Supersymmetric unification and R symmetries

Michael Ratz

TECHNISCHE UNIVERSITÄT MÜNCHEN

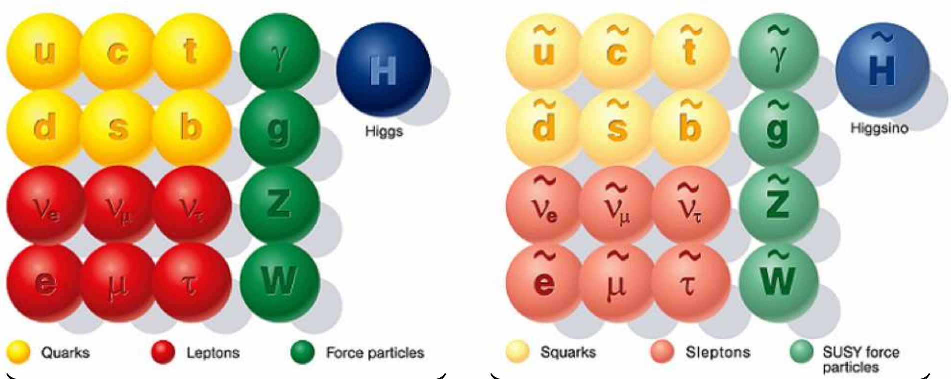
PACIFIC, September 5, 2012

Based on:

- H.M. Lee, S. Raby, G. Ross, M.R., R. Schieren, K. Schmidt–Hoberg & P. Vaudrevange, Phys. Lett. **B** 694, 491-495 (2011) & Nucl. Phys. **B** 850, 1-30 (2011)
- R. Kappl, B. Petersen, S. Raby, M.R., R. Schieren & P. Vaudrevange, Nucl. Phys. **B** 847, 325-349 (2011)
- M. Fallbacher, M.R. & P. Vaudrevange, Phys. Lett. **B** 705, 503-506 (2011)
- M.–C. Chen, M.R., C. Staudt & P. Vaudrevange, arXiv:1206.5375

(Minimal) supersymmetric standard model

- The minimal supersymmetric standard model (MSSM) provides an attractive scheme for physics beyond the SM



SM particles

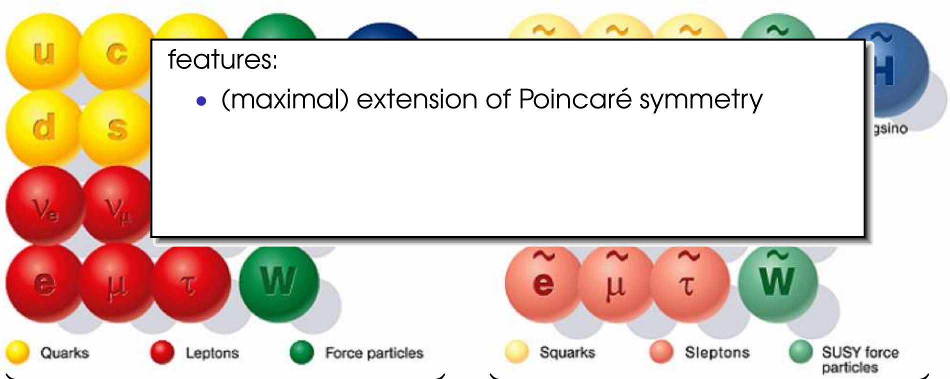
superpartners

(Minimal) supersymmetric standard model

- The minimal supersymmetric standard model (MSSM) provides an attractive scheme for physics beyond the SM

features:

- (maximal) extension of Poincaré symmetry

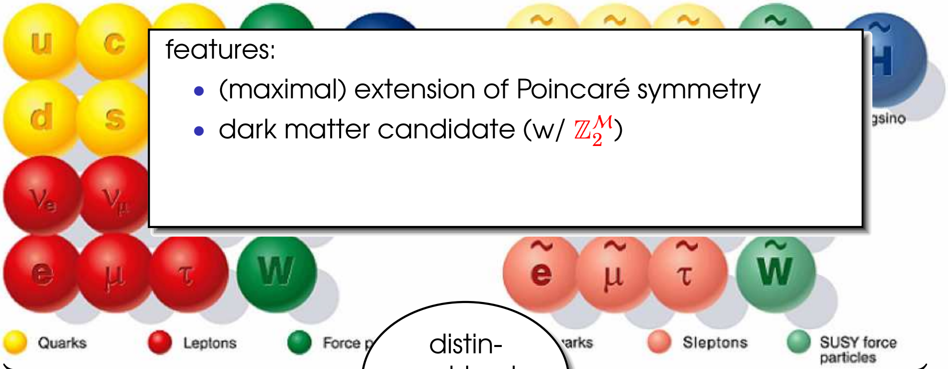


SM particles

superpartners

(Minimal) supersymmetric standard model

- The minimal supersymmetric standard model (MSSM) provides an attractive scheme for physics beyond the SM



features:

- (maximal) extension of Poincaré symmetry
- dark matter candidate (w/ $\mathbb{Z}_2^{\mathcal{M}}$)

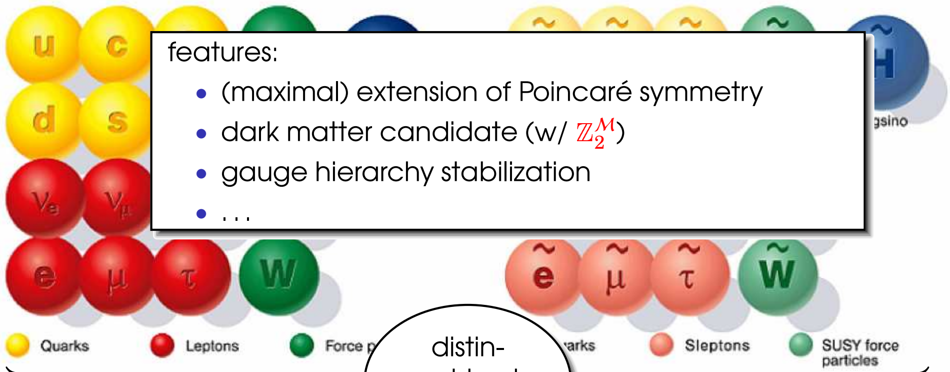
distin-guished by $\mathbb{Z}_2^{\mathcal{M}}$

SM particles
matter parity even

superpartners
matter parity odd

(Minimal) supersymmetric standard model

☞ The minimal supersymmetric standard model (MSSM) provides an attractive scheme for physics beyond the SM



features:

- (maximal) extension of Poincaré symmetry
- dark matter candidate (w/ $\mathbb{Z}_2^{\mathcal{M}}$)
- gauge hierarchy stabilization
- ...

