Topics on Galileons and generalized Galileons

Pacific 2016, Moorea, Sept the 13th

1. What are scalar Galileons ?

2. What are they useful for ?

3. Some recent developments and extensions.

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1. What are scalar Galileons ?

1.1. Scalar Galileon in flat and curved 4 D space-time

Galileon

Originally (Nicolis, Rattazzi, Trincherini 2009) defined in flat space-time as the most general scalar theory which has **(strictly) second order fields equations**

In 4D, there is only 4 non trivial such theories

(with $\pi_\mu = \partial_\mu \pi \quad \pi_{\mu\nu} = \partial_\mu \partial_\nu \pi$)

$$
\begin{aligned}
\mathcal{L}_{(2,0)} &= \pi_\mu \pi^\mu \\
\mathcal{L}_{(3,0)} &= \pi^\mu \pi_\mu \Box \pi \\
\mathcal{L}_{(4,0)} &= (\Box\pi)^2 (\pi_\mu \pi^\mu) - 2 (\Box\pi) (\pi_\mu \pi^{\mu\nu} \pi_\nu) \\
&\quad - (\pi_{\mu\nu} \pi^{\mu\nu}) (\pi_\rho \pi^\rho) + 2 (\pi_\mu \pi^{\mu\nu} \pi_{\nu\rho} \pi^\rho) \\
\mathcal{L}_{(5,0)} &= (\Box\pi)^3 (\pi_\mu \pi^\mu) - 3 (\Box\pi)^2 (\pi_\mu \pi^{\mu\nu} \pi_\nu) - 3 (\Box\pi) (\pi_{\mu\nu} \pi^{\mu\nu}) (\pi_\rho \pi^\rho) \\
&\quad + 6 (\Box\pi) (\pi_\mu \pi^{\mu\nu} \pi_{\nu\rho} \pi^\rho) + 2 \left(\pi_\mu{}^\nu \pi_\nu{}^\rho \pi_\rho{}^\mu\right) \left(\pi_\lambda \pi^\lambda\right) \\
&\quad + 3 (\pi_{\mu\nu} \pi^{\mu\nu}) \left(\pi_\rho \pi^{\rho\lambda} \pi_\lambda\right) - 6 \left(\pi_\mu \pi^{\mu\nu} \pi_{\nu\rho} \pi^{\rho\lambda} \pi_\lambda\right)
\end{aligned}
$$

Simple rewriting of those Lagrangians with epsilon tensors
(up to integrations by part):

(C.D., S.Deser, G.Esposito-Farese, 2009)

$$
\begin{aligned}
\mathcal{L}_{(2,0)} &= \epsilon^{\mu_1\lambda_1\lambda_2\lambda_3}\epsilon^{\nu_1}{}_{\lambda_1\lambda_2\lambda_3}\pi_{\mu_1}\pi_{\nu_1} \\
\mathcal{L}_{(3,0)} &= \epsilon^{\mu_1\mu_2\lambda_1\lambda_2}\epsilon^{\nu_1\nu_2}{}_{\lambda_1\lambda_2}\pi_{\mu_1}\pi_{\nu_1}\pi_{\mu_2\nu_2} \\
\mathcal{L}_{(4,0)} &= \epsilon^{\mu_1\mu_2\mu_3\lambda_1}\epsilon^{\nu_1\nu_2\nu_3}{}_{\lambda_1}\pi_{\mu_1}\pi_{\nu_1}\pi_{\mu_2\nu_2}\pi_{\mu_3\nu_3} \\
\mathcal{L}_{(5,0)} &= \epsilon^{\mu_1\mu_2\mu_3\mu_4}\epsilon^{\nu_1\nu_2\nu_3\nu_4}\pi_{\mu_1}\pi_{\nu_1}\pi_{\mu_2\nu_2}\pi_{\mu_3\nu_3}\pi_{\mu_4\nu_4}
\end{aligned}
$$

This leads to (exactly) second order field equations

Having the « Galilean » symmetry

$$\pi \longrightarrow \pi + C + D_\mu x^\mu$$

Indeed, consider e.g.

$$\begin{aligned}
\mathcal{L}_{(4,0)} &= \epsilon^{\mu_1\mu_2\mu_3\lambda_1}\,\epsilon^{\nu_1\nu_2\nu_3}{}_{\lambda_1}\,\pi_{\mu_1}\,\pi_{\nu_1}\,\pi_{\mu_2\nu_2}\,\pi_{\mu_3\nu_3} \\
&\propto \left(\Box\pi\right)^2\left(\pi_\mu\,\pi^\mu\right) - 2\left(\Box\pi\right)\left(\pi_\mu\,\pi^{\mu\nu}\,\pi_\nu\right) \\
&\quad - \left(\pi_{\mu\nu}\,\pi^{\mu\nu}\right)\left(\pi_\rho\,\pi^\rho\right) + 2\left(\pi_\mu\pi^{\mu\nu}\,\pi_{\nu\rho}\,\pi^\rho\right)
\end{aligned}$$

Indeed, consider e.g.

