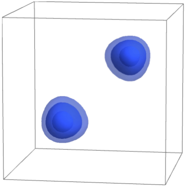
Dark matter boson star collisions

based on [arXiv:1608.00547]

E. Cotner[1]

[1] Department of Physics and Astronomy
University of California, Los Angeles

PACIFIC 2016

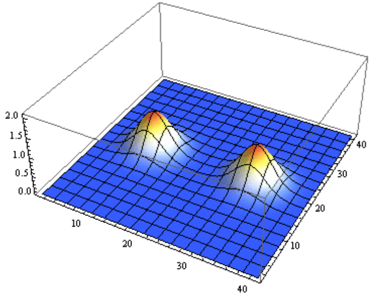


Outline

1. What's a boson star?
2. Motivation for BS dark matter
   - Matches observations
   - Resolves outstanding problems with CDM
3. Boson star properties
   - Radius
   - Maximum mass
4. Effective potential analysis
   - Motivation/derivation
   - Results/predictions
5. Numerical results
   - Effect of kinetic energy
   - Repulsive collisions
   - Tidal effects

What's a boson star?

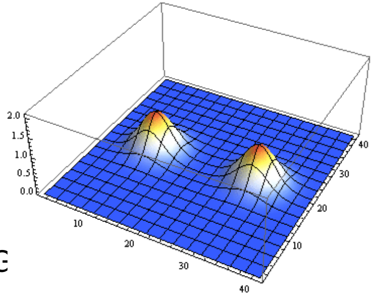
- Classical scalar field (either real or complex) bound by gravity



What's a boson star?

- Classical scalar field (either real or complex) bound by gravity
- Scalar field is a Bose-Einstein condensate, needs to be sufficiently "cold":

$$kT < \frac{2\pi}{m}\left(\frac{n}{\zeta(3/2)}\right)^{2/3} \approx$$
$$578\left(\frac{10^{-9}\text{ eV}}{m}\right)^{5/3}\left(\frac{\rho}{0.3\text{ GeV/cm}^3}\right)^{2/3}\ \text{G}$$

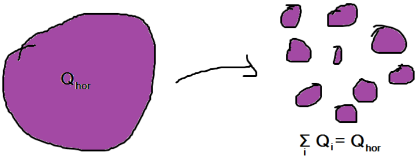


What’s a boson star?

- Subclassification based on form of scalar potential:
  - $V(\phi) = m^2|\phi|^2 \rightarrow$ “mini boson star"
  - $V(\phi) = m^2|\phi|^2 + \lambda|\phi|^4 \rightarrow$ “self-interacting boson star"
  - $V(\phi) - \omega^2|\phi|^2 < 0 \,|\, \exists\omega : 0 < \omega < m \rightarrow$ “soliton star" (Q-ball in absence of gravity)

What's a boson star?

- Subclassification based on form of scalar potential:
  - $V(\phi) = m^2|\phi|^2 \rightarrow$ "mini boson star"
  - $V(\phi) = m^2|\phi|^2 + \lambda|\phi|^4 \rightarrow$ "self-interacting boson star"
  - $V(\phi) - \omega^2|\phi|^2 < 0 \,|\, \exists\omega : 0 < \omega < m \rightarrow$ "soliton star" (Q-ball in absence of gravity)
- Could be formed from variety of processes such as fragmentation of a charged scalar condensate, standard growth of density perturbations in early universe



Equations of motion

- Start with scalar field coupled to gravity

$$S = \int d^4x\,\sqrt{-g}\left[\frac{1}{2\kappa}R + \nabla_\mu\varphi^\dagger\nabla^\mu\varphi - m^2|\varphi|^2 - \lambda|\varphi|^4\right]$$

Equations of motion

- Start with scalar field coupled to gravity

$$S = \int d^4x \sqrt{-g} \left[\frac{1}{2\kappa} R + \nabla_\mu \varphi^\dagger \nabla^\mu \varphi - m^2 |\varphi|^2 - \lambda |\varphi|^4 \right]$$

