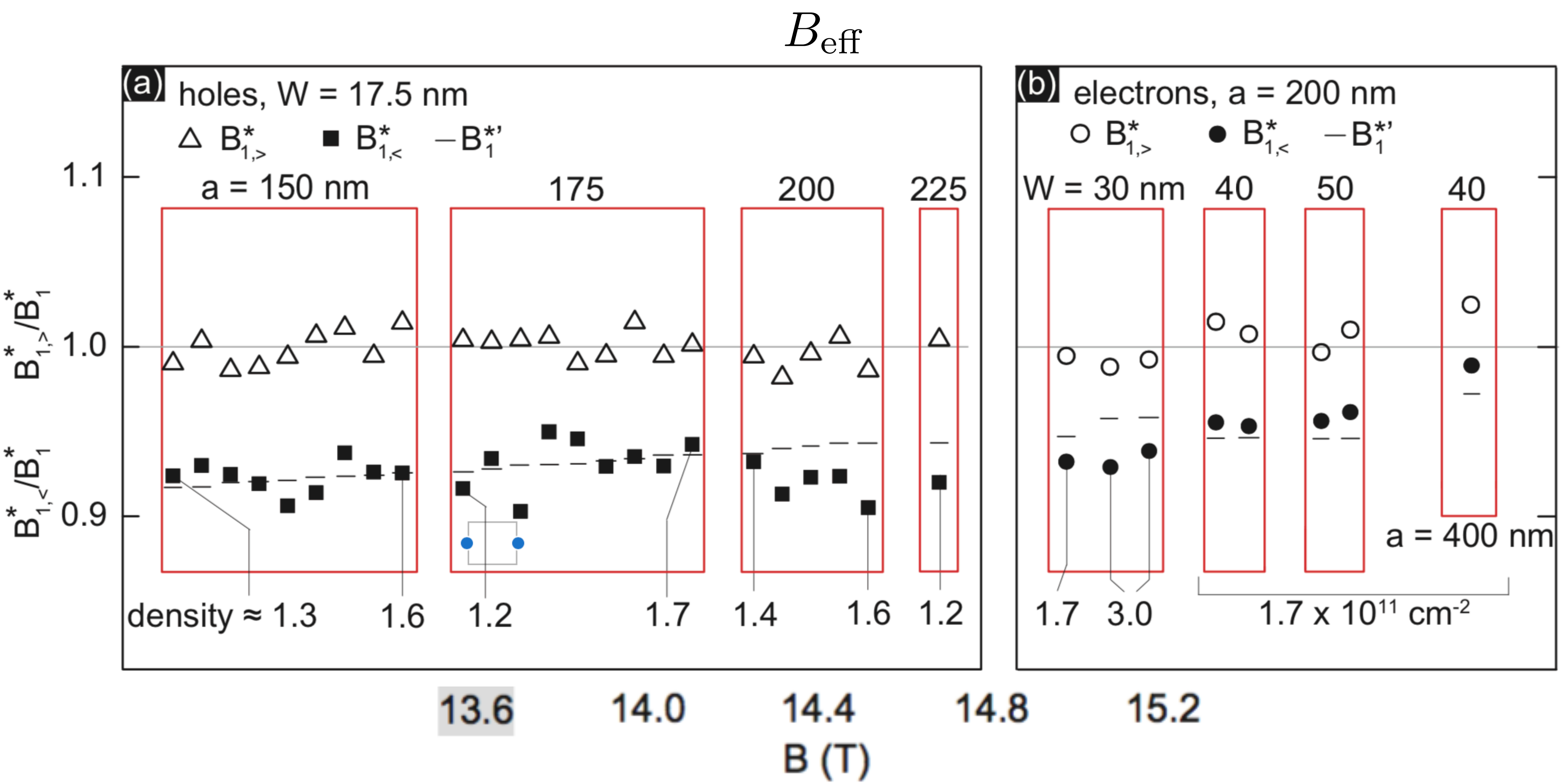

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