SM particles
matter parity even

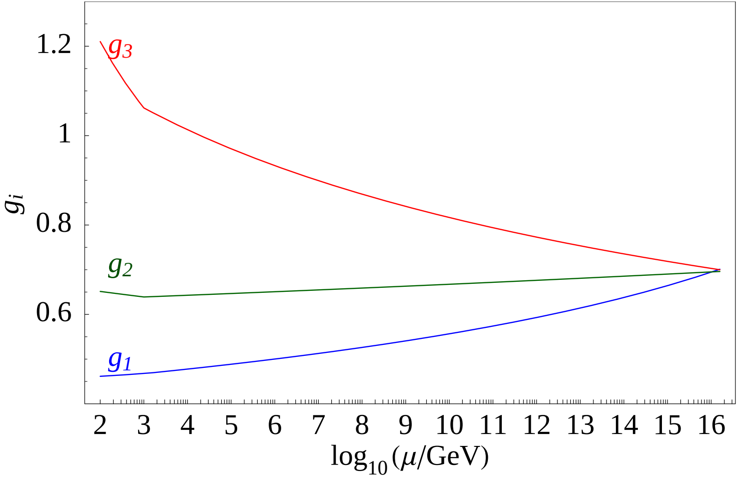
distinguished by $\mathbb{Z}_2^{\mathcal{M}}$

superpartners
matter parity odd

Gauge coupling unification in the MSSM

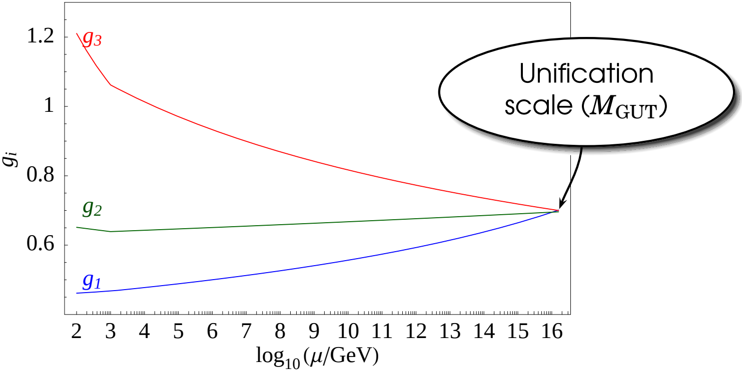
- ☞ Running couplings in the (minimal) supersymmetric standard model (MSSM)

Dimopoulos, Raby, Wilczek (1981)



Gauge coupling unification in the MSSM

- Running couplings in the (minimal) supersymmetric standard model (MSSM) Dimopoulos, Raby, Wilczek (1981)

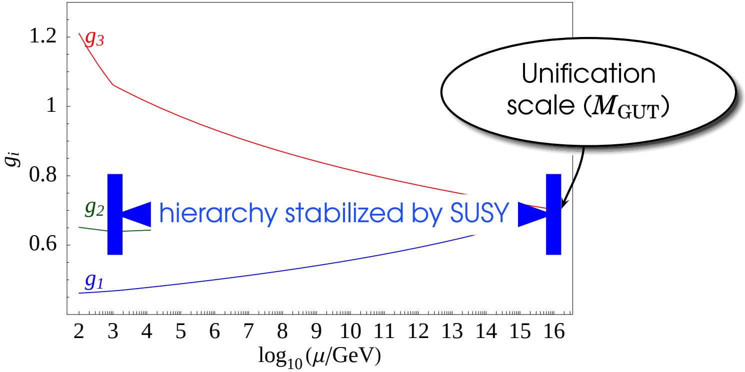


- Gauge coupling unification might be a consequence of $G_{\mathrm{SM}} = \mathrm{SU}(3) \times \mathrm{SU}(2) \times \mathrm{U}(1) \subset \mathrm{SU}(5)$

Gauge coupling unification in the MSSM

- Running couplings in the (minimal) supersymmetric standard model (MSSM)

Dimopoulos, Raby, Wilczek (1981)



- Gauge coupling unification might be a consequence of $G_{\mathrm{SM}} = \mathrm{SU}(3) \times \mathrm{SU}(2) \times \mathrm{U}(1) \subset \mathrm{SU}(5)$

SU(5) and SO(10)

$\mathrm{SU}(5)$ grand unified theories (GUTs) ...

- explain charge quantization
- simplify matter content

  SM generation $= \mathbf{10} + \overline{\mathbf{5}}$

further simplification of matter sector

Fritzsch & Minkowski (1975)

$$\begin{aligned} \mathrm{SO}(10) &\supset \mathrm{SU}(5) \\ \mathbf{16} &= \mathbf{10} \oplus \overline{\mathbf{5}} \oplus \mathbf{1} \\ &= \text{SM generation with `right–handed' neutrino} \end{aligned}$$

- One of the main assumptions in this talk: this is not an accident

Problems of the MSSM

☞ Gauge invariant superpotential terms up to order 4 include

$$\begin{aligned}\mathscr{W} &= \mu\, H_d H_u + \kappa_i\, L_i H_u \\ &\quad + Y_e^{ij}\, L_i H_d \overline{E}_j + Y_d^{ij}\, Q_i H_d \overline{D}_j + Y_u^{ij}\, Q_i H_u \overline{U}_j \\ &\quad + \lambda_{ijk}\, L_i L_j \overline{E}_k + \lambda'_{ijk}\, L_i Q_j \overline{D}_k + \lambda''_{ijk}\, \overline{U}_i \overline{D}_j \overline{D}_k \\ &\quad + \kappa_{ij}^{(0)}\, H_u L_i\, H_u L_j + \kappa_{ijk\ell}^{(1)}\, Q_i Q_j Q_k L_\ell + \kappa_{ijk\ell}^{(2)}\, \overline{U}_i \overline{U}_j \overline{D}_k \overline{E}_\ell\end{aligned}$$