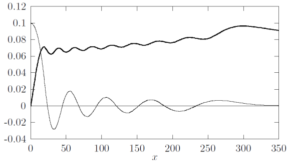
- Take non-relativistic limit and factor out harmonic time-dependence due to particle mass $\psi = \frac{1}{\sqrt{2m}} e^{-imt} \varphi$, results in equations of motion (Schrödinger-Poisson system):

$$i\dot{\psi} = -\frac{1}{2m} \nabla^2 \psi + \frac{\lambda}{8m^2} |\psi|^2 \psi + m\phi\psi$$
$$\nabla^2 \phi = 4\pi G m |\psi|^2$$

Outline

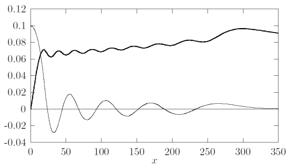
Rotation curves

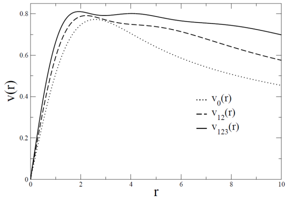
- Galactic-scale boson stars can provide rotation curves with correct long-distance behavior [Lee, Koh, arXiv:hep-ph/9507385]
  - Requires very small mass: $m \sim 10^{-23}$ eV



Rotation curves

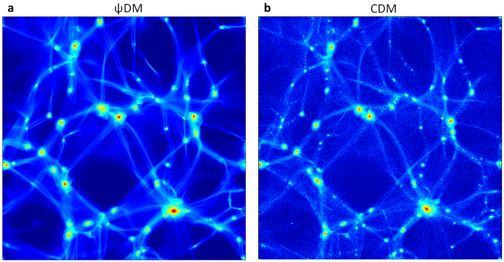
- Galactic-scale boson stars can provide rotation curves with correct long-distance behavior [Lee, Koh, arXiv:hep-ph/9507385]
  - Requires very small mass: $m \sim 10^{-23}$ eV
- Superpositions of multiply-excited states can provide even better fits [Ureña-Lopez, Bernal, arXiv:1008.1231]





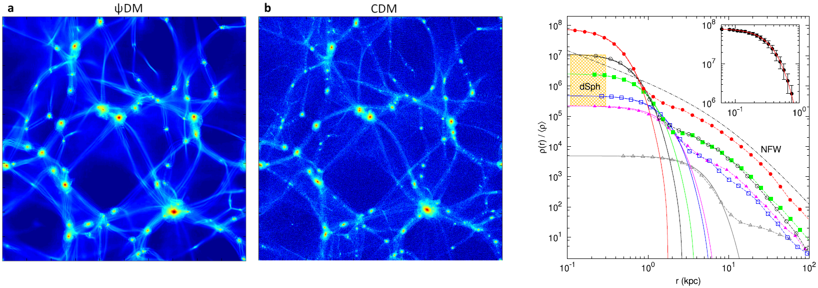
Structure formation

- Boson stars form same large-scale structure as ΛCDM [Schive, Chiueh, Broadhurst, arXiv:1406.6586]



Structure formation

- Boson stars form same large-scale structure as ΛCDM [Schive, Chiueh, Broadhurst, arXiv:1406.6586]
- Reproduces NFW-like density profile with cored center



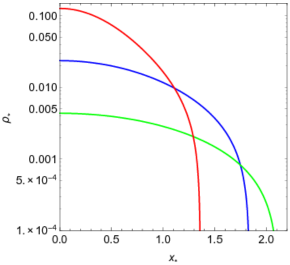
Outline

Cusp-core problem: resolution by galactic-scale BS

- Galactic scale boson stars naturally have non-singular density profiles due to uncertainty principle and wave function spreading

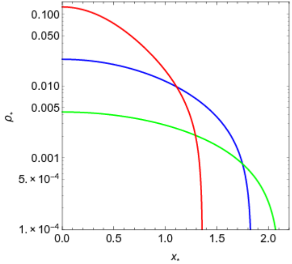
Cusp-core problem: resolution by galactic-scale BS

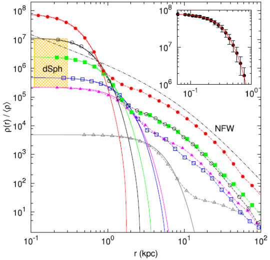
- Galactic scale boson stars naturally have non-singular density profiles due to uncertainty principle and wave function spreading
  - Single stars have core-like profile but sharp cutoff [Eby, Kouvaris, Nielsen, Wijewardhana, arXiv:1511.04474]