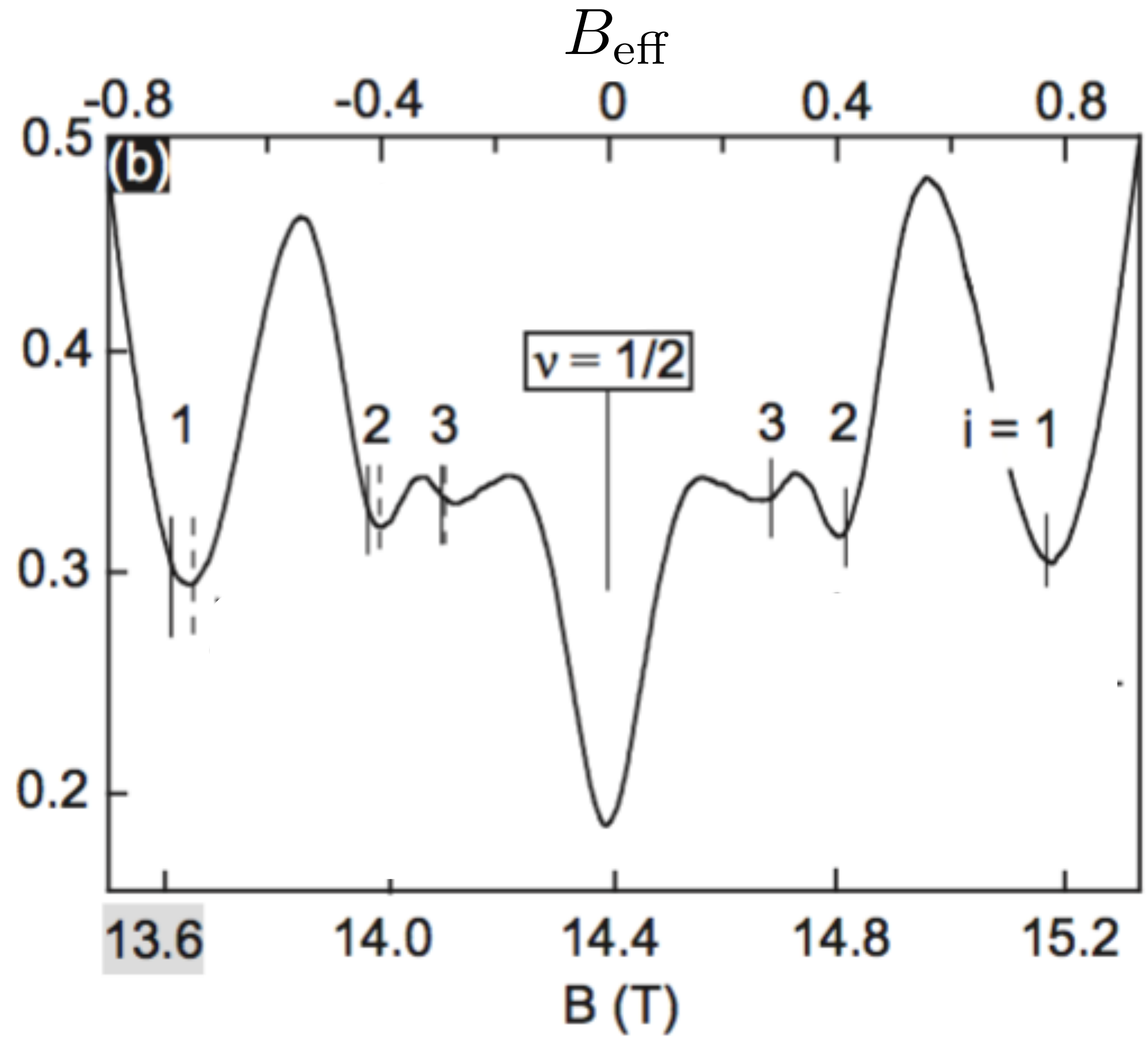


Kamburov, Liu, Mueed, Shayegan, Pfeiffer, West, & Baldwin (2014)