$\stackrel{!}{\sim} 1/10^{15}$ GeV
in order to
explain see–saw
suppressed
ν masses

Problems of the MSSM

- Gauge invariant superpotential terms up to order 4 include

$$\begin{aligned}\mathscr{W} \;=\; & \mu H_d H_u + \kappa_i L_i H_u \quad \overset{!}{\sim}\text{TeV} \\ & + Y_e^{ij} L_i H_d \overline{E}_j + Y_d^{ij} Q_i H_d \overline{D}_j + Y_u^{ij} Q_i H_u \overline{U}_j \\ & + \lambda_{ijk} L_i L_j \overline{E}_k + \lambda'_{ijk} L_i Q_j \overline{D}_k + \lambda''_{ijk} \overline{U}_i \overline{D}_j \overline{D}_k \\ & + \kappa_{ij}^{(0)} H_u L_i H_u L_j + \kappa_{ijk\ell}^{(1)} Q_i Q_j Q_k L_\ell + \kappa_{ijk\ell}^{(2)} \overline{U}_i \overline{U}_j \overline{D}_k \overline{E}_\ell\end{aligned}$$

- Problematic terms
  - ☹ $\mu/B\mu$ problem(s)
    Why does μ know about the electroweak scale?

Problems of the MSSM

- Gauge invariant superpotential terms up to order 4 include

$$\begin{aligned}
\mathscr{W} &= \mu\, H_d H_u + \kappa_i\, L_i H_u \\
&\quad + Y_e^{ij}\, L_i H_d \overline{E}_j + Y_d^{ij}\, Q_i H_d \overline{D}_j + Y_u^{ij}\, Q_i H_u \overline{U}_j \\
&\quad + \lambda_{ijk}\, L_i L_j \overline{E}_k + \lambda'_{ijk}\, L_i Q_j \overline{D}_k + \lambda''_{ijk}\, \overline{U}_i \overline{D}_j \overline{D}_k \\
&\quad + \kappa_{ij}^{(0)}\, H_u L_i\, H_u L_j + \kappa_{ijk\ell}^{(1)}\, Q_i Q_j Q_k L_\ell + \kappa_{ijk\ell}^{(2)}\, \overline{U}_i \overline{U}_j \overline{D}_k \overline{E}_\ell
\end{aligned}$$

$$\kappa_{1121}^{(1)} \overset{!}{\lesssim} \frac{10^{-8}}{M_\mathrm{P}}$$

- Problematic terms
  - ☹ $\mu/B\mu$ problem(s)
  - ☹ dimension four and five proton decay operators

Problems of the MSSM

- Gauge invariant superpotential terms up to order 4 include

$$
\begin{aligned}
\mathscr{W} &= \mu H_d H_u + \kappa_i L_i H_u \\
&\quad + Y_e^{ij} L_i H_d \overline{E}_j + Y_d^{ij} Q_i H_d \overline{D}_j + Y_u^{ij} Q_i H_u \overline{U}_j \\
&\quad + \lambda_{ijk} L_i L_j \overline{E}_k + \lambda'_{ijk} L_i Q_j \overline{D}_k + \lambda''_{ijk} \overline{U}_i \overline{D}_j \overline{D}_k \\
&\quad + \kappa_{ij}^{(0)} H_u L_i H_u L_j + \kappa_{ijk\ell}^{(1)} Q_i Q_j Q_k L_\ell + \kappa_{ijk\ell}^{(2)} \overline{U}_i \overline{U}_j \overline{D}_k \overline{E}_\ell
\end{aligned}
$$

- Problematic terms
  - ☹ $\mu/B\mu$ problem(s)
  - ☹ dimension four and five proton decay operators
  - ☹ CP and flavor problems not addressed in this talk

Problems of the MSSM

- Gauge invariant superpotential terms up to order 4 include

$$
\begin{aligned}
\mathscr{W} &= \mu H_d H_u + \kappa_i L_i H_u \\
&\quad + Y_e^{ij} L_i H_d \overline{E}_j + Y_d^{ij} Q_i H_d \overline{D}_j + Y_u^{ij} Q_i H_u \overline{U}_j \\
&\quad + \lambda_{ijk} L_i L_j \overline{E}_k + \lambda'_{ijk} L_i Q_j \overline{D}_k + \lambda''_{ijk} \overline{U}_i \overline{D}_j \overline{D}_k \\
&\quad + \kappa_{ij}^{(0)} H_u L_i H_u L_j + \kappa_{ijk\ell}^{(1)} Q_i Q_j Q_k L_\ell + \kappa_{ijk\ell}^{(2)} \overline{U}_i \overline{U}_j \overline{D}_k \overline{E}_\ell
\end{aligned}
$$

- Problematic terms
  - ☹ $\mu/B\mu$ problem(s)
  - ☹ dimension four and five proton decay operators
  - ☹ CP and flavor problems not addressed in this talk
  - ☹ ...

Problems of the MSSM

- Gauge invariant superpotential terms up to order 4 include

$$\begin{aligned}\mathscr{W} &= \mu H_d H_u + \kappa_i L_i H_u \\ &\quad + Y_e^{ij} L_i H_d \overline{E}_j + Y_d^{ij} Q_i H_d \overline{D}_j + Y_u^{ij} Q_i H_u \overline{U}_j \\ &\quad + \lambda_{ijk} L_i L_j \overline{E}_k + \lambda'_{ijk} L_i Q_j \overline{D}_k + \lambda''_{ijk} \overline{U}_i \overline{D}_j \overline{D}_k \\ &\quad + \kappa_{ij}^{(0)} H_u L_i H_u L_j + \kappa_{ijk\ell}^{(1)} Q_i Q_j Q_k L_\ell + \kappa_{ijk\ell}^{(2)} \overline{U}_i \overline{U}_j \overline{D}_k \overline{E}_\ell\end{aligned}$$

- Problematic terms
  - ☹ $\mu/B\mu$ problem(s)
  - ☹ dimension four and five proton decay operators
  - ☹ CP and flavor problems not addressed in this talk
  - ☹ ...