Cusp-core problem: resolution by galactic-scale BS

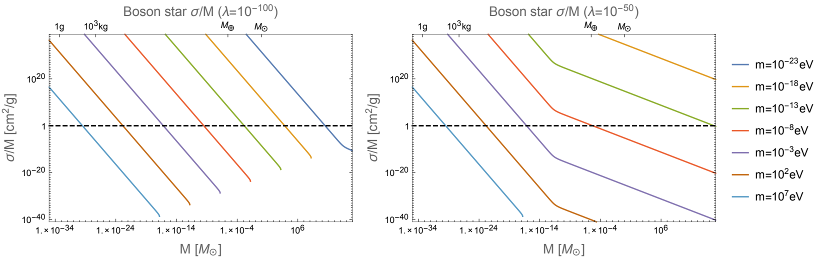
- Galactic scale boson stars naturally have non-singular density profiles due to uncertainty principle and wave function spreading
    - Single stars have core-like profile but sharp cutoff [Eby, Kouvaris, Nielsen, Wijewardhana, arXiv:1511.04474]
    - Realistic halos created as result of multiple mergers





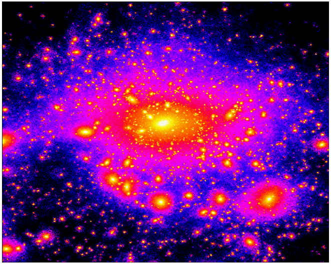
Cusp-core problem: resolution by SIDM-like BS

- Sub-galactic scale boson stars act like SIDM with a geometric cross section $\sigma \sim \pi R^2$ [EC, unpublished]



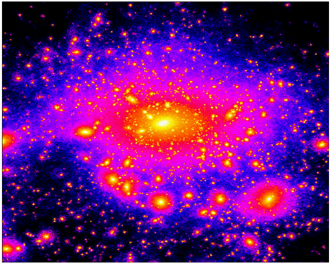
Missing satellite problem

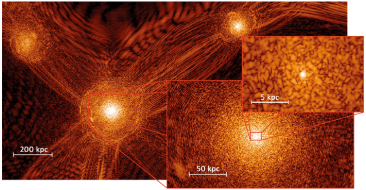
- Simulations of CDM vastly overpredict number of satellite galaxies



Missing satellite problem

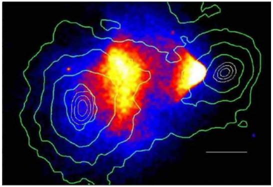
- Simulations of CDM vastly overpredict number of satellite galaxies
- Simulations of scalar dark matter predict small number of satellites
  - Instead have large amount of small density fluctuations





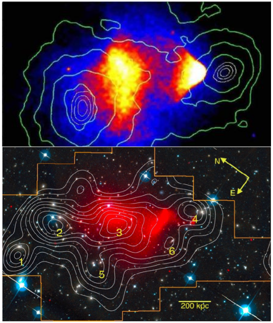
Apparent cluster merger contradictions?

- Bullet cluster shows halos pass right through each other



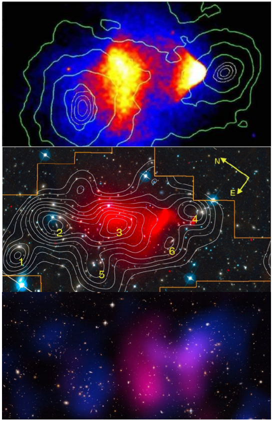
Apparent cluster merger contradictions?

- Bullet cluster shows halos pass right through each other
- Abell 520 shows repulsive or drag force during infall



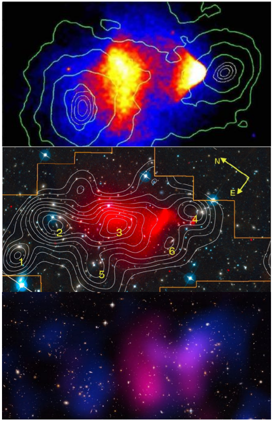
Apparent cluster merger contradictions?

