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N=4 Super-Yang-Mills in ‘t Hooft limit as Exactly Solvable 4D Conformal Field Theory

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Outline

- Yang-Mills gauge theories in 4 dimensions describe fundamental forces of Nature, but they are very difficult for practical computations. Treated so far only perturbatively (weak coupling → high energies, Regge limit → BFKL), or by computer lattice Monte-Carlo simulations, with all advantages and disadvantages of "experimental" approach.

- Some exact results in topological, BPS sectors of supersymmetric gauge theories: Examples: Seiberg-Witten prepotentials for N=2 SYM, Dijkgraaf-Vafa matrix models for N=1 SYM, AGT-correspondence, etc. Does not work for non-BPS physical quantities given by sums of genuine Feynman graphs (gluons, quarks etc.)

- N=4 SYM theory – a unique exactly solvable (=integrable) 4D QFT, at least $N_c=\infty$. Correlators, scattering amplitudes, Wilson loops etc., are "in principal" computable at any value of 't Hooft coupling $g^2= N_c\, \alpha_{YM}$. Tools: AdS/CFT-duality + Integrability

- Already achieved: exact Riemann-Hilbert equations for scaling dimensions of any local operators at any 't Hooft coupling g. Sums up non-trivial 4D Feynman diagrams!

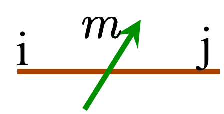
$$\langle \mathcal{O}_i(x)\mathcal{O}_j(0)\rangle = \frac{\delta_{ij}}{|x|^{2\Delta_j(g)}}$$

CFT: N=4 SYM as a superconformal 4d QFT

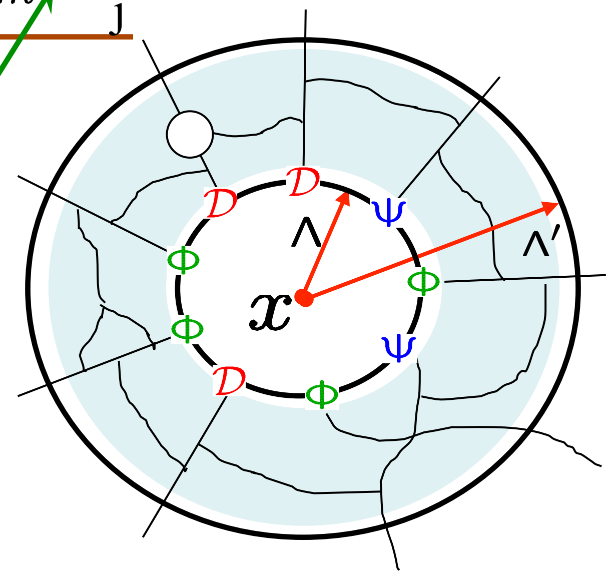
Gliozzi,Scherk,Olive'77

$$\mathcal{S}_{SYM} = \frac{N_c}{g^2}\int d^4x \, \mathrm{Tr}\left(F^2 + [\mathcal{D},\Phi]^2 + \bar{\Psi}[\not{\mathcal{D}}\Psi] + \bar{\Psi}[\not{\Phi},\Psi] + [\Phi,\Phi]^2\right)$$

- Zero β-function! Global 4d superconformal symmetry PSU(2,2|4)
- Operators from local fields, e.g. Φ_m^{ij} $\Longleftrightarrow$



$\mathcal{O}(x) = \mathrm{Tr}\,[\mathcal{D}\mathcal{D}\Psi\Psi\Phi\Phi\mathcal{D}\Psi\ldots](x)$ +permutations



- Dilatation operator $\hat{D}$ from point-splitting and renormalization

$$\mathcal{O}_j^{\Lambda'}(x) = \left[\left(\frac{\Lambda'}{\Lambda}\right)^{\hat{D}}\right]_{jk} \mathcal{O}_k^{\Lambda}(x)$$

$$\hat{D} = \hat{D}^{(0)} + g^2\hat{D}^{(2)} + g^4\hat{D}^{(4)} + \ldots$$

- Conformal dimensions are eigenvalues of dilatation operator

$$\hat{D}_{jk}\mathcal{O}_k(x) = \Delta_j\mathcal{O}_j \qquad \Delta = \Delta^{(0)} + g^2\Delta^{(2)} + g^4\Delta^{(4)} + \ldots$$

- $\tau \sim \log\Lambda$ corresponds to AdS time for dual string on AdS/CFT

*

Perturbative Integrability, su(2)-sector:

$X = \Phi_1 + i\Phi_2$

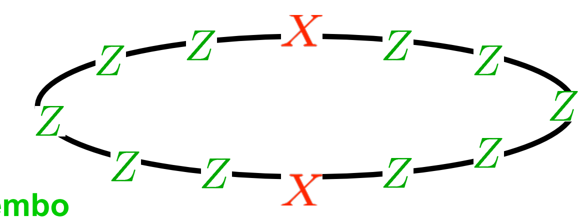
$Z = \Phi_5 + i\Phi_6$

- 1-loop dilatation operator = Hamiltonian of Heisenberg quantum spin chain.
  Integrable by Bethe ansatz! Bethe'31

$\mathrm{Tr}\, Z^L(x)$ - vacuum:

$$\hat{D} = L + g^2 \sum_{l=1}^{L} \left(1 - \sigma_l \cdot \sigma_{l+1}\right)$$

Minahan, Zarembo



also integrable!