$$\mathcal{L}_{(4,0)} = \epsilon^{\mu_1\mu_2\mu_3\lambda_1}\epsilon^{\nu_1\nu_2\nu_3}{}_{\lambda_1}\pi_{\mu_1}\pi_{\nu_1}\pi_{\mu_2\nu_2}\pi_{\mu_3\nu_3}$$

Varying this Lagrangian with respect to π yields (after integrating by part)

$$\delta\mathcal{L}_{(4,0)} \supset -\epsilon^{\mu_1\mu_2\mu_3\lambda_1}\epsilon^{\nu_1\nu_2\nu_3}{}_{\lambda_1}\delta\pi\partial_{\mu_1}\left\{\pi_{\nu_1}\pi_{\mu_2\nu_2}\pi_{\mu_3\nu_3}\right\}$$

Second order
derivative

Indeed, consider e.g.

$$\mathcal{L}_{(4,0)} = \epsilon^{\mu_1\mu_2\mu_3\lambda_1}\epsilon^{\nu_1\nu_2\nu_3}{}_{\lambda_1}\pi_{\mu_1}\pi_{\nu_1}\pi_{\mu_2\nu_2}\pi_{\mu_3\nu_3}$$

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Third order derivative…

… killed by the contraction with epsilon tensor

Indeed, consider e.g.

$$\mathcal{L}_{(4,0)} = \epsilon^{\mu_1\mu_2\mu_3\lambda_1}\epsilon^{\nu_1\nu_2\nu_3}{}_{\lambda_1}\pi_{\mu_1}\pi_{\nu_1}\pi_{\mu_2\nu_2}\pi_{\mu_3\nu_3}$$

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Similarly, one also have in the field equations

$$\delta\mathcal{L}_{(4,0)} \supset \epsilon^{\mu_1\mu_2\mu_3\lambda_1}\epsilon^{\nu_1\nu_2\nu_3}{}_{\lambda_1}\delta\pi\partial_{\nu_2}\partial_{\mu_2}\left\{\pi_{\mu_1}\pi_{\nu_1}\pi_{\mu_3\nu_3}\right\}$$

Yields third and fourth
order derivative…
killed by the epsilon tensor

Indeed, consider e.g.

$$\mathcal{L}_{(4,0)} = \epsilon^{\mu_1\mu_2\mu_3\lambda_1}\epsilon^{\nu_1\nu_2\nu_3}{}_{\lambda_1}\pi_{\mu_1}\pi_{\nu_1}\pi_{\mu_2\nu_2}\pi_{\mu_3\nu_3}$$

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Similarly, one also have in the field equations

$$\delta\mathcal{L}_{(4,0)} \supset \epsilon^{\mu_1\mu_2\mu_3\lambda_1}\epsilon^{\nu_1\nu_2\nu_3}{}_{\lambda_1}\delta\pi\partial_{\nu_2}\partial_{\mu_2}\left\{\pi_{\mu_1}\pi_{\nu_1}\pi_{\mu_3\nu_3}\right\}$$

Hence the field equations are proportional to

$$\mathcal{E}_{(4,0)} = \epsilon^{\mu_1\mu_2\mu_3\lambda_1}\epsilon^{\nu_1\nu_2\nu_3}{}_{\lambda_1}\pi_{\mu_1\nu_1}\pi_{\mu_2\nu_2}\pi_{\mu_3\nu_3}$$

Which does only contain second derivatives

NB: the field equations are linear in time derivatives (Cauchy problem ?)

This can easily be generalized to an arbitrary number of dimensions

And …. (not so easily) to curved space-time ….
this requires a non trivial non minimal coupling to curvature

C.D., G. Esposito-Farese, A. Vikman PRD 79 (2009) 084003
C.D., S.Deser, G.Esposito-Farese, PRD 82 (2010) 061501, PRD 80 (2009) 064015

E.g. Adding to

$$\mathcal{L}_{(4,0)} = (\Box\pi)^2 (\pi_\mu \pi^\mu) - 2 (\Box\pi) (\pi_\mu \pi^{\mu\nu} \pi_\nu) - (\pi_{\mu\nu}\pi^{\mu\nu}) (\pi_\rho \pi^\rho) + 2 (\pi_\mu \pi^{\mu\nu} \pi_{\nu\rho} \pi^\rho)$$

The Lagrangian

$$\mathcal{L}_{(4,1)} = (\pi_\lambda \pi^\lambda)\, \pi_\mu \left[R^{\mu\nu} - \frac{1}{2} g^{\mu\nu} R \right] \pi_\nu$$

Yields second order field equations for the scalar and the metric
(but loss of the « Galilean » symmetry)

1.2. Further generalization: from k-essence to generalized Galileons

C.D. Xian Gao, Daniele Steer, George Zahariade
arXiv:1103.3260 [hep-th] (PRD)

What is the most general **scalar theory** which has (not necessarily exactly) second order field equations in **flat space** ?