➥ Supersymmetry alone seems not to be enough

Traditional cure of proton decay problems

- Gauge invariant superpotential terms up to order 4 include

$$
\begin{aligned}
\mathscr{W} &= \mu\, H_d H_u + \kappa_i\, L_i H_u \\
&\quad + Y_e^{ij}\, L_i H_d \overline{E}_j + Y_d^{ij}\, Q_i H_d \overline{D}_j + Y_u^{ij}\, Q_i H_u \overline{U}_j \\
&\quad + \lambda_{ijk}\, L_i L_j \overline{E}_k + \lambda'_{ijk}\, L_i Q_j \overline{D}_k + \lambda''_{ijk}\, \overline{U}_i \overline{D}_j \overline{D}_k \\
&\quad + \kappa_{ij}^{(0)}\, H_u L_i\, H_u L_j + \kappa_{ijk\ell}^{(1)}\, Q_i Q_j Q_k L_\ell + \kappa_{ijk\ell}^{(2)}\, \overline{U}_i \overline{U}_j \overline{D}_k \overline{E}_\ell
\end{aligned}
$$

Traditional cure of proton decay problems

☞ Gauge invariant superpotential terms up to order 4 include

$$
\begin{aligned}
\mathscr{W} \;=\; & \mu\, H_d H_u + \kappa_i\, L_i H_u \\
& + Y_e^{ij}\, L_i H_d \overline{E}_j + Y_d^{ij}\, Q_i H_d \overline{D}_j + Y_u^{ij}\, Q_i H_u \overline{U}_j \\
& + \lambda_{ijk}\, L_i L_j \overline{E}_k + \lambda'_{ijk}\, L_i Q_j \overline{D}_k + \lambda''_{ijk}\, \overline{U}_i \overline{D}_j \overline{D}_k \\
& + \kappa_{ij}^{(0)}\, H_u L_i\, H_u L_j + \kappa_{ijk\ell}^{(1)}\, Q_i Q_j Q_k L_\ell + \kappa_{ijk\ell}^{(2)}\, \overline{U}_i \overline{U}_j \overline{D}_k \overline{E}_\ell
\end{aligned}
$$

need to be strongly suppressed

Traditional cure of proton decay problems

☞ Gauge invariant superpotential terms up to order 4 include

$$
\begin{aligned}
\mathscr{W} \;=\; & \mu H_d H_u + \kappa_i L_i H_u \\
& + Y_e^{ij} L_i H_d \overline{E}_j + Y_d^{ij} Q_i H_d \overline{D}_j + Y_u^{ij} Q_i H_u \overline{U}_j \\
& + \lambda_{ijk} L_i L_j \overline{E}_k + \lambda'_{ijk} L_i Q_j \overline{D}_k + \lambda''_{ijk} \overline{U}_i \overline{D}_j \overline{D}_k \\
& + \kappa_{ij}^{(0)} H_u L_i H_u L_j + \kappa_{ijk\ell}^{(1)} Q_i Q_j Q_k L_\ell + \kappa_{ijk\ell}^{(2)} \overline{U}_i \overline{U}_j \overline{D}_k \overline{E}_\ell
\end{aligned}
$$

forbidden by matter parity

Farrar and Fayet (1978) ; Dimopoulos et al. (1982)

Traditional cure of proton decay problems

☞ Gauge invariant superpotential terms up to order 4 include

$$\begin{aligned}\mathscr{W} &= \mu H_d H_u + \kappa_i L_i H_u \\ &\quad + Y_e^{ij} L_i H_d \overline{E}_j + Y_d^{ij} Q_i H_d \overline{D}_j + Y_u^{ij} Q_i H_u \overline{U}_j \\ &\quad + \lambda_{ijk} L_i L_j \overline{E}_k + \lambda'_{ijk} L_i Q_j \overline{D}_k + \lambda''_{ijk} \overline{U}_i \overline{D}_j \overline{D}_k \\ &\quad + \kappa_{ij}^{(0)} H_u L_i H_u L_j + \kappa_{ijk\ell}^{(1)} Q_i Q_j Q_k L_\ell + \kappa_{ijk\ell}^{(2)} \overline{U}_i \overline{U}_j \overline{D}_k \overline{E}_\ell\end{aligned}$$

forbidden by baryon triality

Ibáñez and Ross (1992)

Traditional cure of proton decay problems

☞ Gauge invariant superpotential terms up to order 4 include

$$\begin{aligned} \mathscr{W} &= \mu H_d H_u + \kappa_i L_i H_u \\ &+ Y_e^{ij} L_i H_d \overline{E}_j + Y_d^{ij} Q_i H_d \overline{D}_j + Y_u^{ij} Q_i H_u \overline{U}_j \\ &+ \lambda_{ijk} L_i L_j \overline{E}_k + \lambda'_{ijk} L_i Q_j \overline{D}_k + \lambda''_{ijk} \overline{U}_i \overline{D}_j \overline{D}_k \\ &+ \kappa_{ij}^{(0)} H_u L_i H_u L_j + \kappa_{ijk\ell}^{(1)} Q_i Q_j Q_k L_\ell + \kappa_{ijk\ell}^{(2)} \overline{U}_i \overline{U}_j \overline{D}_k \overline{E}_\ell \end{aligned}$$

forbidden by proton hexality

Babu et al. (2003b) ; Dreiner et al. (2006)

☞ Proton hexality = matter parity + baryon triality

Ibáñez and Ross (1992)

Dreiner et al. (2006)

Traditional cure: proton hexality

Ibáñez and Ross (1992) ; Babu et al. (2003b) ; Dreiner et al. (2006)

☞ Proton hexality P_6 = matter parity $\mathbb{Z}_2^{\mathcal{M}}$ × baryon triality B_3

	Q	U	D	L	E	H_u	H_d	$\overline{\nu}$
$\mathbb{Z}_2^{\mathcal{M}}$	1	1	1	1	1	0	0	1
B_3	0	−1	1	−1	2	1	−1	0
P_6	0	1	−1	−2	1	−1	1	3

Traditional cure: proton hexality

Ibáñez and Ross (1992) ; Babu et al. (2003b) ; Dreiner et al. (2006)

- Proton hexality P_6 = matter parity $\mathbb{Z}_2^{\mathcal{M}}$ × baryon triality B_3

	Q	U	D	L	E	H_u	H_d	$\overline{\nu}$
$\mathbb{Z}_2^{\mathcal{M}}$	1	1	1	1	1	0	0	1
B_3	0	−1	1	−1	2	1	−1	0
P_6	0	1	−1	−2	1	−1	1	3

- Appealing features
  - ☺ forbids dimension four and five proton decay operators