Beisert, Kristijansen,Staudacher

- One loop Baxter equations for the N=4 SYM spectrum:

$$T(u)Q(u) = \left(u - \frac{i}{2}\right)^L Q(u+i) + \left(u + \frac{i}{2}\right)^L Q(u-i)$$

- Polynomiality of $T(u)$ fixes Bethe roots in $Q(u) = \prod_{k=1}^{J} (u - u_k)$

Anomalous dimensions: $\Delta - L = \frac{g^2}{8\pi^2} \partial_u \log \frac{Q(u+\frac{i}{2})}{Q(u-\frac{i}{2})}\Big|_{u=0} + \mathcal{O}(g^4)$

Maldacena
Gubser, Klebanov, Polyakov
Witten

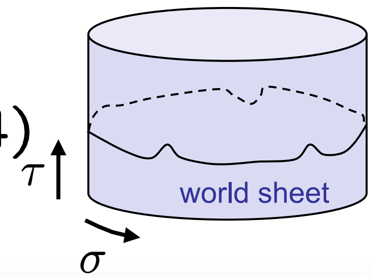
SYM is dual to superstring σ-model on $AdS_5 \times S^5$

- Super-conformal symmetry PSU(2,2|4) → isometry of string target space

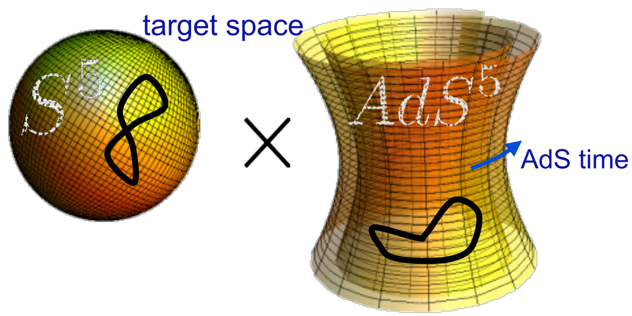
- 2D σ-model on a coset $\dfrac{PSU(2,2|4)}{SO(1,4)\times SO(5)}$ $\quad G(\sigma,\tau) = \left(\begin{array}{c|c} A & B \\ \hline C & D \end{array}\right) \in SL(4|4)$

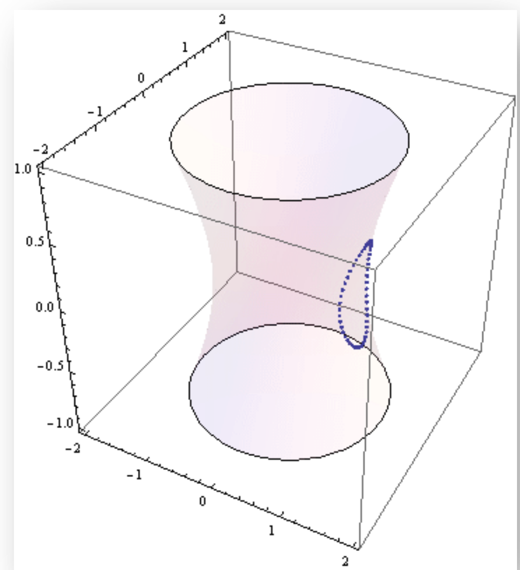
fermions

$$J = -G^{-1}dG = J^{(0)} + J^{(1)} + J^{(2)} + J^{(3)} \in su(2,2|4)$$



target space





- Metsaev-Tseytlin action

$$S_{MT} = g\ \mathrm{str} \int_{\mathcal{M}_2} \left[J^{(2)} \wedge *J^{(2)} - J^{(1)} \wedge J^{(3)} \right]$$

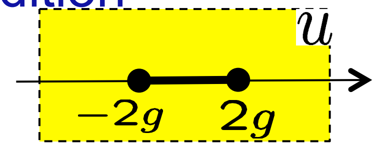
Dimension of YM operator $\Delta(g)$ = Energy of a string state

Classical integrability of superstring on $AdS_5 \times S^5$

- String eqs. of motion and constraints recast into flatness condition

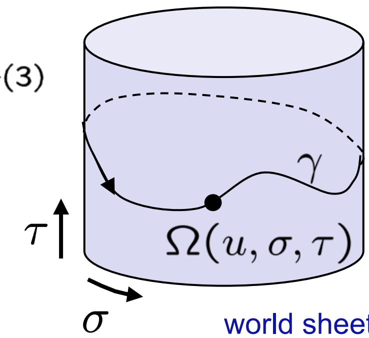
Mikhailov,Zakharov
Bena,Roiban,Polchinski

$$[(\partial_0 + \mathcal{A}_0(u)), (\partial_1 + \mathcal{A}_1(u))] = 0$$



for Lax connection ***double valued*** w.r.t. spectral parameter u

$$\mathcal{A}(u) = J^{(0)} + \frac{u}{\sqrt{u^2-4g^2}} J^{(2)} + \frac{g}{\sqrt{u^2-4g^2}} *J^{(2)} + \left(\frac{u+2g}{u-2g}\right)^{1/4} J^{(1)} + \left(\frac{u-2g}{u+2g}\right)^{1/4} J^{(3)}$$



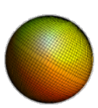
- Monodromy matrix $\Omega(u) = P \exp \oint_\gamma \mathcal{A}(u) \quad \in PSU(2,2|4)$
  encodes infinitely many conservation lows

- Eigenvalues define quasi-momenta:

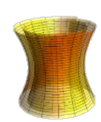
V.K.,Marshakov,Minahan,Zarembo
Beisert,V.K.,Sakai,Zarembo

$$\Omega(u) = U^{-1} \{e^{i\hat{p}_1(u)}, e^{i\hat{p}_2(u)}, e^{i\hat{p}_3(u)}, e^{i\hat{p}_4(u)} || e^{i\check{p}_1(u)}, e^{i\check{p}_2(u)}, e^{i\check{p}_3(u)}, e^{i\check{p}_4(u)}\} U$$

- Asymptotics fixed by Cartan charges of PSU(2,2|4): $\{J_1, J_2, J_3 \,|\, \Delta, S_1, S_2\}$



$$\begin{pmatrix} \hat{p}_1 \\ \hat{p}_2 \\ \hat{p}_3 \\ \hat{p}_4 \end{pmatrix} \simeq \frac{1}{2u} \begin{pmatrix} +J_1 + J_2 - J_3 \\ +J_1 - J_2 + J_3 \\ -J_1 + J_2 + J_3 \\ -J_1 - J_2 - J_3 \end{pmatrix}$$



$$\begin{pmatrix} \check{p}_1 \\ \check{p}_2 \\ \check{p}_3 \\ \check{p}_4 \end{pmatrix} \simeq \frac{1}{2u} \begin{pmatrix} +\Delta_1 - S_1 + S_2 \\ +\Delta_1 + S_1 - S_2 \\ -\Delta_1 - S_1 - S_2 \\ -\Delta_1 + S_1 + S_2 \end{pmatrix}$$

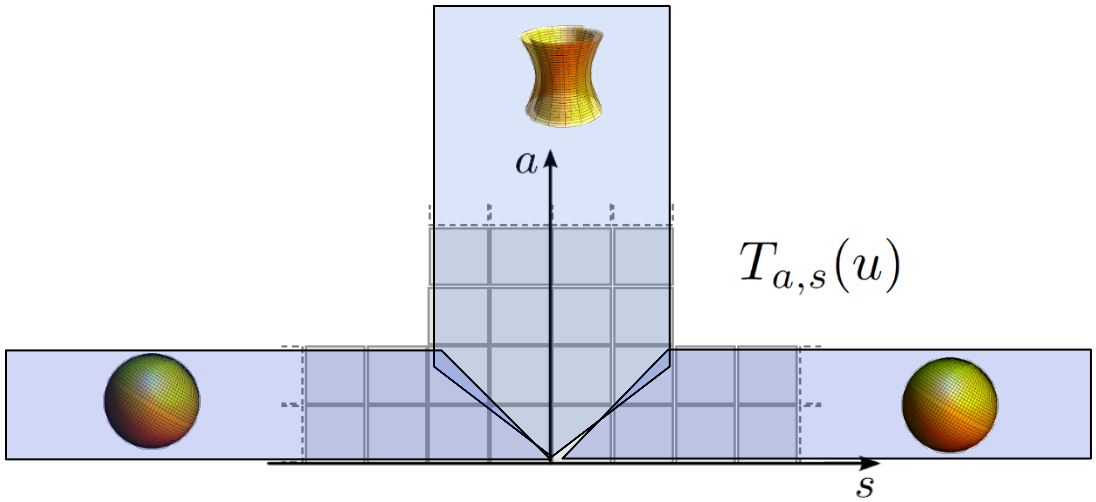
- Each quasi-momentum inherits the double-valuedness of Lax connection.

Gromov,V.K.,Vieira

Exact integrability: Y-system, T-system, Q-system…

- Exact quantum equations boil down to T-system (Hirota-Miwa eq.) on discrete T-shaped lattice (“T-hook”):

$$T_{a,s}\left(u+\frac{i}{2}\right)\,T_{a,s}\left(u-\frac{i}{2}\right)=T_{a,s-1}(u)\,T_{a,s+1}(u)+T_{a+1,s}(u)T_{a-1,s}(u)$$



- Integrable system, solvable in terms of Wronskians of Baxter’s Q-functions
- Example: solution for right band via two arbitrary functions:

$$T_{1,s}(u)=\mathbf{P}_1(u+\frac{is}{2})\mathbf{P}_2(u-\frac{is}{2})-\mathbf{P}_1(u-\frac{is}{2})\mathbf{P}_2(u+\frac{is}{2})$$

- Complete solution described by Q-system – full set of 2^8 Q-functions
  All of them can be expressed through 8 basic Q-functions

- Important: it should be supplemented by analyticity conditions

Exact quantum analogues of quasimomenta

- One assumes exact quantum integrability of this sigma model. Then standard tools can be applied: thermodynamic Bethe ansatz (TBA), exact Y-system, T-system (Hirota-Miwa eq.).

Gromov,V.K.,Vieira

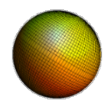
Bombardelli,Fioravanti,Tateo
Gromov,V.K.,Kozak,Vieira
Arutyunov,Frolov

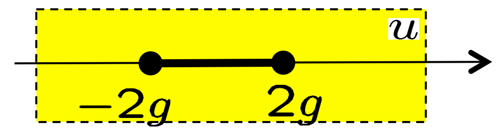
- Consequence: a set of exact Riemann-Hilbert equations for AdS/CFT spectrum - quantum spectral curve.

Gromov, V.K., Leurent, Volin 2013

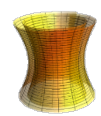
- 4+4 basic functions, with relatively simple analytic properties (one cut!)