Specifically we looked for the most general scalar theory such that (in flat space-time)

i/ Its **Lagrangian contains derivatives of order 2** or less of the scalar field π

ii/ Its **Lagrangian is polynomial in second derivatives of π** (can be relaxed: Padilla, Sivanesan; Sivanesan)

iii/ The **field equations are of order 2 or lower** in derivatives

(NB: those hypothesis cover k-essence, simple Galileons ,…)

Answer: the most general such theory is given by a linear combination of the Lagrangians $\mathcal{L}_n\{f\}$

Free function of π and X

defined by
$$\mathcal{L}_n\{f\} = f(\pi, X) \times \mathcal{L}^{\mathrm{Gal},3}_{N=n+2},$$
$$= f(\pi, X) \times \left(X \mathcal{A}_{(2n)}^{\mu_1 \ldots \mu_n \nu_1 \ldots \nu_n} \pi_{\mu_1 \nu_1} \ldots \pi_{\mu_n \nu_n} \right)$$

where $X \equiv \pi_\mu \pi^\mu$

Which can also be « covariantized » using the previous technique …

Doing so, one ends up on a previously known set of theories … « Horndeski theories » (1972) … most general scalar tensor (ST) theories with second order field equations

2. What are Galileons useful for ?

2. 1 Vainshtein mechanism and k-mouflage

A (new) way to hide the scalar of a scalar-tensor (S-T) theory

$$S = M_P^2 \int d^4x \sqrt{-g} \left(\underbrace{\frac{R}{2} + \frac{\gamma}{2} m^2 \phi R}_{\text{Standard piece (S-T) in the Jordan frame}} + m^2 H(\phi) \right) + \underbrace{S_m}_{\text{Matter is minimally coupled to the metric } g_{\mu\nu}}$$

Derivative self-interactions

$$H(\phi)_{DGP} = m^2 \Box\phi \, \phi_{;\mu}\phi^{;\mu}$$

$$H(\phi)_{Gal} = m^2 \left(\phi_{;\lambda}\,\phi^{;\lambda}\right) \left[2\left(\Box\phi\right)^2 - 2\left(\phi_{;\mu\nu}\,\phi^{;\mu\nu}\right)\right]$$

$$H(\phi)_{CovGal} = m^2 \left(\phi_{;\lambda}\,\phi^{;\lambda}\right) \left[2\left(\Box\phi\right)^2 - 2\left(\phi_{;\mu\nu}\,\phi^{;\mu\nu}\right) - \frac{1}{2}\left(\phi_{;\mu}\,\phi^{;\mu}\right) R\right]$$

$\vdots$

$$S = M_P^2 \int d^4x\sqrt{-g}\left(\frac{R}{2}+\frac{\gamma}{2}m^2\phi R + m^2 H(\phi)\right) + S_m$$

Generates O(1) correction to GR by a scalar exchange

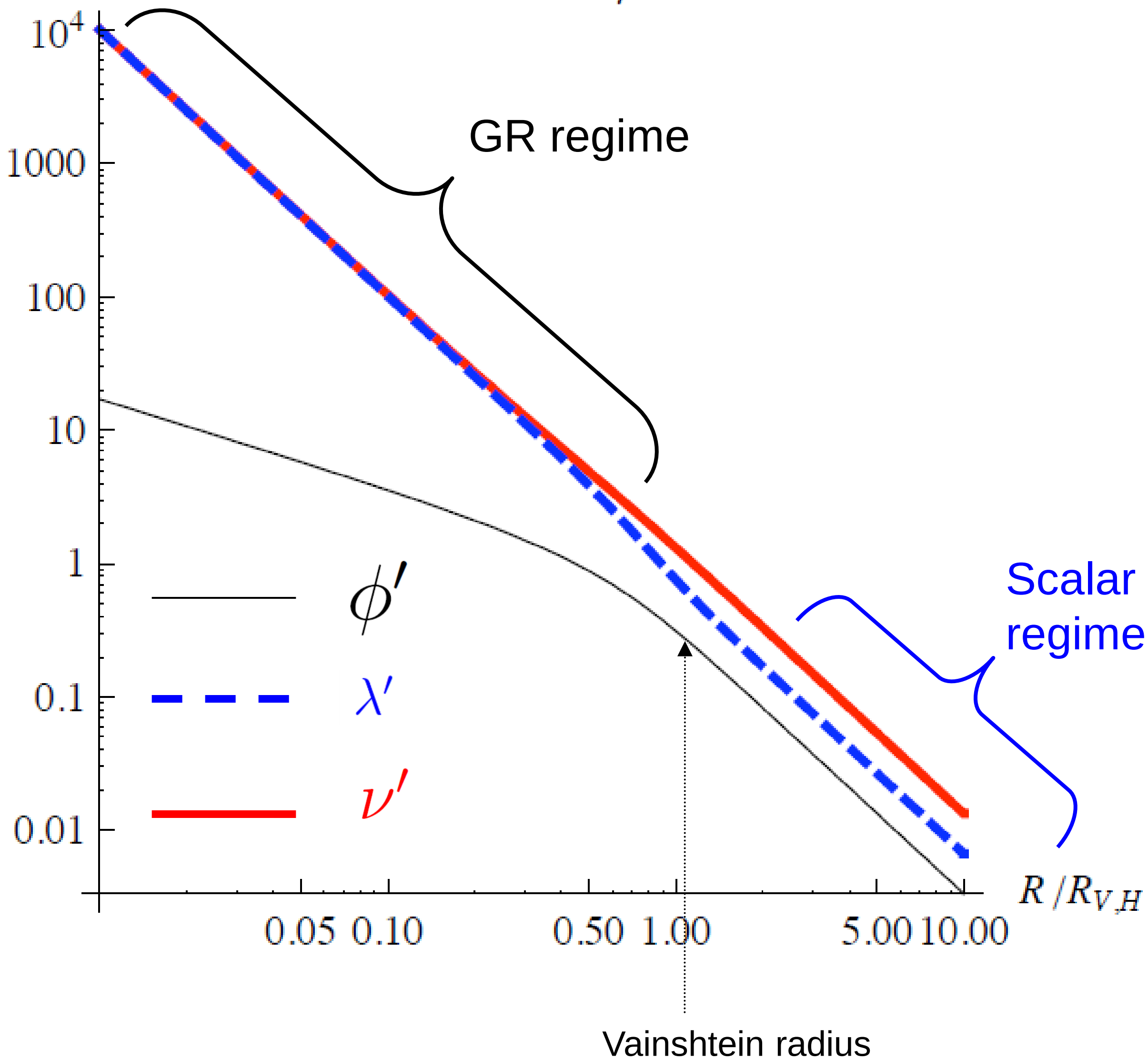
Standard piece (S-T) in the Jordan frame