Traditional cure: proton hexality

Ibáñez and Ross (1992) ; Babu et al. (2003b) ; Dreiner et al. (2006)

- Proton hexality P_6 = matter parity $\mathbb{Z}_2^{\mathcal{M}}$ × baryon triality B_3

	Q	U	D	L	E	H_u	H_d	$\overline{\nu}$
$\mathbb{Z}_2^{\mathcal{M}}$	1	1	1	1	1	0	0	1
B_3	0	−1	1	−1	2	1	−1	0
P_6	0	1	−1	−2	1	−1	1	3

- Appealing features
  - ☺ forbids dimension four and five proton decay operators
  - ☺ allows Yukawa couplings & Weinberg operator $\kappa_{ij}^{(0)} H_u L_i H_u L_j$

Traditional cure: proton hexality

Ibáñez and Ross (1992) ; Babu et al. (2003b) ; Dreiner et al. (2006)

☞ Proton hexality P_6 = matter parity $\mathbb{Z}_2^{\mathcal{M}}$ × baryon triality B_3

	Q	U	D	L	E	H_u	H_d	$\bar{\nu}$
$\mathbb{Z}_2^{\mathcal{M}}$	1	1	1	1	1	0	0	1
B_3	0	−1	1	−1	2	1	−1	0
P_6	0	1	−1	−2	1	−1	1	3

☞ Appealing features

- ☺ forbids dimension four and five proton decay operators
- ☺ allows Yukawa couplings & Weinberg operator $\kappa_{ij}^{(0)} H_u L_i H_u L_j$
- ☺ unique anomaly–free symmetry with the above features
  . . . with the common notion of anomaly freedom

Traditional cure: proton hexality

Ibáñez and Ross (1992) ; Babu et al. (2003b) ; Dreiner et al. (2006)

- Proton hexality P_6 = matter parity $\mathbb{Z}_2^{\mathcal{M}}$ × baryon triality B_3

	Q	U	D	L	E	H_u	H_d	$\bar{\nu}$
$\mathbb{Z}_2^{\mathcal{M}}$	1	1	1	1	1	0	0	1
B_3	0	-1	1	-1	2	1	-1	0
P_6	0	1	-1	-2	1	-1	1	3

- Appealing features
  - ☺ forbids dimension four and five proton decay operators
  - ☺ allows Yukawa couplings & Weinberg operator $\kappa_{ij}^{(0)} H_u L_i H_u L_j$
  - ☺ unique anomaly–free symmetry with the above features

- However:
  - ☹ not consistent with unification for matter (i.e. inconsistent with universal discrete charges for all matter fields)

Proton hexality

- Disturbing aspects of proton hexality
  - ☹ not consistent with (grand) unification for matter

Proton hexality

- Disturbing aspects of proton hexality
  - ☹ not consistent with (grand) unification for matter
  - ☹ does not address μ problem

Proton hexality

- Disturbing aspects of proton hexality
  - ☹ not consistent with (grand) unification for matter
  - ☹ does not address μ problem

$$
\begin{aligned}
\mathscr{W} &= \mu H_d H_u + \kappa_i L_i H_u \\
&\quad + Y_e^{ij} L_i H_d \overline{E}_j + Y_d^{ij} Q_i H_d \overline{D}_j + Y_u^{ij} Q_i H_u \overline{U}_j \\
&\quad + \lambda_{ijk} L_i L_j \overline{E}_k + \lambda'_{ijk} L_i Q_j \overline{D}_k + \lambda''_{ijk} \overline{U}_i \overline{D}_j \overline{D}_k \\
&\quad + \kappa_{ij}^{(0)} H_u L_i H_u L_j + \kappa_{ijk\ell}^{(1)} Q_i Q_j Q_k L_\ell + \kappa_{ijk\ell}^{(2)} \overline{U}_i \overline{U}_j \overline{D}_k \overline{E}_\ell + \ldots
\end{aligned}
$$

Proton hexality

- Disturbing aspects of proton hexality
  - ☹ not consistent with (grand) unification for matter
  - ☹ does not address μ problem

$$
\begin{aligned}
\mathscr{W} = {} & \mu H_d H_u + \kappa_i L_i H_u \\
& + Y_e^{ij} L_i H_d \overline{E}_j + Y_d^{ij} Q_i H_d \overline{D}_j + Y_u^{ij} Q_i H_u \overline{U}_j \\
& + \lambda_{ijk} L_i L_j \overline{E}_k + \lambda'_{ijk} L_i Q_j \overline{D}_k + \lambda''_{ijk} \overline{U}_i \overline{D}_j \overline{D}_k \\
& + \kappa_{ij}^{(0)} H_u L_i H_u L_j + \kappa_{ijk\ell}^{(1)} Q_i Q_j Q_k L_\ell + \kappa_{ijk\ell}^{(2)} \overline{U}_i \overline{U}_j \overline{D}_k \overline{E}_\ell + \dots
\end{aligned}
$$

need to be strongly suppressed

Proton hexality

- Disturbing aspects of proton hexality
  - ☹ not consistent with (grand) unification for matter
  - ☹ does not address μ problem

$$
\begin{aligned}
\mathscr{W} \;=\; & \mu\, H_d H_u + \kappa_i\, L_i H_u \\
& + Y_e^{ij}\, L_i H_d \overline{E}_j + Y_d^{ij}\, Q_i H_d \overline{D}_j + Y_u^{ij}\, Q_i H_u \overline{U}_j \\
& + \lambda_{ijk}\, L_i L_j \overline{E}_k + \lambda'_{ijk}\, L_i Q_j \overline{D}_k + \lambda''_{ijk}\, \overline{U}_i \overline{D}_j \overline{D}_k \\
& + \kappa_{ij}^{(0)}\, H_u L_i\, H_u L_j + \kappa_{ijk\ell}^{(1)}\, Q_i Q_j Q_k L_\ell + \kappa_{ijk\ell}^{(2)}\, \overline{U}_i \overline{U}_j \overline{D}_k \overline{E}_\ell + \dots
\end{aligned}
$$

needs to be suppressed as well. . .