- Bullet cluster shows halos pass right through each other
- Abell 520 shows repulsive or drag force during infall
- Musket ball cluster shows slowing of halo wrt baryons after first passing



Apparent cluster merger contradictions?

- Bullet cluster shows halos pass right through each other
- Abell 520 shows repulsive or drag force during infall
- Musket ball cluster shows slowing of halo wrt baryons after first passing
- Boson star dark matter halos can pass through, merge, or scatter inelastically, depending on kinetic energy and relative velocity



Outline

1. What's a boson star?
2. Motivation for BS dark matter
   - Matches observations
   - Resolves outstanding problems with CDM
3. Boson star properties
   - Radius
   - Maximum mass
4. Effective potential analysis
   - Motivation/derivation
   - Results/predictions
5. Numerical results
   - Effect of kinetic energy
   - Repulsive collisions
   - Tidal effects
   - Attractive instability

Variational method and characteristic radius

- Use the Green's function for the Poisson equation to solve for ϕ, then calculate expectation value of Hamiltonian

$$\langle H\rangle = \frac{1}{2m}\int d^3x\,|\nabla\psi|^2 + \frac{\lambda}{16m^2}\int d^3x\,|\psi|^4 - \frac{Gm^2}{2}\int d^3x\int d^3x'\,\frac{|\psi(\boldsymbol{x})|^2|\psi(\boldsymbol{x}')|^2}{|\boldsymbol{x}-\boldsymbol{x}'|}$$

Variational method and characteristic radius

- Use the Green's function for the Poisson equation to solve for ϕ, then calculate expectation value of Hamiltonian

$$\langle H\rangle = \frac{1}{2m}\int d^3x\,|\nabla\psi|^2 + \frac{\lambda}{16m^2}\int d^3x\,|\psi|^4 - \frac{Gm^2}{2}\int d^3x \int d^3x' \frac{|\psi(\boldsymbol{x})|^2|\psi(\boldsymbol{x}')|^2}{|\boldsymbol{x}-\boldsymbol{x}'|}$$

- Use variational method with Gaussian variational state $\psi \sim e^{-(r/R)^2}$ to find approximate ground state radius R

$$R = \frac{3\sqrt{\pi}}{2Gm^3N}\left(1+\sqrt{1+\xi}\right) \qquad \xi \equiv \frac{1}{12\pi^2}\lambda G m^2 N^2$$

- [Chavanis, arXiv:1103.2050],

Variational method and characteristic radius

- Use the Green's function for the Poisson equation to solve for ϕ, then calculate expectation value of Hamiltonian

$$\langle H\rangle = \frac{1}{2m}\int d^3x\,|\nabla\psi|^2 + \frac{\lambda}{16m^2}\int d^3x\,|\psi|^4 - \frac{Gm^2}{2}\int d^3x\int d^3x'\,\frac{|\psi(\boldsymbol{x})|^2|\psi(\boldsymbol{x}')|^2}{|\boldsymbol{x}-\boldsymbol{x}'|}$$

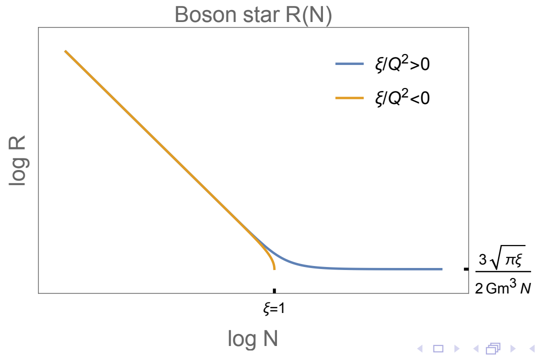
- Use variational method with Gaussian variational state $\psi \sim e^{-(r/R)^2}$ to find approximate ground state radius R

$$R = \frac{3\sqrt{\pi}}{2Gm^3N}\left(1+\sqrt{1+\xi}\right) \qquad \xi \equiv \frac{1}{12\pi^2}\lambda G m^2 N^2 - \frac{20}{9\pi}G^2 m^4 N^2$$