minority hypothesis

inferred from solid line

$$k_F(\nu < 1/2) = \sqrt{4\pi n_e}$$

inferred from dotted line

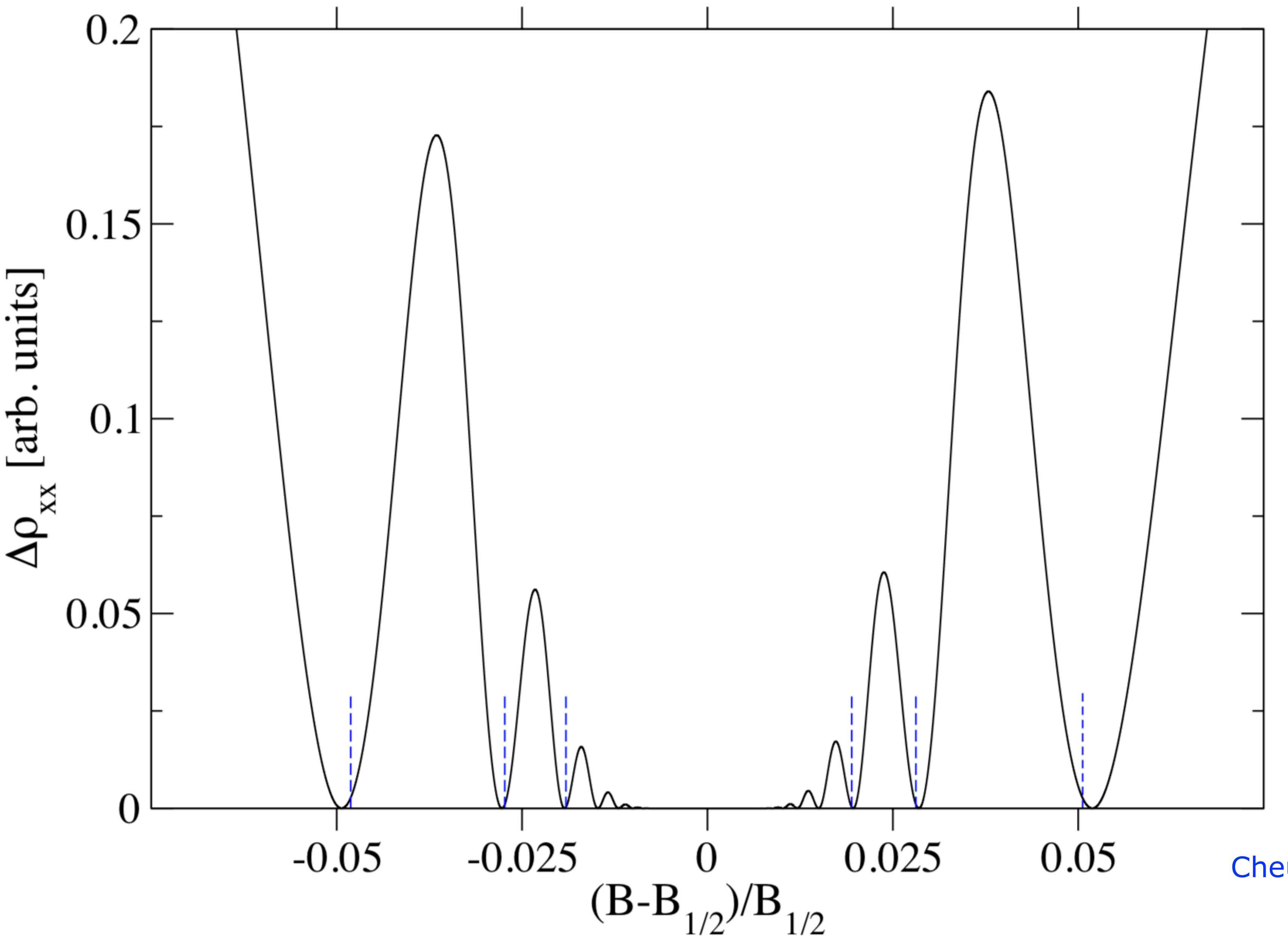
$$k_F(\nu > 1/2) = \sqrt{4\pi(B/2\pi - n_e)} = \sqrt{4\pi n_h}$$

composite fermion theories of holes and electrons …?

Barkeshli, MM, and Fisher (2015)

In this talk, I will argue:

(i) for a different interpretation. In mean-field theory, where fluctuations of the emergent gauge field are ignored, HLR composite fermions experience BOTH periodic scalar and vector potentials.