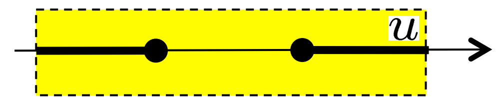
$$\mathbf{P}_a(u) \sim \exp\left(-\int^u \widehat{p}_a(u')du'\right)$$





$$\mathbf{Q}_j(u) \sim \exp\left(+\int^u \breve{p}_j(u')du'\right)$$





- Large u asymptotics defined by classical quasimomenta:

$$\mathbf{P}_b \simeq A_b\, u^{\frac{\pm J_1 \pm J_2 \pm J_3}{2}}, \qquad \mathbf{Q}_j \simeq B_j\, u^{\frac{\pm \Delta \pm S_1 \pm S_2}{2}}, \qquad b, j = 1, 2, 3, 4.$$

- On the next sheets: infinite ladder of such cuts spaced by ***i***.
- No other singularities are admitted
- To fix these functions completely we have to know the monodromy around the branch points.

Gromov, V.K., Leurent, Volin 2013

Pμ-system: example of SL(2) sector $\mathrm{Tr}(\nabla^S Z^L)$

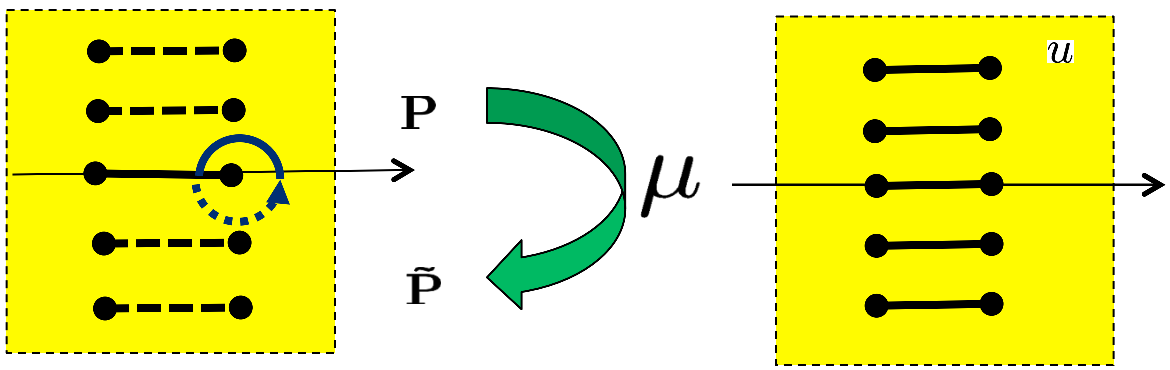
- Monodromy around the branch point matrix:

$$\tilde{\mathbf{P}}_a = \mu_{ab}\mathbf{P}^b$$

- Antisymmetric pseudo-periodic coefficient matrix:

$$\tilde{\mu}_{ab}(u) = \mu_{ab}(u+i)\,, \qquad \mathrm{Pf}(\mu) = 0$$

$\tilde{\mathbf{P}}$ is the analytic continuation of $\mathbf{P}$ through the cut:



SL(2) reduction:

$$\mathbf{P}^a = -\chi^{ab}\mathbf{P}_b$$

$$\chi = \begin{pmatrix} 0 & 0 & 0 & -1 \\ 0 & 0 & 1 & 0 \\ 0 & -1 & 0 & 0 \\ 1 & 0 & 0 & 0 \end{pmatrix}$$

- Pμ -system contains also an equation for monodromy of μ:

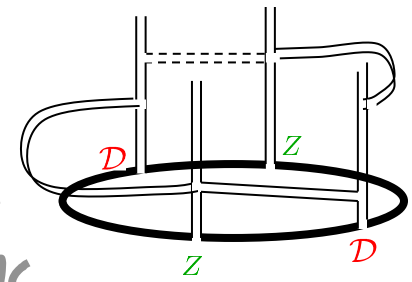
$$\tilde{\mu}_{ab} - \mu_{ab} = \mathbf{P}_a\tilde{\mathbf{P}}_b - \mathbf{P}_b\tilde{\mathbf{P}}_a$$

- Asymptotics at $u \to \infty$

$$\begin{pmatrix} \mathbf{P}_1 \\ \mathbf{P}_2 \\ \mathbf{P}_3 \\ \mathbf{P}_4 \end{pmatrix} \sim \begin{pmatrix} A_1 u^{-\frac{L}{2}} \\ A_2 u^{-\frac{L+2}{2}} \\ A_3 u^{\frac{L}{2}} \\ A_4 u^{\frac{L-2}{2}} \end{pmatrix} \qquad \begin{pmatrix} \mu_{12} \\ \mu_{13} \\ \mu_{14} \\ \mu_{24} \\ \mu_{34} \end{pmatrix} \sim \begin{pmatrix} u^{\Lambda-L} \\ u^{\Lambda+1} \\ u^{\Lambda} \\ u^{\Lambda-1} \\ u^{\Lambda+L} \end{pmatrix}, \qquad \Lambda = 0,\ \pm\Delta,\ \pm(S-1)$$

Perturbative Konishi: integrability versus Feynman graphs

$\mathcal{O}_{\text{Konishi}} = \text{Tr}\ [\mathcal{D}, Z]^2$



- Integrability allows to sum up exactly enormous numbers of Feynman diagrams of N=4 SYM