- [Chavanis, arXiv:1103.2050], [EC, unpublished]

Boson star properties: radius

$$R = \frac{3\sqrt{\pi}}{2Gm^3N}\left(1+\sqrt{1+\xi}\right) \qquad \xi \equiv \frac{1}{12\pi^2}\lambda G m^2 N^2 - \frac{20}{9\pi}G^2 m^4 N^2$$



Boson star properties: radius

- Results of variational method gives

$$R = \frac{3\sqrt{\pi}}{2Gm^3N}\left(1+\sqrt{1+\xi}\right) \qquad \xi \equiv \frac{\lambda G m^2 N^2}{12\pi^2}$$

Boson star properties: radius

- Results of variational method gives

$$R = \frac{3\sqrt{\pi}}{2Gm^3 N}\left(1+\sqrt{1+\xi}\right) \qquad \xi \equiv \frac{\lambda G m^2 N^2}{12\pi^2}$$

- Weak self-interaction limit ($|\xi| \ll 1$)

$$R \approx \frac{3\sqrt{\pi}}{Gm^2 M} = 0.88\left(\frac{m}{10^{-9}\ \mathrm{eV}}\right)^{-2}\left(\frac{M}{1\ \mathrm{M}_\odot}\right)^{-1}\ \mathrm{km}$$

$$= 120\left(\frac{m}{2\times 10^{-25}\ \mathrm{eV}}\right)^{-2}\left(\frac{M}{10^{12}\ \mathrm{M}_\odot}\right)^{-1}\ \mathrm{kpc}$$

Boson star properties: radius

- Strong self-interaction limit ($\xi \gg 1$)

$$R \approx \frac{1}{4m^2}\sqrt{\frac{3\lambda}{G}} = 103\left(\frac{m}{10^{-9}\ \mathrm{eV}}\right)^{-2}\left(\frac{\lambda}{10^{-35}}\right)^{1/2}\ \mathrm{kpc}$$

Boson star properties: radius

- Strong self-interaction limit ($\xi \gg 1$)

$$R \approx \frac{1}{4m^2}\sqrt{\frac{3\lambda}{G}} = 103\left(\frac{m}{10^{-9}\text{ eV}}\right)^{-2}\left(\frac{\lambda}{10^{-35}}\right)^{1/2}\text{ kpc}$$

- Can be strongly self-interacting even for extremely small values of the coupling

Boson star properties: radius

- Strong self-interaction limit ($\xi \gg 1$)

$$R \approx \frac{1}{4m^2}\sqrt{\frac{3\lambda}{G}} = 103 \left(\frac{m}{10^{-9}\ \mathrm{eV}}\right)^{-2} \left(\frac{\lambda}{10^{-35}}\right)^{1/2} \mathrm{kpc}$$

- Can be strongly self-interacting even for extremely small values of the coupling
- As $N \to \infty$, R approaches constant value

Outline

Boson star properties: maximum mass

- Maximum mass achieved when radius comparable to Schwarzschild radius: $R \approx 2GM$

Boson star properties: maximum mass

- Maximum mass achieved when radius comparable to Schwarzschild radius: $R \approx 2GM$

$$M_{\max} \sim \sqrt{\frac{3\pi^{1/2}}{2}\frac{M_p^2}{m}} \approx 10^{-1}\left(\frac{m}{10^{-9}\text{ eV}}\right)^{-1} \text{M}_{\odot} \qquad (|\xi| \ll 1)$$

Boson star properties: maximum mass

- Maximum mass achieved when radius comparable to Schwarzschild radius: $R \approx 2GM$

$$M_{\max} \sim \sqrt{\frac{3\pi^{1/2}}{2}\frac{M_p^2}{m}} \approx 10^{-1}\left(\frac{m}{10^{-9}\ \mathrm{eV}}\right)^{-1}\ \mathrm{M}_\odot \qquad (|\xi| \ll 1)$$

$$M_{\max} \sim \frac{M_p^3}{8m^2}\sqrt{\frac{3\lambda}{\pi}} \approx 10^{32}\left(\frac{\lambda}{10^{-6}}\right)^{1/2}\left(\frac{m}{10^{-9}\ \mathrm{eV}}\right)^{-2}\mathrm{M}_\odot \quad (\xi \gg 1)$$