Matter is minimally coupled to the metric $g_{\mu\nu}$

The derivative self- interactions can screen the effect of the scalar at distances below the « Vainshtein Radius » R_V
(Vainshtein mechanism or « k-Mouflage »)

E.g. for a static and spherically symmetric solution

$$g_{\mu\nu}dx^\mu dx^\nu = -e^{\nu(R)}dt^2 + e^{\lambda(R)}dR^2 + R^2 d\Omega^2$$



Babichev, C.D., Ziour 2009

This can be used to screen other interactions than GR

E.g. : A simple (well maybe not so !) model for MOND

Babichev, C.D., Esposito-Farese 1106.2538 (PRD)

MOND can be obtained by considering a scalar with the non standard kinetic term

$$\mathcal{L}_{\mathrm{MOND}} = -\frac{c^2}{3a_0} s\sqrt{|s|} \quad \text{with} \quad s \equiv g^{\mu\nu}\varphi_{,\mu}\varphi_{,\nu}$$

And matter coupled « disformally » to the metric

$$\tilde{g}_{\mu\nu} \approx e^{2\varphi} g_{\mu\nu} + B(\varphi, s)\varphi_{,\mu}\varphi_{,\nu}$$

(using some appropriately chosen function *B*)

Or $$\tilde{g}_{\mu\nu} \equiv e^{-2\varphi} g_{\mu\nu} - 2\sinh(2\varphi) U_\mu U_\nu$$

(using some time like vector field U)

The recovery of GR at « small distances » (rather large accelerations) requires usually the introduction of a very tuned « interpolating function » (as well as difficulties with the vector field)

The screening of MOND effects at small distances can be rather obtained by a suitable k-Mouflage

$$S = \frac{c^3}{4\pi G}\int d^4x\sqrt{-g}\left(\frac{R}{4} + \mathcal{L}_{\text{standard}} + \mathcal{L}_{\text{MOND}} + \mathcal{L}_{\text{Galileon}}\right) + S_{\text{matter}}\left[\psi_{\text{matter}}; \tilde{g}_{\mu\nu}\right],$$

« standard » MOND piece (TeVeS)

$$\begin{cases} \mathcal{L}_{\text{standard}} = -\frac{\epsilon}{2}\,s = -\frac{\epsilon}{2}(\partial_\lambda\varphi)^2, \\ \mathcal{L}_{\text{MOND}} = -\frac{c^2}{3a_0}\,s\sqrt{|s|}\,, \end{cases}$$

New ingredient :

$$\mathcal{L}_{\text{Galileon}} = -\frac{k}{3}\varepsilon^{\alpha\beta\gamma\delta}\varepsilon^{\mu\nu\rho\sigma}\varphi_{,\alpha}\varphi_{,\mu}\varphi_{;\beta\nu}R_{\gamma\delta\rho\sigma}$$