131000 graphs

$$
\begin{aligned}
\Delta = \ & 4 + 12\,g^2 - 48\,g^4 + 336\,g^6 + 96\,g^8\,(-26 + 6\,\zeta_3 - 15\,\zeta_5) \\
& -96\,g^{10}\,(-158 - 72\,\zeta_3 + 54\,\zeta_3^2 + 90\,\zeta_5 - 315\,\zeta_7) \\
& -48\,g^{12}\,(160 + 5472\,\zeta_3 - 3240\,\zeta_3\,\zeta_5 + 432\,\zeta_3^2 - 2340\,\zeta_5 - 1575\,\zeta_7 + 10206\,\zeta_9) \\
& +48\,g^{14}\,(-44480 + 108960\,\zeta_3 + 8568\,\zeta_3\,\zeta_5 - 40320\,\zeta_3\,\zeta_7 - 8784\,\zeta_3^2 + 2592\,\zeta_3^3 \\
& \qquad - 4776\,\zeta_5 - 20700\,\zeta_5^2 - 26145\,\zeta_7 - 17406\,\zeta_9 + 152460\,\zeta_{11}) \\
& +96\,g^{16}\,(566752 - 869760\,\zeta_3 - 45360\,\zeta_3\,\zeta_5 - 64890\,\zeta_3\,\zeta_7 + 241920\,\zeta_3\,\zeta_9 + 82656\,\zeta_3^2 - 33912\,\zeta_3^2\,\zeta_5 + 20736\,\zeta_3^3 \\
& \qquad - 204984\,\zeta_5 + 231840\,\zeta_5\,\zeta_7 + 24840\,\zeta_5^2 + 227799\,\zeta_7 + 97164\,\zeta_9 + 135927\,\zeta_{11} - 1104246\,\zeta_{13} \\
& \qquad + 7128\,\frac{\zeta_{11} - \zeta_3\,\zeta_{3,5} + \zeta_{3,5,3}}{5}) \\
& -96\,g^{18}\,(10568224 - 11884608\,\zeta_3 + 148896\,\zeta_3\,\zeta_5 - 177768\,\zeta_3\,\zeta_5^2 - 354384\,\zeta_3\,\zeta_7 - 1244484\,\zeta_3\,\zeta_9 + 2901096\,\zeta_{11}\,\zeta_3 \\
& \qquad + 533952\,\zeta_3^2 + 284904\,\zeta_3^2\,\zeta_5 - 229824\,\zeta_3^2\,\zeta_7 + 209952\,\zeta_3^3 - 5993280\,\zeta_5 + 963954\,\zeta_5\,\zeta_7 + 2553120\,\zeta_5\,\zeta_9 - 576000\,\zeta_5^2 \\
& \qquad + 2324196\,\zeta_7 + 1184274\,\zeta_7^2 + 2573892\,\zeta_9 + 355266\,\zeta_{11} + 2644434\,\zeta_{13} - 15810795\,\zeta_{15} \\
& \qquad + 163296\,\frac{\zeta_{11} - \zeta_3\,\zeta_{3,5} + \zeta_{3,5,3}}{5} - 13608\,(\zeta_3\,\zeta_{3,7} - \zeta_{3,7,3} + \zeta_3^2\,\zeta_5 - \zeta_5\,\zeta_{5,3} + \zeta_{5,3,5}))
\end{aligned}
$$

Bajnok,Janik
Leurent,Serban,Volin

Bajnok,Janik,Lukowski
Lukowski,Rej,
Velizhanin,Orlova
Leurent, Volin

Leurent, Volin
(8 loops from FiNLIE)

Volin
(9-loops from spectral curve)

- Confirmed up to 5 loops by direct graph calculus (6 loops promised)

Fiamberti,Santambrogio,Sieg,Zanon
Velizhanin
Eden,Heslop,Korchemsky,Smirnov,Sokatchev