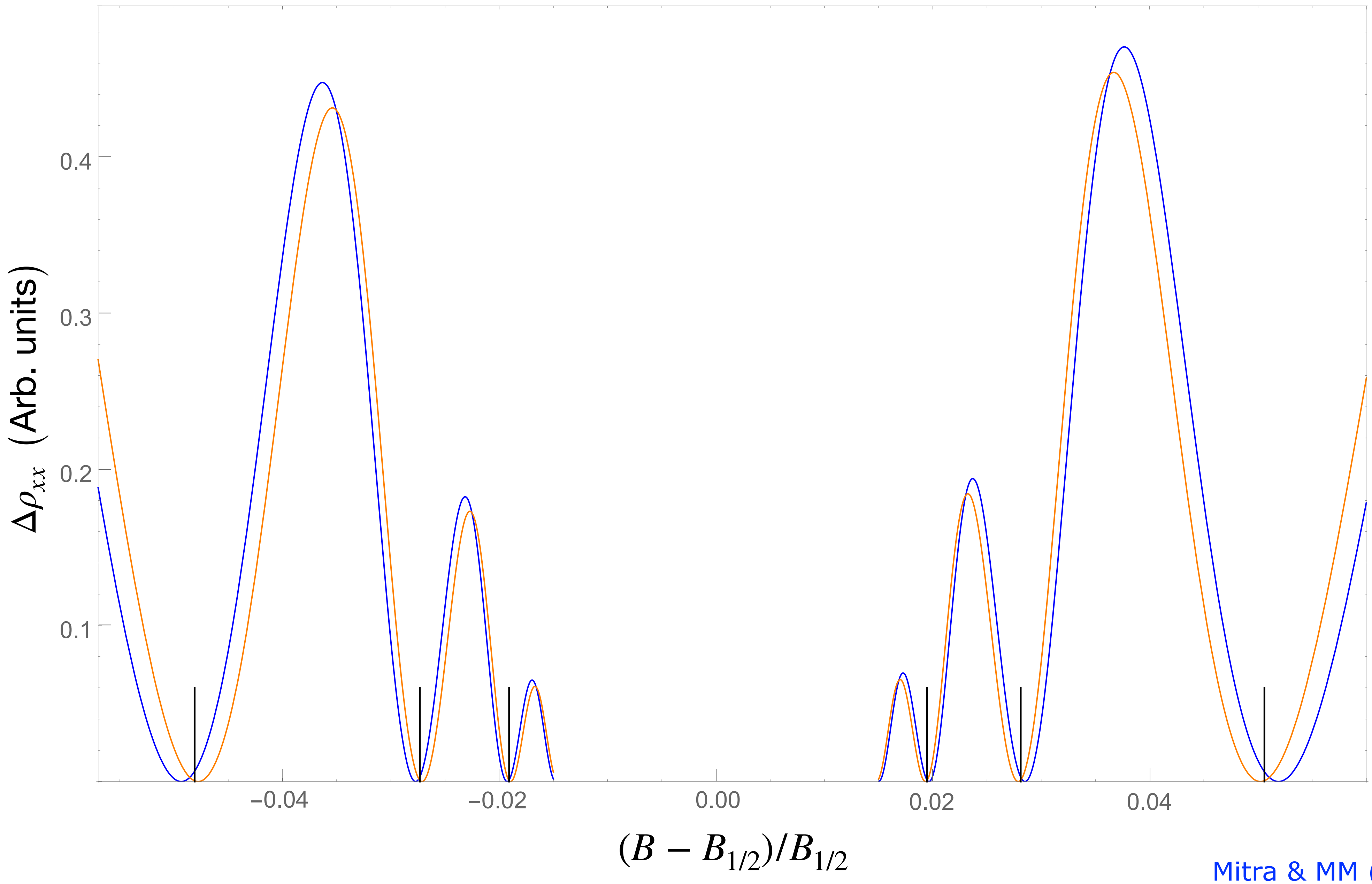


Cheung, Raghu, & MM (2017)

see also: Wang, Cooper, Halperin, & Stern (2017); Levin & Son (2017)

In this talk, I will argue:

(ii) In the Dirac composite fermion theory (to be described), emergent gauge fluctuations improve the comparison between theory and experiment (we expect similar agreement in the HLR theory).



Mitra & MM (2019)

see also: Wang, Cooper, Halperin, & Stern (2017); Levin & Son (2017)

Dirac electrons

singular LLL $\frac{B}{m_e} \to \infty$ is "regularized" by a Dirac electron Lagrangian

half-filled lowest Landau "interpreted" as a half-filled zeroth Landau level of a Dirac fermion (similar to graphene)

Son (2015)

$$\mathcal{L}_e = \bar{\Psi}_e \gamma^\mu (i\partial_\mu + A_\mu)\Psi_e + \frac{1}{8\pi}\epsilon^{\mu\nu\rho} A_\mu \partial_\nu A_\rho + \ldots$$

$$n_e \equiv \frac{\delta \mathcal{L}_e}{\delta A_t} = \Psi_e^\dagger \Psi_e + \frac{B}{4\pi}$$

$$\nu_e = \frac{1}{2} \iff \Psi_e^\dagger \Psi_e = 0$$

Dirac electrons

$$\mathcal{L}_e = \bar{\Psi}_e \gamma^\mu (i\partial_\mu + A_\mu)\Psi_e + \frac{1}{8\pi}\epsilon^{\mu\nu\rho} A_\mu \partial_\nu A_\rho + \ldots$$

$$\text{particle} - \text{hole} = \mathbf{CT} + \mathcal{T}$$

$$\mathbf{CT} = \quad \begin{array}{c} t \mapsto -t \\ \Psi_e \mapsto -\gamma^t \Psi_e^* \\ (A_t, A_x, A_y) \mapsto (-A_t, A_x, A_y) \end{array}$$

$$+$$

$$\mathcal{T} = \quad \mathcal{L}_e \mapsto \mathcal{L}_e + \frac{1}{4\pi}\epsilon^{\mu\nu\rho} A_\mu \partial_\nu A_\rho$$

Witten (2003)