Boson star properties: maximum mass

- Maximum mass achieved when radius comparable to Schwarzschild radius: $R \approx 2GM$

$$M_{\rm max} \sim \sqrt{\frac{3\pi^{1/2}}{2}\frac{M_p^2}{m}} \approx 10^{-1}\left(\frac{m}{10^{-9}\ \text{eV}}\right)^{-1}\ \text{M}_\odot \qquad (|\xi| \ll 1)$$

$$M_{\rm max} \sim \frac{M_p^3}{8m^2}\sqrt{\frac{3\lambda}{\pi}} \approx 10^{32}\left(\frac{\lambda}{10^{-6}}\right)^{1/2}\left(\frac{m}{10^{-9}\ \text{eV}}\right)^{-2}\text{M}_\odot \quad (\xi \gg 1)$$

$$M_{\rm max} \sim \frac{3\pi}{\sqrt{2G|\lambda|}} \approx 6.7\times 10^3\left(\frac{|\lambda|}{10^{-6}}\right)^{-1/2} M_p \qquad (\xi = -1)$$

Outline

Motivation for effective potential

- Current method of resolving scattering outcome is to perform numerical simulation
  - Time-consuming and computationally expensive
  - Numerical instability makes simulation of strongly self-interacting boson stars intractable

Motivation for effective potential

- Current method of resolving scattering outcome is to perform numerical simulation
  - Time-consuming and computationally expensive
  - Numerical instability makes simulation of strongly self-interacting boson stars intractable
- Advantages of effective potential:
  - Much less computationally expensive; most of the effective potential can be computed analytically
  - Idea can be generalized to include angular momentum, “electronic" excitations, etc. as extra degrees of freedom

Effective potential derivation

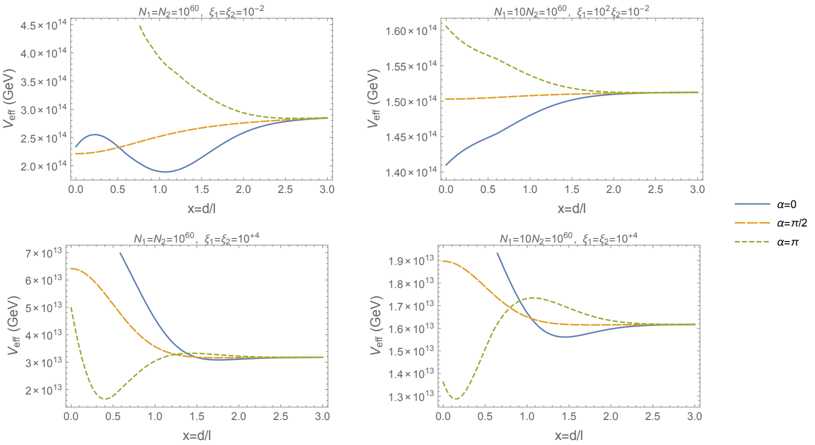
- Calculate expectation value of Hamiltonian in a state which is a superposition of two boson stars at rest, separated by vector $\boldsymbol{d}$:

$$|\Psi(\boldsymbol{r})\rangle = A\left[|\psi(\boldsymbol{r}-\boldsymbol{d}/2)\rangle + e^{i\alpha}\,|\psi(\boldsymbol{r}+\boldsymbol{d}/2)\rangle\right]$$

- Individual wave functions ψ are variational ground states derived earlier

Outline

Effective potential results [EC, arXiv:1608.00547]



Effective potential predictions I

- Weak-interaction regime:
  - Attractive/repulsive when in-phase/out-of-phase
  - Could pass through each other if kinetic energy is high enough, but difficult when $\alpha = \pi$

Effective potential predictions I

- Weak-interaction regime:
  - Attractive/repulsive when in-phase/out-of-phase
  - Could pass through each other if kinetic energy is high enough, but difficult when $\alpha = \pi$
- Strong-interaction regime:
  - Repulsive when in phase, only mildly attractive when out of phase

Effective potential predictions II

- Since phase difference is dynamical variable itself, we expect it to evolve
    - Initially out-of-phase configurations will rotate to a mutual value, then merge