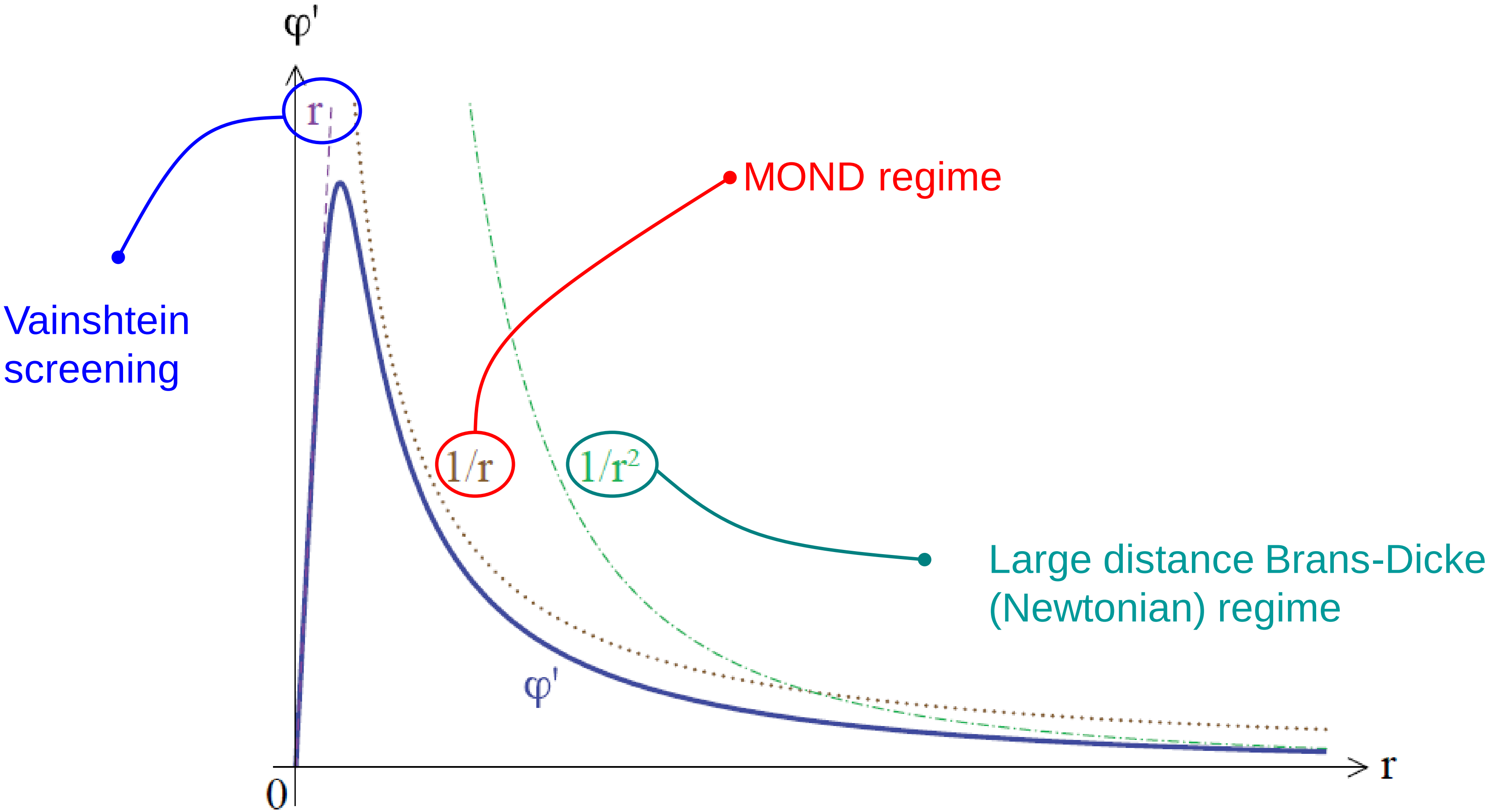
$$\mathcal{L}_{\text{Galileon}} = -\frac{k}{3}\varepsilon^{\alpha\beta\gamma\delta}\varepsilon^{\mu\nu\rho\sigma}\varphi_{,\alpha}\varphi_{,\mu}\varphi_{;\beta\nu}R_{\gamma\delta\rho\sigma}$$

- Covariant version of a « generalized » Galileon
- This simple Lagrangian has second order e.o.m.

Note that other terms also provide (not quite as) efficient screenings , such as the covariant $\mathcal{L}_5$ given by

$$\varepsilon^{\alpha\beta\gamma\delta}\varepsilon^{\mu\nu\rho\sigma}\varphi_{,\alpha}\varphi_{,\mu}\varphi_{;\beta\nu}\left[\varphi_{;\gamma\rho}\varphi_{;\delta\sigma} - \frac{3}{4}(\varphi_{,\lambda})^2 R_{\gamma\delta\rho\sigma}\right]$$

This yields the following profile for φ’



2. 2 Self acceleration, homogeneous cosmology

Consider a Scalar Tensor theory in the Einstein frame, Matter is coupled to the metric $\tilde{g}_{\mu\nu} = \mathcal{A}^2(\varphi) g_{\mu\nu}$ where $g_{\mu\nu}$ has a standard Einstein-Hilbert action.