Dirac composite fermions

$$\mathcal{L}_e = \bar{\Psi}_e \gamma^\mu (i\partial_\mu + A_\mu)\Psi_e + \frac{1}{8\pi}\epsilon^{\mu\nu\rho}A_\mu \partial_\nu A_\rho + \ldots$$

$$\updownarrow$$

$$\mathcal{L} = \bar{\psi}\gamma^\mu (i\partial_\mu + a_\mu)\psi - \frac{1}{4\pi}\epsilon^{\mu\nu\rho}a_\mu \partial_\nu A_\rho + \frac{1}{8\pi}\epsilon^{\mu\nu\rho}A_\mu \partial_\nu A_\rho + \ldots$$

Son (2015)

Dirac composite fermions

Son (2015)

Dirac composite fermions

$$\mathcal{L} = \bar{\psi}\gamma^\mu(i\partial_\mu + a_\mu)\psi - \frac{1}{4\pi}\epsilon^{\mu\nu\rho}a_\mu\partial_\nu A_\rho + \frac{1}{8\pi}\epsilon^{\mu\nu\rho}A_\mu\partial_\nu A_\rho + \ldots$$

Son (2015)

Dirac composite fermions

$$\mathcal{L} = \bar{\psi}\gamma^{\mu}(i\partial_{\mu} + a_{\mu})\psi - \frac{1}{4\pi}\epsilon^{\mu\nu\rho}a_{\mu}\partial_{\nu}A_{\rho} + \frac{1}{8\pi}\epsilon^{\mu\nu\rho}A_{\mu}\partial_{\nu}A_{\rho} + \ldots$$

Son (2015)

$$\text{particle} - \text{hole} = \mathbf{T} + \mathcal{T}$$

unbroken particle-hole symmetry forbids a Dirac mass and Chern-Simons term for the emergent gauge field

$$n_e = \frac{1}{4\pi}(-b + B)$$

$$\nu_e = \frac{1}{2} \iff b = 0$$

$$\frac{\delta\mathcal{L}}{\delta a_t} = 0 \iff \psi^{\dagger}\psi = \frac{B}{4\pi}$$

setup: electrical conductivity dictionary

$$\sigma_{ij} = \frac{1}{4\pi}\left(\epsilon_{ij} - \frac{1}{2}\epsilon_{ik}(\sigma^{\psi})^{-1}_{kl}\epsilon_{lj}\right)$$

(aside: particle-hole symmetric conductivity if the composite fermions exhibit zero Hall response)

Kivelson, Lee, Krotov, & Gan (1997)

periodic scalar potential gives rise to a periodic vector potential

$$\delta\vec{a} = \Big(0, W\sin(Kx_1)\Big),$$

geometric resonances appear as corrections to resistivity due to periodic vector potential; largest corrections in x_1 direction

$$\Delta\rho_{ii} \propto |\epsilon_{ij}|\Delta\sigma^{\psi}_{jj}$$

setup: Kubo formula

$$\Delta\sigma^{\psi}_{ij} = \frac{1}{L_1 L_2}\Sigma_M \Big(\partial_{E_M} f_D(E_M)\Big)\tau(E_M) v_i^M v_j^M ,$$

Charbonneau, van Vliet & Vasilopoulos (1982);
Peeters & Vasilopoulos (1993)

we assume:

$$\tau(E_M) = \tau$$

$$v_i^M = \frac{\partial E_M}{\partial p_i}$$

and we calculate the spectrum of the quadratic Hamiltonian

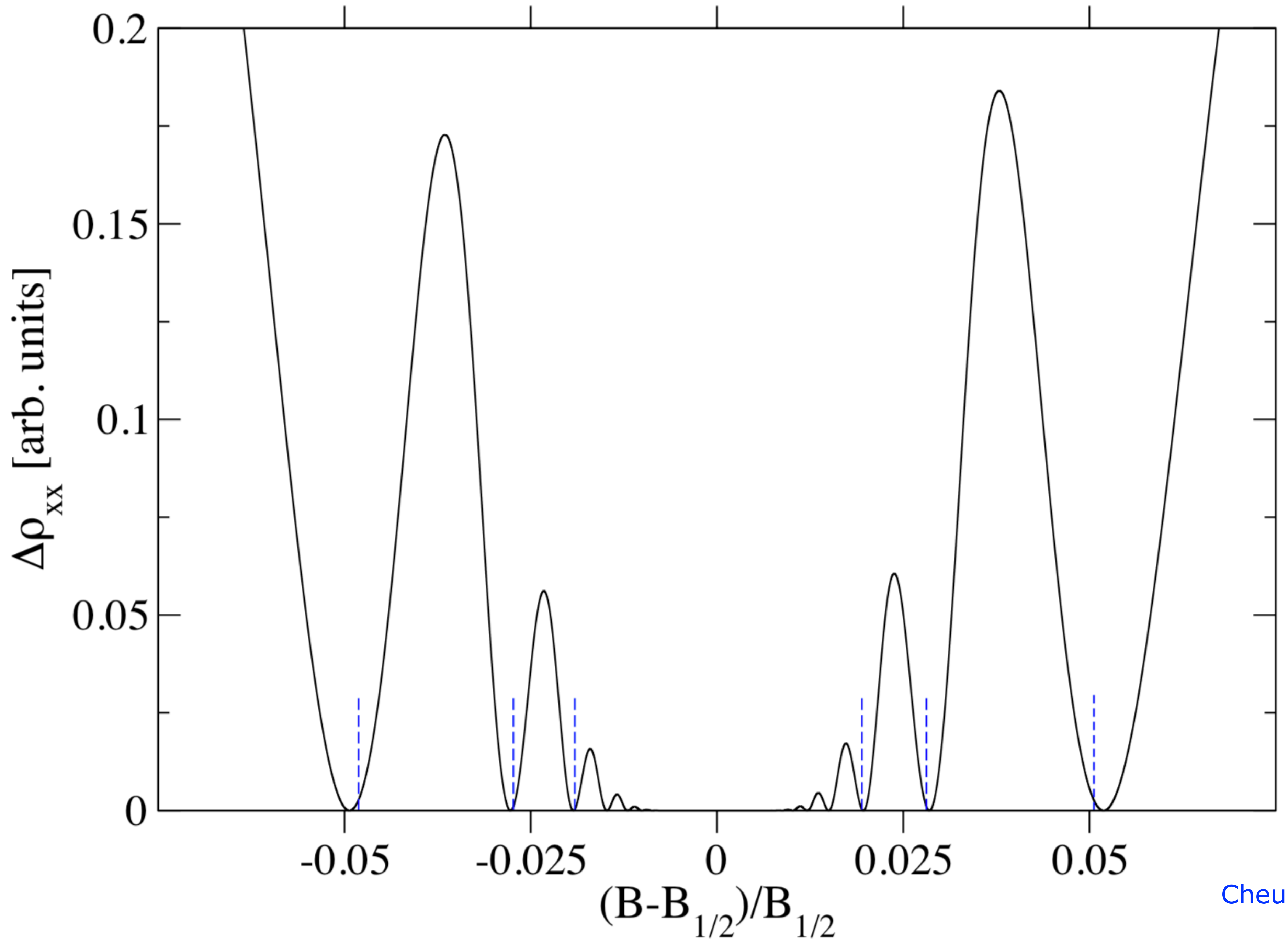
$$H = \vec{\sigma}\cdot\left(\frac{\partial}{\partial\vec{x}} + \vec{a}\right) + m\sigma_3$$

in mean-field theory:

$$m = 0$$

$$\mu = \sqrt{B}$$

mean-field theory result



Cheung, Raghu, & MM (2017)

(i) systematic 2% or less "inward shift" from the mean-field result

(ii) IR equivalence?
in the regime of experimental parameters: HLR and the Dirac composite fermion theories have the same mean-field predictions

going beyond mean-field theory

strategy: (i) determine fluctuation corrections to quadratic Hamiltonian

$$H = \vec{\sigma} \cdot \left(\frac{\partial}{\partial \vec{x}} + \vec{a} \right) + m\sigma_3$$

by determining the fluctuation corrections to the (inverse of the) Dirac composite fermion propagator G(x,y) that approximately (RPA) solves the Schwinger-Dyson equations,

$$iG^{-1}(x,y) - iG_0^{-1}(x,y) = \gamma^\alpha G(x,y) \gamma^\beta \Pi^{-1}_{\alpha\beta}(x-y),$$

$$i\Pi^{\alpha\beta}(x-y) = \mathrm{tr}\Big[\gamma^\alpha G(x,y)\gamma^\beta G(y,x)\Big]$$

(we use the bare vertices to leading order; consistent with our later expansion of the fermion self-energy)

away from half-filling, particle-hole symmetry is broken (since there's an effective magnetic field) and so we expect fluctuations to generate all terms consistent with symmetry: this means a mass is no longer forbidden in the composite fermion Hamiltonian

(ii) recompute the contribution to the resistivity from the periodic potential using the corrected quadratic Hamiltonian

Dirac fermions in a magnetic field

tree-level fermion propagator

$$G_0(x,y) = e^{i\Phi(x,y)} \int \frac{d^3p}{(2\pi)^3} e^{ip_\alpha (x-y)^\alpha} G_0(p)$$

Schwinger (1951); Ritus (1978)

with Schwinger phase

$$\Phi(x,y) = -\frac{b}{2}(x_2 - y_2)(x_1 + y_1)$$

and pseudo-momentum tree-level fermion propagator

$$-iG_0(p) \equiv \frac{(p_\alpha + \mu_0 \delta_{\alpha,0})\gamma^\alpha + m_0 \mathbb{I}}{(p_0 + \mu_0 + i\epsilon_{p_0})^2 - p_i^2 - m_0^2} + b\frac{(p_0 + \mu_0)\mathbb{I} + m_0\gamma^0}{\left((p_0 + \mu_0 + i\epsilon_{p_0})^2 - p_i^2 - m_0^2\right)^2} + \mathcal{O}(b^2)$$