Outline

1. Introduction & Motivation ✓
2. Anomaly–free discrete symmetries & unification
   - anomaly cancellation
   - consistency with unification
   - unique $\mathbb{Z}_4^R$ symmetry
   - no–go theorems in 4D
3. String model(s)
4. Summary

Anomaly–free

discrete symmetries

and

grand unification

- anomaly cancellation
- consistency with unification
- unique $\mathbb{Z}_4^R$ symmetry
- no–go theorems in 4D

Prejudices and assumptions

Assumptions:

- ☞ $\mathrm{SO}(10)$ unification of matter is not an accident
- ☞ μ term is forbidden by a symmetry
- ☞ symmetries need to be anomaly–free

Important ingredient :

- ☞ Green–Schwarz anomaly cancellation

Anomaly freedom

Chen et al. (2012)

Anomaly freedom

+

Grand unification

+

Green–Schwarz anomaly cancellation

Anomaly freedom

Chen et al. (2012)

$$\left.\begin{array}{c}\text{Anomaly freedom}\\ +\\ \text{Grand unification}\\ +\\ \text{Green–Schwarz}\\ \text{anomaly cancellation}\end{array}\right\} \to \text{“Anomaly universality”}$$

Anomaly freedom

Chen et al. (2012)

$$\left.\begin{array}{c}\text{Anomaly freedom}\\ +\\ \text{Grand unification}\\ +\\ \text{Green–Schwarz}\\ \text{anomaly cancellation}\end{array}\right\} \rightarrow \text{“Anomaly universality”}$$

Example: anomaly coefficients for $\mathbb{Z}_N$ symmetry

$$A_{G^2-\mathbb{Z}_N} = \sum_f \ell^{(f)} \cdot q^{(f)}$$

$$A_{\mathrm{grav}^2-\mathbb{Z}_N} = \sum_m q^{(m)}$$

Anomaly freedom

Chen et al. (2012)

$$\left.\begin{array}{c}\text{Anomaly freedom}\\+\\\text{Grand unification}\\+\\\text{Green–Schwarz}\\\text{anomaly cancellation}\end{array}\right\}\to \text{“Anomaly universality”}$$

Example: anomaly coefficients for $\mathbb{Z}_N$ symmetry

$$A_{G^2-\mathbb{Z}_N} = \sum_f \ell^{(f)} \cdot q^{(f)}$$

$$A_{\mathrm{grav}^2-\mathbb{Z}_N} = \sum_m q^{(m)}$$

sum over all representations of G

sum over all fermions

Anomaly freedom

Chen et al. (2012)

$$\left.\begin{array}{c}\text{Anomaly freedom}\\ +\\ \text{Grand unification}\\ +\\ \text{Green–Schwarz}\\ \text{anomaly cancellation}\end{array}\right\} \to \text{“Anomaly universality”}$$

Example: anomaly coefficients for $\mathbb{Z}_N$ symmetry

$$A_{G^2-\mathbb{Z}_N} = \sum_f \ell^{(f)} \cdot q^{(f)}$$

$$A_{\text{grav}^2-\mathbb{Z}_N} = \sum_m q^{(m)}$$

Dynkin index

discrete charges

Anomaly freedom

Chen et al. (2012)

$$\left.\begin{array}{c}\text{Anomaly freedom}\\ +\\ \text{Grand unification}\\ +\\ \text{Green–Schwarz}\\ \text{anomaly cancellation}\end{array}\right\} \to \text{“Anomaly universality”}$$

traditional anomaly freedom:

all A coefficients vanish

Example: anomaly coefficients for $\mathbb{Z}_N$ symmetry

$$A_{G^2-\mathbb{Z}_N} = \sum_f \ell^{(f)} \cdot q^{(f)} \stackrel{!}{=} 0 \mod \eta$$

$$A_{\mathrm{grav}^2-\mathbb{Z}_N} = \sum_m q^{(m)} \stackrel{!}{=} 0 \mod \eta$$

$$\eta := \begin{cases} N & \text{for } N \text{ odd} \\ N/2 & \text{for } N \text{ even} \end{cases}$$

Ibáñez and Ross (1991)
Banks and Dine (1992)

Anomaly freedom

Chen et al. (2012)

Anomaly freedom
+
Grand unification
+
Green–Schwarz anomaly cancellation

→ "Anomaly universality"

Example: anomaly coefficients for $\mathbb{Z}_N$ symmetry

$$A_{G^2-\mathbb{Z}_N} = \sum_f \ell^{(f)} \cdot q^{(f)} \stackrel{!}{=} \rho \mod \eta$$

$$A_{\mathrm{grav}^2-\mathbb{Z}_N} = \sum_m q^{(m)} \stackrel{!}{=} \rho \mod \eta$$

traditional anomaly freedom:

all A coefficients vanish

⬇ ⬇ ⬇ ⬇ ⬇

anomaly "universality":

$A_{\mathrm{SU}(3)^2-\mathbb{Z}_N} = A_{\mathrm{SU}(2)^2-\mathbb{Z}_N}$ if $\mathrm{SU}(3) \times \mathrm{SU}(2) \subset \mathrm{SU}(5)$ or E_8

Anomaly–free symmetries, μ and unification

☞ Working assumptions:

(i) anomaly freedom (allow for GS anomaly cancellation)

Anomaly–free symmetries, μ and unification

☞ Working assumptions:

(i) anomaly freedom (allow for GS anomaly cancellation)

(ii) μ term forbidden at perturbative level

Anomaly–free symmetries, μ and unification

☞ Working assumptions:

(i) anomaly freedom (allow for GS anomaly cancellation)

(ii) μ term forbidden at perturbative level

(iii) Yukawa couplings and Weinberg neutrino mass operator allowed

Anomaly–free symmetries, μ and unification

☞ Working assumptions:

(i) anomaly freedom (allow for GS anomaly cancellation)

(ii) μ term forbidden at perturbative level

(iii) Yukawa couplings and Weinberg neutrino mass operator allowed

(iv) $\mathrm{SU}(5)$ or $\mathrm{SO}(10)$ GUT relations for quarks and leptons

Anomaly–free symmetries, μ and unification

☞ Working assumptions:

(i) anomaly freedom (allow for GS anomaly cancellation)

(ii) μ term forbidden at perturbative level

(iii) Yukawa couplings and Weinberg neutrino mass operator allowed

(iv) $\mathrm{SU}(5)$ or $\mathrm{SO}(10)$ GUT relations for quarks and leptons

☞ Will prove:

1. assuming (i) & $\mathrm{SU}(5)$ relations:
   $\curvearrowright$ only R symmetries can forbid the μ term