Expanding $\tilde{g}_{\mu\nu}$ around a flat space time as $\tilde{g}_{\mu\nu} \sim \eta_{\mu\nu}\left[1 + \pi\left(x^\rho\right)\right]$

De Sitter space-time can be defined locally as an expansion around Minkowski of the form

$$d\tilde{s}^2 = \tilde{g}_{\mu\nu} dx^\mu dx^\nu \sim (1 + \underbrace{H^2 x^\rho x_\rho}_{\pi\left(x^\rho\right)} + \cdots)\, \eta_{\mu\nu} dx^\mu dx^\nu$$

Quadratic form of the coordinates …

… and one of the original motivations for the Galileons

That there is such a solution in vacuum (self- acceleration) will be garanteed if the field equations are of the type

$$\Box\pi - \frac{1}{3\Lambda^3} \underbrace{\left[\left(\Box\pi\right)^2 - \pi_{;\mu\nu}\pi^{;\mu\nu}\right]}_{\text{Or, any pure second order operator}} = \frac{T}{3M_P}$$

Hence, a linear combination of the Galileons

$$
\begin{aligned}
\mathcal{L}_{(2,0)} &= \pi_\mu \pi^\mu \\
\mathcal{L}_{(3,0)} &= \pi^\mu \pi_\mu \Box \pi \\
\mathcal{L}_{(4,0)} &= (\Box\pi)^2 \left(\pi_\mu\, \pi^\mu\right) - 2\left(\Box\pi\right)\left(\pi_\mu\, \pi^{\mu\nu}\, \pi_\nu\right) \\
&\quad - \left(\pi_{\mu\nu}\, \pi^{\mu\nu}\right)\left(\pi_\rho\, \pi^\rho\right) + 2\left(\pi_\mu \pi^{\mu\nu}\, \pi_{\nu\rho}\, \pi^\rho\right) \\
\mathcal{L}_{(5,0)} &= (\Box\pi)^3 \left(\pi_\mu\, \pi^\mu\right) - 3\,(\Box\pi)^2 \left(\pi_\mu\, \pi^{\mu\nu}\, \pi_\nu\right) - 3\left(\Box\pi\right)\left(\pi_{\mu\nu}\, \pi^{\mu\nu}\right)\left(\pi_\rho\, \pi^\rho\right) \\
&\quad + 6\left(\Box\pi\right)\left(\pi_\mu \pi^{\mu\nu}\, \pi_{\nu\rho}\, \pi^\rho\right) + 2\left(\pi_\mu{}^\nu\, \pi_\nu{}^\rho\, \pi_\rho{}^\mu\right)\left(\pi_\lambda\, \pi^\lambda\right) \\
&\quad + 3\left(\pi_{\mu\nu}\, \pi^{\mu\nu}\right)\left(\pi_\rho\, \pi^{\rho\lambda}\, \pi_\lambda\right) - 6\left(\pi_\mu\, \pi^{\mu\nu}\, \pi_{\nu\rho}\, \pi^{\rho\lambda}\, \pi_\lambda\right)
\end{aligned}
$$

should yield branches of self-accelerating solutions

Nicolis, Rattazzi, Trincherini

Many works have studied application to late and early cosmology, where the Galileon drives the cosmological expansion

Chow, Khoury 0905.1325; de Rahm, Heisenberg 1106.3312, de Rahm, Tolley; 1003.5917; Creminelli, Nicolis, Trincherini, 1007.0027; Padilla, Saffin, Zhou 1007.5424; C.D., Pujolas, Sawicki, Vikman, 1008.0048; Hinterbichler, Trodden, Wesley; 1008.1305; Mizuno, Koyama, 1009.0677; Kobayashi, Yamaguchi, Yokoyama, 1105.5723; Charmousis, Copeland, Padilla, Saffin, 1106.2000; Perreault Levasseur, Brandenberger, David, 1105.5649; Renaux-Petel, Mizuno, Koyama, 1108.0305; Gao, Steer, 1107.2642;…

Note that it might not be so easy to screen « à la Vainshtein » an (interestingly cosmologically) evolving Galileon like scalar

One ends up generically with $\left|\dot{G}/G\right| \approx 2\alpha\dot{\varphi}_{\mathrm{cosm}}(t)$

with $\begin{cases} \dot{\varphi}_{cosm}(t_0) \sim \alpha H_0 \\ \dot{\varphi}_{cosm}(t_0) \sim H_0 \end{cases}$

However, the most stringent bound on $\left|\dot{G}/G\right|$ is

$$|\dot{G}/G| < 1.3 \times 10^{-12}\,\mathrm{yr}^{-1} \quad \Leftrightarrow \quad |\dot{G}/G| < 0.02 H_0$$

Incompatible with a gravitationnally coupled φ

3. Some recent developments

3.1. Generalization to p-forms

C.D., S.Deser, G.Esposito-Farese,
arXiv 1007.5278 [gr-qc] (PRD)

E.g. consider

$$I = \int d^D x \, \varepsilon^{\mu\nu\dots} \varepsilon^{\alpha\beta\dots} \, \omega_{\mu\nu\dots} \omega_{\alpha\beta\dots} (\partial_\rho \omega_{\gamma\delta\dots} \, \dots)(\partial_\epsilon \omega_{\sigma\tau\dots} \, \dots)$$

With $A_{\mu\nu\dots}$ a *p*-form of field strength $\omega_{\lambda\mu\nu\dots} = \partial_{[\lambda} A_{\mu\nu\dots]}$

In the field equations, Bianchi identities annihilate any $\partial_\mu \partial_{[\alpha} \omega_{\beta\gamma\dots]}$