Watson & Reinhardt (2014)

assuming the Schwinger phase is uncorrected, we may work in pseudo-momentum space with standard methods

Schwinger-Dyson equations

in pseudo-momentum space

$$i\Sigma_\alpha(q)\gamma^\alpha + i\Sigma_m(q)\mathbb{I} = \int \frac{d^3p}{(2\pi)^3}\gamma^\alpha G(p+q)\gamma^\beta \Pi^{-1}_{\alpha\beta}(p),$$

$$i\Pi^{\alpha\beta}(\delta q) = N\int \frac{d^3p}{(2\pi)^3}\mathrm{tr}\Big[\gamma^\alpha G(p)\gamma^\beta G(p+\delta q)\Big]$$

N=1 is the number of fermion flavors; 1/N is an artificial small parameter used to organize the calculation

$$\Sigma_\alpha = \Sigma_\alpha^{(1)} + \Sigma_\alpha^{(2)} + \ldots = 0 + \Sigma_\alpha^{(2)} + \ldots,$$

$$\Sigma_m = \Sigma_m^{(1)} + \Sigma_m^{(2)} + \ldots = \Sigma_m^{(1)} + \Sigma_m^{(2)} + \ldots.$$

as a result, (i) the chemical potential is un-corrected and (ii) the fermion obtains the dynamically generated mass (at leading order)

$$m = \Sigma_m^{(1)}$$

gauge field self-energy

$$i\Pi^{\alpha\beta}_{\rm even}(\delta q) - \epsilon^{\alpha\beta\sigma}\delta q_\sigma \Pi_{\rm odd}(\delta q) = N\int \frac{d^3p}{(2\pi)^3}{\rm tr}\left[\gamma^\alpha G^{(0)}(p)\gamma^\beta G^{(0)}(p+\delta q)\right]$$

we **ignore** the particle-hole even component (Landau damping etc)

logic:
(i) finite effective field leads to generation of a Chern-Simons term for the gauge field, which then induces a fermion mass, consistent with the broken particle-hole symmetry

(ii) the particle-hole odd component is enhanced at small, but finite b relative to the particle-hole even component

$$\Pi_{\rm odd}(0) = \frac{N}{4\pi}\left(\Theta(|\Sigma^{(1)}_m| - \mu_0)\frac{\Sigma^{(1)}_m}{|\Sigma^{(1)}_m|} + \Theta(\mu_0 - |\Sigma^{(1)}_m|)\frac{\Sigma^{(1)}_m}{\mu_0}\right)$$

fermion self-energy

the Schwinger-Dyson equation implies the mass (self-energy) solves

$$\Sigma_m^{(1)} = -\frac{2\mu_0 \mathrm{sign}(\Sigma_m^{(1)})}{N} + \frac{2}{3}\frac{b\mu_0^2}{N|\Sigma_m^{(1)}|^3}$$

dimensional analysis implies

$$\Sigma_m^{(1)} = \frac{\mu_0}{N} f\Big(\frac{bN^3}{\mu_0^2}\Big)$$

in perturbation theory, we can determine the wave function renormalization and verify

$$\lim_{N\to\infty} \Sigma_\alpha^{(2)} = 0$$

(recall the 1/N expansion begins with $\Sigma_\alpha^{(1)} = 0.$)

summary of analysis

the result of this rather complicated and **approximate** analysis

$$H = \vec{\sigma} \cdot \left(\frac{\partial}{\partial \vec{x}} + \vec{a} \right) + m\sigma_3$$

with chemical potential

$$\mu = \sqrt{B}$$

and dynamically-generated mass (given by $\Sigma_m^{(1)}$ continued to N=1)