Effective potential predictions II

- Since phase difference is dynamical variable itself, we expect it to evolve
  - Initially out-of-phase configurations will rotate to a mutual value, then merge
- Downsides: assumption that boson stars are rigid leads to mispredictions
  - Doesn't capture the effects of “friction" and excitation
  - Doesn't predict tidal effects in asymmetric-mass systems
  - Successful predictions are at best qualitative

Numerical simulation

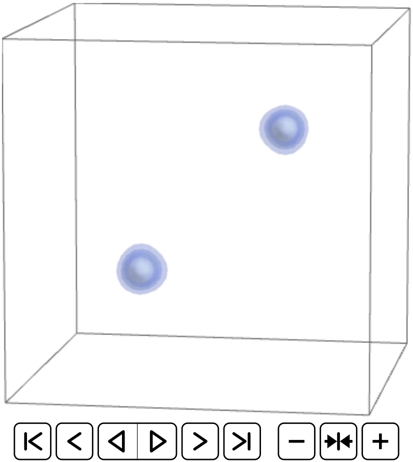
- To confirm predictions of effective potential, I ran a number of numerical simulations

Numerical simulation

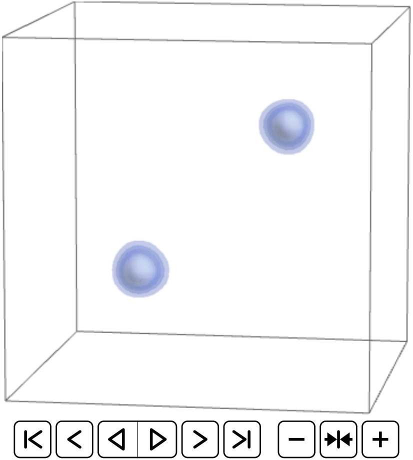
- To confirm predictions of effective potential, I ran a number of numerical simulations
- Numerical recipe:
  - Discretized Schrödinger-Poisson equations on a $50 \times 50 \times 50$ grid
  - Transform coordinates to bring spatial infinity to the boundary of the grid and impose Dirichlet conditions
  - Used first-order time, second-order space grid method in transformed coordinates
  - Initial states were superpositions of two boson stars

Outline

Effect of kinetic energy: low kinetic energy

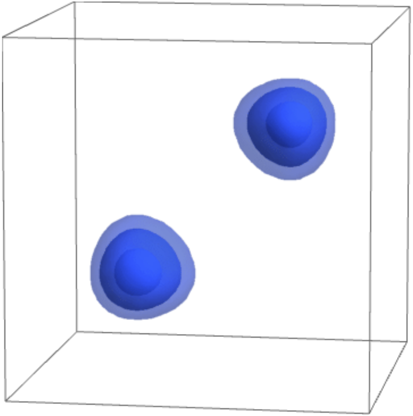


Effect of kinetic energy: high kinetic energy

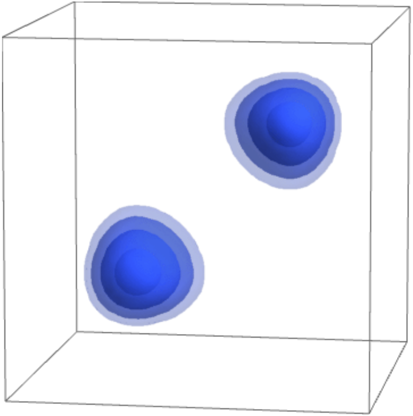


Outline

Repulsive collisions: Weak self-interaction ($\xi = 10^{-2}$), out of phase

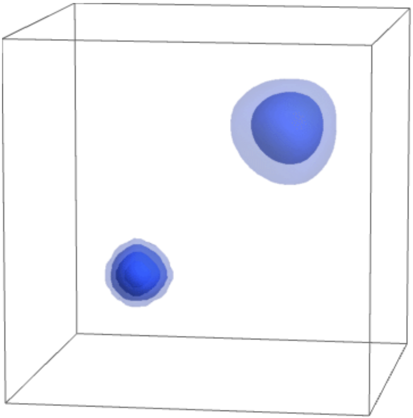


Repulsive collisions: Intermediate-strength ($\xi = 10$) self-interaction, out of phase