E.o.m. are (purely) second order

E.g. for a 2-form

$$I = \int d^7 x \, \varepsilon^{\mu\nu\rho\sigma\tau\varphi\chi} \varepsilon^{\alpha\beta\gamma\delta\epsilon\zeta\eta} \, \omega_{\mu\nu\rho} \, \omega_{\alpha\beta\gamma} \, \partial_\sigma \omega_{\delta\epsilon\zeta} \, \partial_\eta \omega_{\tau\varphi\chi}$$

Note that one must go to 7 dimensions (in general one has $D \geq 2p+3$)
and that this construction does not work for odd p as we show now

For odd p the previous construction does not work

Indeed, the action

$$I = \int d^D x\, \varepsilon^{\mu\nu\dots} \varepsilon^{\alpha\beta\dots}\, \omega_{\mu\nu\dots}\omega_{\alpha\beta\dots}(\partial_\rho \omega_{\gamma\delta\dots}\ \dots)(\partial_\epsilon \omega_{\sigma\tau\dots}\ \dots)$$

With $A_{\mu\nu\dots}$ an (odd *p)*-form of field strength $\omega_{\lambda\mu\nu\dots} = \partial_{[\lambda} A_{\mu\nu\dots]}$

Yields vanishing e.o.m. (the action is a total derivative)

Integration by part

$$I = -\int d^D x\, \varepsilon^{\mu\nu\dots} \varepsilon^{\alpha\beta\dots}\, \omega_{\mu\nu\dots}\partial_\rho(\omega_{\alpha\beta\dots})\omega_{\gamma\delta\dots}\ \dots(\partial_\epsilon \omega_{\sigma\tau\dots}\ \dots)$$

Renumbering of an even (for odd p) number of indices

$$I = -\int d^D x\, \varepsilon^{\mu\nu\dots} \varepsilon^{\alpha\beta\dots}\, \omega_{\mu\nu\dots}\partial_\rho(\omega_{\gamma\delta\dots})\omega_{\alpha\beta\dots}\ \dots(\partial_\epsilon \omega_{\sigma\tau\dots}\ \dots)$$

Is there an (odd p) Galileon ?

Is there an (odd p) Galileon ?

C. D. , A. E. Gumrukcuoglu, S. Mukohyama and Y.Wang, [arXiv:1312.6690 [hep-th]], JHEP 2014.

No « vector » Galileon

Is there an (odd p) Galileon ?

C.D., S. Mukohyama, V. Sivanesan
[arXiv:1601.01287[hep-th]], PRD 2016.

C.D., S. Mukohyama, V. Sivanesan
In preparation

$p > 1$?

We started from the field equations

$$\mathcal{E}^A \equiv \frac{\delta \mathcal{S}}{\delta \mathcal{A}_A} = 0 \qquad \text{with} \qquad \mathcal{E}^A = \mathcal{E}^A(\mathcal{A}_B; \mathcal{A}_{B,a}; \mathcal{A}_{B,ab})$$

For a p-form $\boldsymbol{\mathcal{A}} \in \bigwedge^{p}$ with components $\mathcal{A}_{a[p]}$

with $a[p] \equiv A$ p antisymmetric indices

We ask these field equations

(i) To derive from an action

$$\mathcal{S} = \int d^D x \ \mathcal{L}[\mathcal{A}_B; \partial_a \mathcal{A}_B; \partial_a \ldots \partial_b \mathcal{A}_B]$$

(ii) To depend only on second derivatives

(iii) to be gauge invariant

$$\boldsymbol{\mathcal{A}} \rightarrow \boldsymbol{\mathcal{A}} + d\boldsymbol{\mathcal{C}} \equiv \boldsymbol{\mathcal{A}}' \qquad \boldsymbol{\mathcal{C}} \in \bigwedge^{p-1}$$

Yes !

E.g. a 3-form in D= 9 dimensions

$$\int d^{\,9}x \epsilon^{a_1 a_2 \cdots} \, \epsilon^{b_1 b_2 \cdots} \, A_{a_1 a_2 b_1 , a_3} \, A_{b_2 b_3 a_4 , b_4} \, \partial_a \omega_B \, \partial_b \omega_A$$

NB: gauge invariance « à la Chern-Simons »

To be contrasted with the p-form action constructed in

C.D., S.Deser, G.Esposito-Farese,
arXiv 1007.5278 [gr-qc] (PRD)

$$\int d^{\,9}x \epsilon^{a_1 a_2 \cdots} \, \epsilon^{b_1 b_2 \cdots} \, A_{a_1 a_2 a_3 , a_4} \, A_{b_1 b_2 b_3 , b_4} \, \partial_a \omega_B \, \partial_b \omega_A$$

This can be generalized …..

…… classification on the way

3.2. D.o.f. counting in Galileons and generalized Galileons theories

C.D., G. Esposito-Farèse, D. Steer, arXiv:1506.01974 [gr-qc], PRD

Horndeski-like theories: Cauchy problem ? Numerical studies (e.g. adressing the Vainshtein mechanism in grav. Collapse) ?

No Hamiltonian analysis so far !