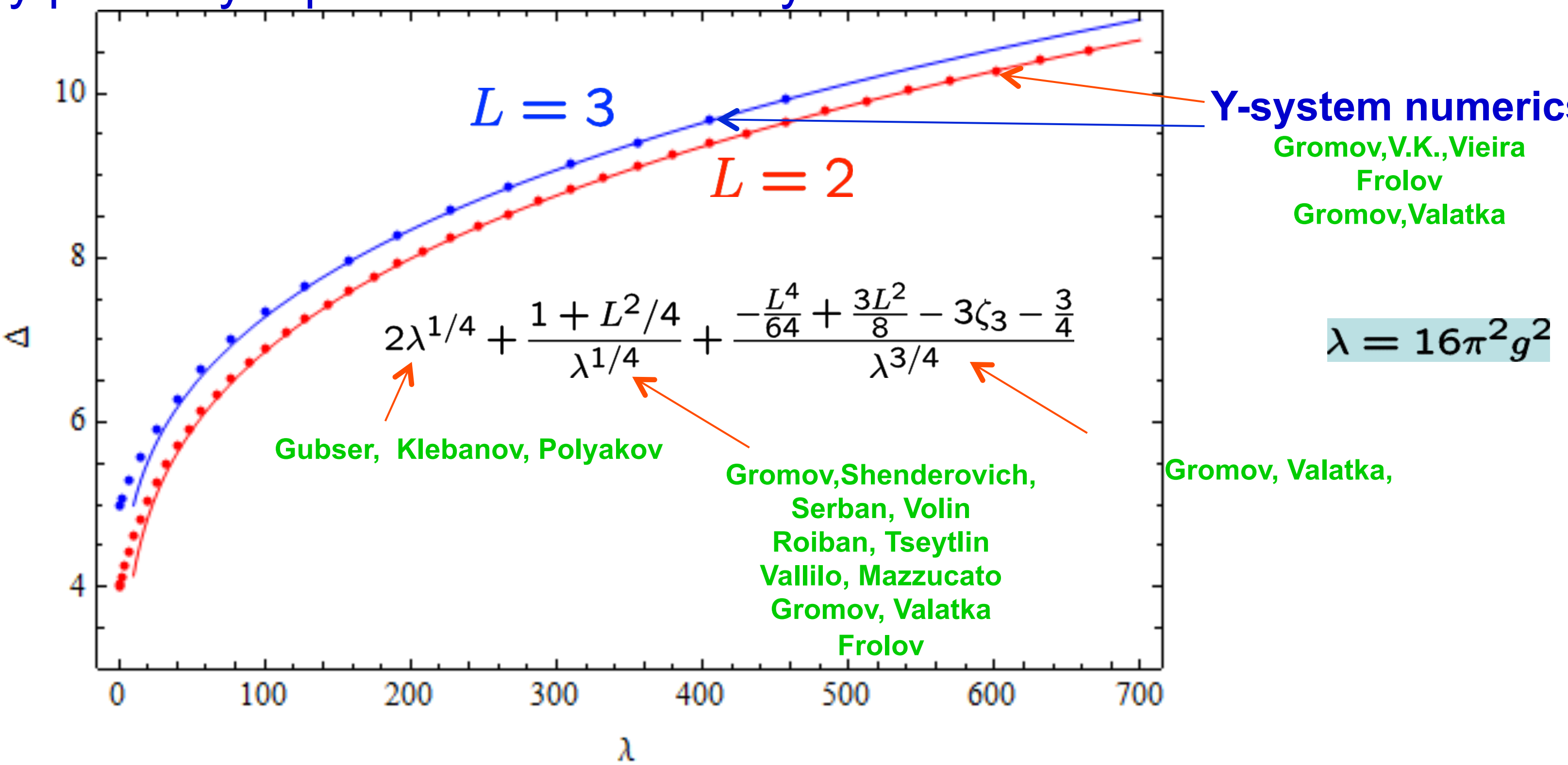
AdS string quasiclassics and numerics in SL(2) sector: twist-L operators of spin S $\mathrm{Tr}\,\mathcal{D}^S Z^L$

- 3 leading strong coupling terms were calculated for any S and L
  - Numerics from Y-system, TBA, FiNLIE, at any coupling:

$S = 2, \quad L = 2, \quad n = 1$ - for Konishi operator

$S = 2, \quad L = 3, \quad n = 1$ - and twist-3 operator

They perfectly reproduce the TBA/Y-system or FiNLIE numerics



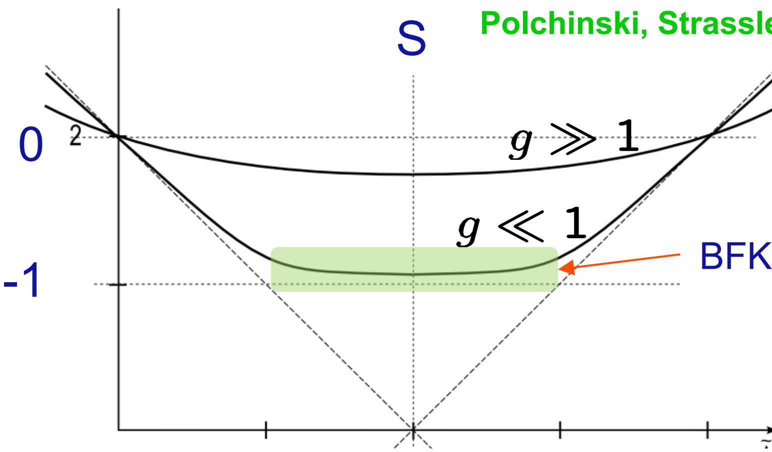
- AdS/CFT Integrability passes all known tests!

Alfimov Gromov V.K. 2014

BFKL Dimension from Quantum Spectral Curve

- QSC allows for analytic continuation of exact dimension $\Delta(S,g)$ to continuous spins $-1 < S < \infty$. We need to find the appropriate analytic continuation of P,Q-functions

Janik   Gromov, V.K.
Gromov, Levkovich-Maslyuk, Sizov, Valatka



- BFKL is a double scaling limit:

$$w = S+1 \to 0, \quad g \to 0, \quad \Lambda = \frac{g^2}{S+1} \quad - \text{fixed}$$

- We will restore from QSC the leading order (LO) BFKL approximation for $\Delta(S,g)$ already known up to NLO from direct summation of Feynman graphs

Balitsky, Fadin, Kuraev, Lipatov
Kotikov, Lipatov

$$\frac{S+1}{4g^2} = \Psi(\Delta) + g^2\delta(\Delta) + \mathcal{O}(g^4)$$

where

$$\Psi(\Delta) = -\psi\left(\frac{1+\Delta}{2}\right) - \psi\left(\frac{1-\Delta}{2}\right) + 2\psi(1)$$

$$\delta(\Delta) = 4\Psi''(\Delta) + 6\zeta_3 + 2\zeta_2\Psi(\Delta) - \frac{\pi^3}{\cos\frac{\pi\Delta}{2}} - 4\Phi(\frac{1}{2} - \frac{\Delta}{2}) - 4\Phi(\frac{1}{2} + \frac{\Delta}{2}),$$

$$\Phi(x) = \sum_{k=0}^{\infty} \frac{(-)^k}{(x+k)^2}[\psi(k+1+x) - \psi(1)]$$

- In particular, near the Regge pole

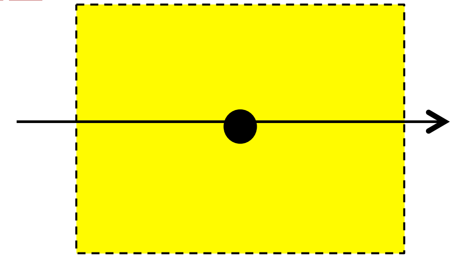
$$\Delta - 1 \simeq \frac{-8g^2}{w} + w\zeta_3\left(\frac{-4g^2}{w}\right)^3 + \mathcal{O}\left(\left(\frac{g^2}{w}\right)^4\right)$$