Anomaly–free symmetries, μ and unification

☞ Working assumptions:

(i) anomaly freedom (allow for GS anomaly cancellation)

(ii) μ term forbidden at perturbative level

(iii) Yukawa couplings and Weinberg neutrino mass operator allowed

(iv) $\mathrm{SU}(5)$ or $\mathrm{SO}(10)$ GUT relations for quarks and leptons

☞ Will prove:

1. assuming (i) & $\mathrm{SU}(5)$ relations:
   $\curvearrowright$ only R symmetries can forbid the μ term
2. assuming (i)–(iii) & $\mathrm{SO}(10)$ relations:
   $\curvearrowright$ unique $\mathbb{Z}_4^R$ symmetry

Anomaly–free symmetries, μ and unification

- Working assumptions:
  - (i) anomaly freedom (allow for GS anomaly cancellation)
  - (ii) μ term forbidden at perturbative level
  - (iii) Yukawa couplings and Weinberg neutrino mass operator allowed
  - (iv) $\mathrm{SU}(5)$ or $\mathrm{SO}(10)$ GUT relations for quarks and leptons

- Will prove:
  1. assuming (i) & $\mathrm{SU}(5)$ relations:
     $\curvearrowright$ only R symmetries can forbid the μ term
  2. assuming (i)–(iii) & $\mathrm{SO}(10)$ relations:
     $\curvearrowright$ unique $\mathbb{Z}_4^R$ symmetry
  3. R symmetries are not available in 4D GUTs

Non–R symmetries do not do the job

- Anomaly coefficients for non–R symmetry with SU(5) relations for matter charges

$$A_{\mathrm{SU}(3)^2-\mathbb{Z}_N} = \frac{1}{2}\sum_{g=1}^{3}\left(3q^g_{\mathbf{10}} + q^g_{\overline{\mathbf{5}}}\right)$$

$$A_{\mathrm{SU}(2)^2-\mathbb{Z}_N} = \frac{1}{2}\sum_{g=1}^{3}\left(3q^g_{\mathbf{10}} + q^g_{\overline{\mathbf{5}}}\right) + \frac{1}{2}\left(q_{H_u} + q_{H_d}\right)$$

charge of g^{th} $\mathbf{10}$–plet

charge of g^{th} $\overline{\mathbf{5}}$–plet

Higgs charges

Non–R symmetries do not do the job

- Anomaly coefficients for non–R symmetry with SU(5) relations for matter charges

$$A_{\mathrm{SU}(3)^2-\mathbb{Z}_N} = \frac{1}{2}\sum_{g=1}^{3}\left(3q_{\mathbf{10}}^{g} + q_{\overline{\mathbf{5}}}^{g}\right)$$

$$A_{\mathrm{SU}(2)^2-\mathbb{Z}_N} = \frac{1}{2}\sum_{g=1}^{3}\left(3q_{\mathbf{10}}^{g} + q_{\overline{\mathbf{5}}}^{g}\right) + \frac{1}{2}\left(q_{H_u} + q_{H_d}\right)$$

- Anomaly universality: $A_{\mathrm{SU}(2)^2-\mathbb{Z}_N} - A_{\mathrm{SU}(3)^2-\mathbb{Z}_N} = 0$

$$\curvearrowright \quad \frac{1}{2}\left(q_{H_u} + q_{H_d}\right) = 0 \mod \begin{cases} N & \text{for } N \text{ odd} \\ N/2 & \text{for } N \text{ even} \end{cases}$$

Non–R symmetries do not do the job

☞ Anomaly coefficients for non–R symmetry with SU(5) relations for matter charges

$$A_{\mathrm{SU}(3)^2-\mathbb{Z}_N} = \frac{1}{2}\sum_{g=1}^{3}\left(3q^g_{\mathbf{10}} + q^g_{\overline{\mathbf{5}}}\right)$$

$$A_{\mathrm{SU}(2)^2-\mathbb{Z}_N} = \frac{1}{2}\sum_{g=1}^{3}\left(3q^g_{\mathbf{10}} + q^g_{\overline{\mathbf{5}}}\right) + \frac{1}{2}\left(q_{H_u} + q_{H_d}\right)$$

☞ Anomaly universality: $A_{\mathrm{SU}(2)^2-\mathbb{Z}_N} - A_{\mathrm{SU}(3)^2-\mathbb{Z}_N} = 0$

$$\curvearrowright \quad \frac{1}{2}\left(q_{H_u} + q_{H_d}\right) = 0 \mod \begin{cases} N & \text{for } N \text{ odd} \\ N/2 & \text{for } N \text{ even} \end{cases}$$

bottom–line:

non–R $\mathbb{Z}_N$ symmetry cannot forbid μ term

Only discrete R symmetries may do the job

- Obvious: if anomaly–free discrete non–R symmetries cannot forbid the μ term, this also applies to continuous non–R symmetries

- There are no anomaly–free continuous R symmetries in the MSSM

Chamseddine and Dreiner (1996)

- Only remaining option: discrete R symmetries

't Hooft anomaly matching for R symmetries

Chen et al. (2012)

☞ Powerful tool: anomaly matching

't Hooft anomaly matching for R symmetries

Chen et al. (2012)

- Powerful tool: anomaly matching
- At the SU(5) level: one anomaly coefficient

$$A_{\mathrm{SU}(5)^2-\mathbb{Z}_M^R} = \underbrace{A^{\text{matter}}_{\mathrm{SU}(5)^2-\mathbb{Z}_M^R}}_{\text{matter}} + \underbrace{A^{\text{extra}}_{\mathrm{SU}(5)^2-\mathbb{Z}_M^R}}_{\text{extra}} + \underbrace{5q_\theta}_{\text{gauginos}}$$

't Hooft anomaly matching for R symmetries

Chen et al. (2012)

- Powerful tool: anomaly matching
- At the $\mathrm{SU}(5)$ level: one anomaly coefficient

$$A_{\mathrm{SU}(5)^2-\mathbb{Z}_M^R} = A^{\mathrm{matter}}_{\mathrm{SU}(5)^2-\mathbb{Z}_M^R} + A^{\mathrm{extra}}_{\mathrm{SU}(5)^2-\mathbb{Z}_M^R} + 5q_\theta$$