Horndeski-like theories: Scalar tensor theories with second order field equations + diffeo invariance

A priori 2 (graviton) + 1 (scalar) d.o.f.

Claimed to be true in an even larger set of theories (« Beyond Horndeski » theories) !

J. Gleyzes, D. Langlois, F. Piazza, and F. Vernizzi, **(GLPV)** arXiv**:**1404.6495, arXiv:1408.1952

Our works aimed at

Provides a first step toward a proper Hamiltonian treatment of Horndeski-like and beyond Horndeski theories

Rexamine the GLPV claim
(arguments of GLPV being not convincing to us)

Our works aimed at

- Provides a first step toward a proper Hamiltonian treatment of Horndeski-like and beyond Horndeski theories
- Rexamine the GLPV claim (arguments of GLPV being not convincing to us)

Along two directions

- Show how in a (large) set of beyond Horndeski theories (matching the one considered by GLPV), the order in time derivatives of the field equations can be reduced **(not presented here).**
- Analyze in details via Hamiltonianian formalism a simple (the simplest ?) non trivial beyond Horndeski theory

Hamiltonian analysis of the quartic Galileon

Consider $$S = \int d^4x \sqrt{-g}\left[R + \mathcal{L}_{(4,0)}\right]$$

With
$$\left\{\begin{aligned} \mathcal{L}_{(4,0)} &= (\Box\pi)^2 (\pi_\mu \pi^\mu) - 2(\Box\pi)(\pi_\mu \pi^{\mu\nu}\pi_\nu) \\ &\quad - (\pi_{\mu\nu}\pi^{\mu\nu})(\pi_\rho \pi^\rho) + 2(\pi_\mu \pi^{\mu\nu}\pi_{\nu\rho}\pi^\rho) \\ &= \epsilon^{\mu_1\mu_3\mu_5\nu_1}\epsilon^{\mu_2\mu_4\mu_6}{}_{\nu_1}\pi_{\mu_1}\pi_{\mu_2}\pi_{\mu_3\mu_4}\pi_{\mu_5\mu_6} \end{aligned}\right.$$

In the ADM parametrization, the action *S* becomes (in an arbitrary gauge)

$$\begin{aligned} S = & \int dt d^3x\, N\sqrt{\gamma}(K_{ij}K^{ij} - K^2 + {}^{(3)}R) \\ & + \int dt d^3x\, \frac{\sqrt{\gamma}}{N}\epsilon^{ijk}\epsilon^{\ell m}{}_k\big[-\dot{\pi}^2 s_{i\ell}s_{jm} - 2\pi_i\pi_\ell s_{00}s_{jm} + 2\pi_i\pi_\ell s_{0m}s_{0j} + 4\dot{\pi}\pi_\ell s_{i0}s_{jm}\big] \\ & + \int dt d^3x\, \frac{\sqrt{\gamma}}{N}\epsilon^{ijk}\epsilon^{\ell mn}N_k\big[2\dot{\pi}\pi_\ell s_{im}s_{jn} - 4\pi_i\pi_\ell s_{0m}s_{jn}\big] \\ & + \int dt d^3x\, N\sqrt{\gamma}\left(1 - \frac{N_pN^p}{N^2}\right)\epsilon^{ijk}\epsilon^{\ell mn}s_{jm}s_{kn}\pi_i\pi_\ell \end{aligned}$$

Where $s_{\mu\nu} \equiv \nabla_\mu\nabla_\nu\pi$

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- Generates third order time derivatives
- Absent in the unitary gauge (used by GLPV)

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- Generates third order time derivatives
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but S depends on $\dot{N}$, $\dot{N}^i$ and non linearly on second derivatives of π

More convenient to work with

$$\tilde{S} = S + \int d^4x \, \tilde{\lambda}^{\mu\nu} \left(s_{\mu\nu} - \nabla_\mu \nabla_\nu \pi \right)$$

31 canonical (Lagrangian) fields $N \, , N^i \, , \gamma_{ij} \, , \pi , \lambda_{\mu\nu} \, , s_{\mu\nu}$

(where $\lambda^{\mu\nu} = N\sqrt{-\gamma}\tilde{\lambda}^{\mu\nu}$)

- 23 primary constraints
- 23 secondary constraint
- At least 8 of them are first class

At most $62 - (2 \times 8) - (46 - 8) = 8$ Hamiltonian d.o.f.

Further analysis shows that there exist a tertiary (and likely also a quaternary) second class constraint, hence less than 8 d.o.f.

Conclusions

Scalar Galileons

Lead to a (re)discovery of a whole family of scalar-tensor theories with various interesting theoretical and phenomenological aspects:

- Vainshtein mechanism and k-mouflaging
- Self acceleration (and self tuning ?)
- Application to early cosmology
  (e.g. « Galilean genesis » thanks to stable NEC violation)
- Uniqueness and non renormalization theorems
- Links with massive gravity, classicalization

⋮ ⋮

Can be generalized e.g. to p-forms or « beyond Horndeski » theories…

Several aspects still needed to be explored / understood / cured ?

(phenomenology, $\dot{G}$,UV completion, superluminal propagation, duality …)

Thank you for your attention !