- BFKL is an excellent test for the whole AdS/CFT integrability: it sums up "wrapped" graphs omitted in asymptotic Bethe ansatz

Kotikov, Lipatov, Rej, Staudacher
Bajnok, Janik, Lukowsky
Lukowski, Rej, Velizhanin,Orlova

P- and μ-functions at LO BFKL

- **P** have a single cut which generates poles at $u = 0$ in the regime $g \ll |u| \ll 1$



$$\sqrt{u^2 - 4g^2} \equiv \sqrt{u^2 - 4\Lambda w} = u - \frac{2\Lambda}{u} w - \frac{2\Lambda^2}{u^3} w^2 + O(w^3)$$

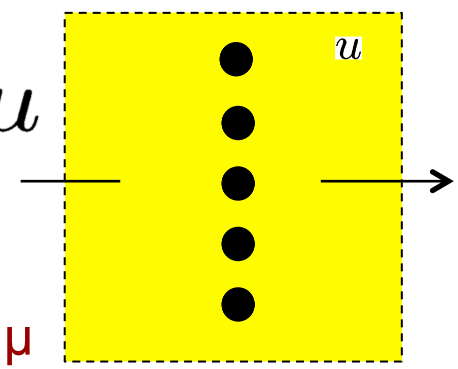
reminder: $w = S + 1$, $\Lambda = \dfrac{g^2}{S+1}$

- Due to asymptotics and parity **P**'s are fixed at leading order (LO) up to a single constant (fixed in NLO):

$$\mathbf{P}_1 = \frac{1}{u},, \qquad \mathbf{P}_2 = \frac{1}{u^2}, \qquad \mathbf{P}_3 = A_3^{(0)} u + \frac{c_{3,1}^{(1)}}{\Lambda u}, \qquad \mathbf{P}_4 = A_4^{(0)}$$

- μ has a "ladder" of cuts generating poles at $u = i\mathbb{Z}$

μ

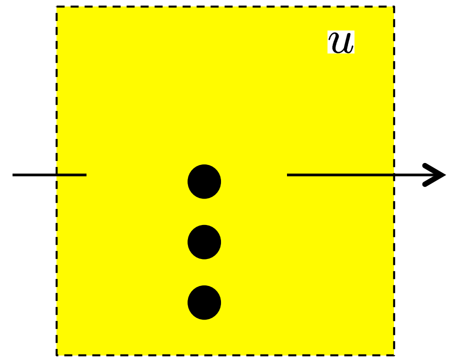


- Asymptotics $u \to \infty$ suggests that at LO μ=polynomials×Sinh^2(πu)
- Combining it with the monodromy equation we can fix μ

$$\mu_{12} = \frac{1}{w^2} \frac{\sinh^2 \pi u}{\pi^2 \Lambda^2} \frac{-4i}{(\Delta^2 - 1)^2}, \qquad \mu_{13} = \cdots, \qquad \cdots$$

- At the same time we fix the missing coefficient in **P** $\quad c_{3,1}^{(1)} = -\dfrac{i(\Delta^2 - 1)^2}{96}$

Analytic properties of Q-functions



- Natural objects for approaching BFKL are **Q**-functions: their asymptotics contain conformal charges, including Δ
- A "ladder" of cuts generates poles at $u = i\mathbb{Z}_-$
- From purely algebraic relations of Q-system we get a 4-th order finite difference equation with 4 solutions giving all 4 **Q**-functions in terms of on **P**-functions:

$$0 = \mathbf{Q}^{[+4]} D_0 - \mathbf{Q}^{[+2]} \left[D_1 - \mathbf{P}_a^{[+2]} \mathbf{P}^{a[+4]} D_0 \right] + \frac{1}{2} \mathbf{Q} \left[D_3 + \mathbf{P}_a \mathbf{P}^{a[+4]} D_0 + \mathbf{P}_a \mathbf{P}^{a[+2]} D_1 \right] + \text{c.c.}$$

- The coefficients depend only on **P**-functions: $D_m = \det_{1 \le a,k \le 4} (\mathbf{P}^a)^{[4-2k+2\delta_{k,m}]}$
- Plugging NLO **P**'s we get factorized eq. for BFKL $\mathbf{Q}$

$$\left[D + D^{-1} - 2 - \frac{1-\Delta^2}{4u^2} \right] \mathbf{Q} = 0$$

- 2-nd order equation is the Faddeev-Korchemsky Baxter eq. for BFKL pomeron !
- Similarly, we find NLO $\mathbf{P}$ and NLO Baxter equation for $\mathbf{Q}$

$$\mathbf{Q}_j \left(\frac{\Delta^2 - 1 - 8u^2}{4u^2} + w \frac{\left(\Delta^2 - 1\right)\Lambda - u^2}{2u^4} \right) + \mathbf{Q}_j^{--} \left(1 - \frac{iw/2}{u-i} \right) + \mathbf{Q}_j^{++} \left(1 + \frac{iw/2}{u+i} \right) = 0$$

- Solution - hypergeometric function. Gives pomeron spectrum and twist-2 dimension!