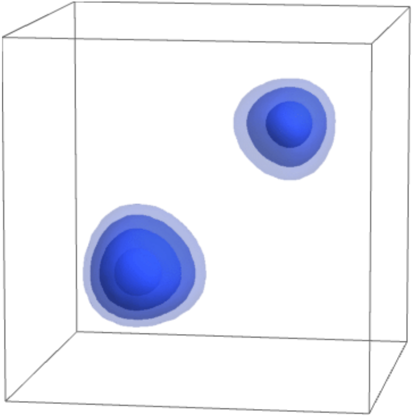


Outline

Tidal effects: asymmetric mass ($N_1 = 10N_2$), in phase

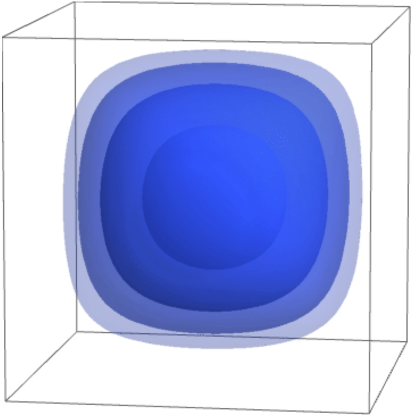


Tidal effects: asymmetric mass ($N_1 = 10N_2$), out of phase



Outline

Attractive instability ($\xi = -10$)



Summary

- Boson stars have a variety of interesting properties when it comes to collisions that make them interesting dark matter candidates
- The effective potential provides decent predictions for scattering of boson stars without resorting to computationally-expensive numerical simulations, can make predictions regarding the $\xi \gg 1$ limit
- However, it fails to capture the tidal deformation present in asymmetric-mass collisions
- Outlook, possible future directions:
  - Look at gravitational waves generated by collisions and oscillations and possible detection
  - Further exploration of boson star dark matter phenomenology

Thank you!

For Further Reading I

- R. Wyse, G. Gilmore, Dark Galaxies & Lost Baryons, Proceedings IAU Symposium No. 244, (2007)
- J. Bullock, arXiv:1009.4505 [astro-ph.CO] (2010).
- J-W. Lee, S. Lim, D. Choi, arXiv:0805.3827v1 [hep-ph] (2008).
- Y. Kobayashi, M. Kasai, T. Futamase, Phys. Rev. D 50, 15 Dec. (1994)
- P-H. Chavanis, Phys. Rev. D 84, 043531 (2011).
- D. J. Kaup, Phys. Rev. 172, 1331 (1968).
- R. Runi, S. Bonazzola, Phys. Rev. 187, 1767 (1969).
- M. Colpi, S. L. Shapiro, I. Wasserman, Phys. Rev. Lett. 57, 2485 (1986).

For Further Reading II

- J. Eby, C. Kouvaris, N. G. Nielsen, L.C.R. Wijewardhana, arXiv:1511.04474 [hep-ph]
- P. Jetzer, Physics Reports 220, # 4 (1992). 163-227

Boson star properties: binding energy

- Binding energy given by

$$E_0 = -\frac{8G^2 m^5 N^3 (3 + 2\xi + 3\sqrt{1+\xi})}{36\pi(1+\sqrt{1+\xi})^3}$$

Boson star properties: binding energy

- Binding energy given by

$$E_0 = -\frac{8G^2 m^5 N^3 (3 + 2\xi + 3\sqrt{1+\xi})}{36\pi(1+\sqrt{1+\xi})^3}$$

- Non-relativistic analysis breaks down once $mN \gtrsim |E_0|$:

Boson star properties: binding energy

- Binding energy given by

$$E_0 = -\frac{8G^2 m^5 N^3 (3 + 2\xi + 3\sqrt{1+\xi})}{36\pi(1+\sqrt{1+\xi})^3}$$

- Non-relativistic analysis breaks down once $mN \gtrsim |E_0|$:
  - $m \lesssim 10^{-21}$ eV for $M \sim 10^{12}$ $M_\odot$
  - $m \lesssim 10^{-9}$ eV for $M \sim 1$ $M_\odot$