$$m \approx .69 \mathrm{sign}(b) |b|^{1/3} B^{1/6}$$

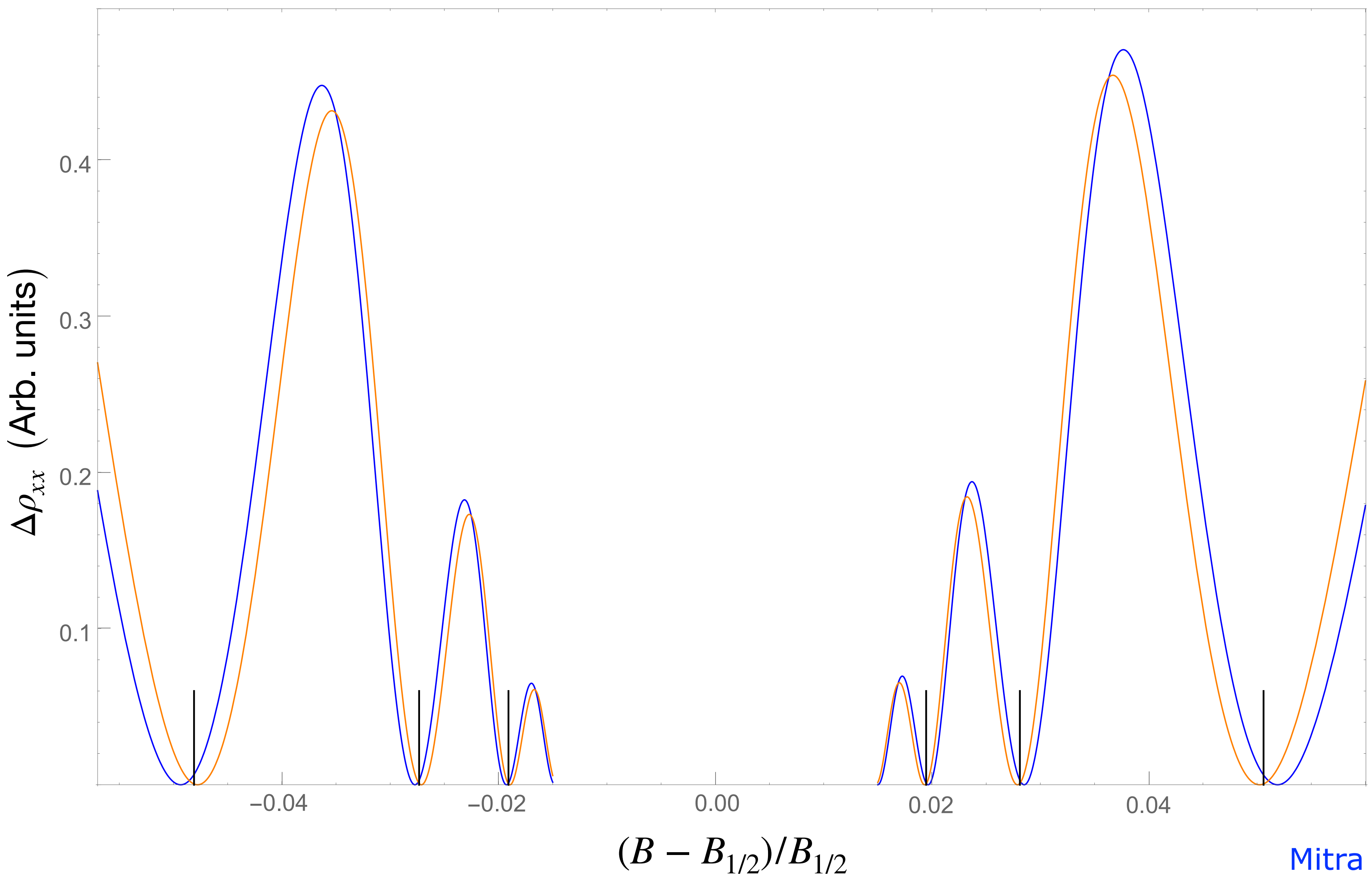
asymmetric at fixed density; symmetric at fixed external field B

commensurability oscillations

$$\Delta\rho_{xx} \propto 1 - \frac{T/T_D}{\sinh(T/T_D)}\left[1 - 2\sin^2\left(\frac{2\pi l_b^2\sqrt{B-m^2}}{d} - \frac{\pi}{4}\right)\right]$$

$$T_D^{-1} = \frac{4\pi^2 l_b^2}{d}\frac{1}{\sqrt{1-\frac{m^2}{B}}}$$

(blue curve with m = 0 is the mean-field result)



Mitra & MM (2019)

“resolving” the IR equivalence?

in HLR **mean-field theory**

$$\Delta\rho_{xx} \propto \frac{T/T_{NR}}{\sinh(T/T_{NR})}$$

where

$$T_{NR}^{-1} = \frac{4\pi^2 l_b^2}{d} \frac{m^*}{\sqrt{4\pi n_e}}$$

If the HLR composite fermion effective mass

$$m^* \propto \sqrt{B}$$

Manoharan, Shayegan, & Klepper (1994)

and if this result holds when fluctuations are included, then the finite-temperature behavior of commensurability oscillations could distinguish the two theories

open questions

a careful analysis on the effects of the particle-hole even component of the gauge field self-energy

fluctuation effects in the HLR composite fermion theory

can this “finite magnetic field technology” be used to study the renormalization group properties of non-Fermi liquids in a non-zero field?