Conclusions and Comments

- We proposed a system of matrix Riemann-Hilbert equations – Quantum Spectral Curve - for the exact spectrum of anomalous dimensions of planar N=4 SYM theory in 4D.
- BFKL dimension in LO is recovered. Consequences for scattering theory in Regge limit and a link to QCD pomeron.
- Hopefully efficient for numerics. In particular, the full curve $\Delta(S,g)$ could be restored numerically.
- Applicable for Wilson loops and quark-antiquark potential in N=4 SYM
- Works for cusped Wilson loops and quark-antiquark potential in N=4 SYM
- Very efficient for various approximations: weak coupling (9 loops!) and strong coupling (3 loops) expansions exact slope and curvature functions

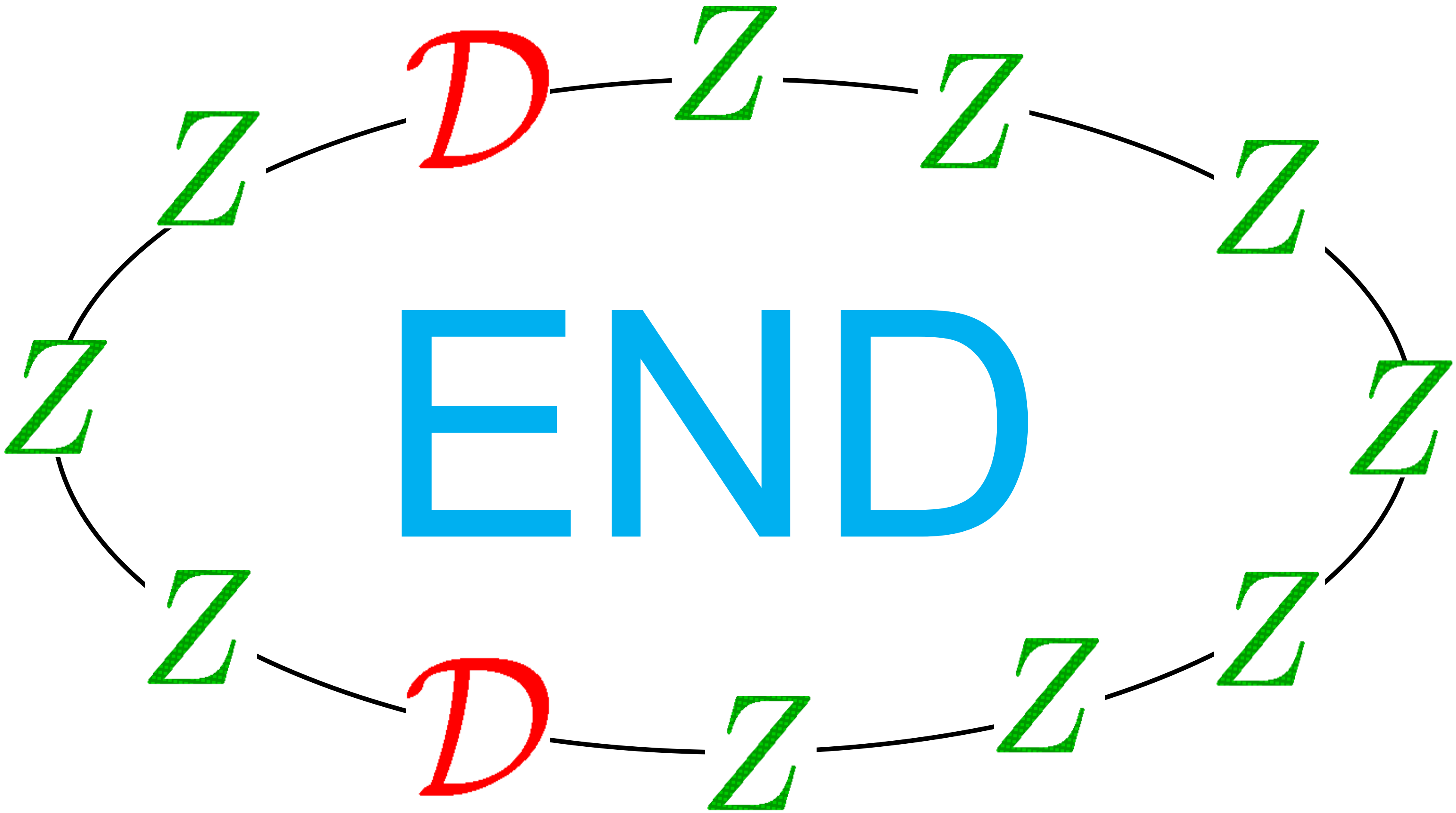
Correa, Maldacena, Sever
Drucker
Gromov, Sever
Gromov, Kazakov, Leurent, Volin

$$\Delta(S,g) - \Delta_0 = \Delta'(g)\, S + \Delta''(g)\, S^2 + O(S^3)$$

Basso
Gromov, Levkovich-Maslyuk, Sizov, Valatka

Future directions

- Similar equations in gluon amplitudes, correlators, Wilson loops, 1/N – expansion ?
- Strong coupling expansion from P-μ -system?
- Same method of Riemann-Hilbert equations and Q-system for other sigma models ?
- Finite size bootstrap for 2D sigma models without S-matrix and TBA ?
- Deep reasons for integrability of planar N=4 SYM ?
- NLO, NNLO, … BFKL (in progress)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