- Consider the $\mathrm{SU}(3)$ and $\mathrm{SU}(2)$ subgroups

$$A^{\mathrm{SU}(5)}_{\mathrm{SU}(3)^2-\mathbb{Z}_M^R} = A^{\mathrm{matter}}_{\mathrm{SU}(3)^2-\mathbb{Z}_M^R} + A^{\mathrm{extra}}_{\mathrm{SU}(3)^2-\mathbb{Z}_M^R} + 3q_\theta + \frac{1}{2}\cdot 2\cdot 2\cdot q_\theta$$

$$A^{\mathrm{SU}(5)}_{\mathrm{SU}(2)^2-\mathbb{Z}_M^R} = A^{\mathrm{matter}}_{\mathrm{SU}(2)^2-\mathbb{Z}_M^R} + A^{\mathrm{extra}}_{\mathrm{SU}(2)^2-\mathbb{Z}_M^R} + 2q_\theta + \frac{1}{2}\cdot 2\cdot 3\cdot q_\theta$$

universal

SM gauginos

extra gauginos from X, Y bosons

't Hooft anomaly matching for R symmetries

Chen et al. (2012)

- Powerful tool: anomaly matching

- At the $\mathrm{SU}(5)$ level: one anomaly coefficient

$$A_{\mathrm{SU}(5)^2-\mathbb{Z}_M^R} = A^{\mathrm{matter}}_{\mathrm{SU}(5)^2-\mathbb{Z}_M^R} + A^{\mathrm{extra}}_{\mathrm{SU}(5)^2-\mathbb{Z}_M^R} + 5q_\theta$$

- Consider the $\mathrm{SU}(3)$ and $\mathrm{SU}(2)$ subgroups

$$A^{\mathrm{SU}(5)}_{\mathrm{SU}(3)^2-\mathbb{Z}_M^R} = A^{\mathrm{matter}}_{\mathrm{SU}(3)^2-\mathbb{Z}_M^R} + A^{\mathrm{extra}}_{\mathrm{SU}(3)^2-\mathbb{Z}_M^R} + 3q_\theta + \xcancel{\tfrac{1}{2}\cdot 2\cdot 2\cdot q_\theta}$$

$$A^{\mathrm{SU}(5)}_{\mathrm{SU}(2)^2-\mathbb{Z}_M^R} = A^{\mathrm{matter}}_{\mathrm{SU}(2)^2-\mathbb{Z}_M^R} + A^{\mathrm{extra}}_{\mathrm{SU}(2)^2-\mathbb{Z}_M^R} + 2q_\theta + \xcancel{\tfrac{1}{2}\cdot 2\cdot 3\cdot q_\theta}$$

- Assume now that some mechanism eliminates the extra gauginos

't Hooft anomaly matching for R symmetries

Chen et al. (2012)

- Powerful tool: anomaly matching
- At the $\mathrm{SU}(5)$ level: one anomaly coefficient

$$A_{\mathrm{SU}(5)^2-\mathbb{Z}_M^R} = A^{\text{matter}}_{\mathrm{SU}(5)^2-\mathbb{Z}_M^R} + A^{\text{extra}}_{\mathrm{SU}(5)^2-\mathbb{Z}_M^R} + 5q_\theta$$

- Consider the $\mathrm{SU}(3)$ and $\mathrm{SU}(2)$ subgroups

$$A^{\mathrm{SU}(5)}_{\mathrm{SU}(3)^2-\mathbb{Z}_M^R} = A^{\text{matter}}_{\mathrm{SU}(3)^2-\mathbb{Z}_M^R} + A^{\text{extra}}_{\mathrm{SU}(3)^2-\mathbb{Z}_M^R} + 3q_\theta + \cancel{\tfrac{1}{2}\cdot 2\cdot 2\cdot q_\theta}$$

$$A^{\mathrm{SU}(5)}_{\mathrm{SU}(2)^2-\mathbb{Z}_M^R} = A^{\text{matter}}_{\mathrm{SU}(2)^2-\mathbb{Z}_M^R} + A^{\text{extra}}_{\mathrm{SU}(2)^2-\mathbb{Z}_M^R} + 2q_\theta + \cancel{\tfrac{1}{2}\cdot 2\cdot 3\cdot q_\theta}$$

- Assume now that some mechanism eliminates the extra gauginos

➥ Extra stuff must be non–universal (split multiplets)

't Hooft anomaly matching for R symmetries

Chen et al. (2012)

- Powerful tool: anomaly matching
- At the $\mathrm{SU}(5)$ level: one anomaly coefficient

$$A_{\mathrm{SU}(5)^2-\mathbb{Z}_M^R} = A^{\text{matter}}_{\mathrm{SU}(5)^2-\mathbb{Z}_M^R} + A^{\text{extra}}_{\mathrm{SU}(5)^2-\mathbb{Z}_M^R} + 5q_\theta$$

- Consider the $\mathrm{SU}(3)$ and $\mathrm{SU}(2)$ subgroups

$$A^{\mathrm{SU}(5)}_{\mathrm{SU}(3)^2-\mathbb{Z}_M^R} = A^{\text{matter}}_{\mathrm{SU}(3)^2-\mathbb{Z}_M^R} + A^{\text{extra}}_{\mathrm{SU}(3)^2-\mathbb{Z}_M^R} + 3q_\theta + \cancel{\tfrac{1}{2}\cdot 2\cdot 2\cdot q_\theta}$$
$$A^{\mathrm{SU}(5)}_{\mathrm{SU}(2)^2-\mathbb{Z}_M^R} = A^{\text{matter}}_{\mathrm{SU}(2)^2-\mathbb{Z}_M^R} + A^{\text{extra}}_{\mathrm{SU}(2)^2-\mathbb{Z}_M^R} + 2q_\theta + \cancel{\tfrac{1}{2}\cdot 2\cdot 3\cdot q_\theta}$$

bottom–line:

't Hooft anomaly matching for (discrete) R symmetries implies the presence of split multiplets below the GUT scale!

SO(10) implies unique symmetry

Lee et al. (2011) ; Chen et al. (2012)

☞ Consider $\mathbb{Z}_M^R$ symmetry which commutes with $\mathrm{SO}(10)$
i.e. quarks and leptons have universal charge q

SO(10) implies unique symmetry

Lee et al. (2011) ; Chen et al. (2012)

- Consider $\mathbb{Z}_M^R$ symmetry which commutes with $\mathrm{SO}(10)$
i.e. quarks and leptons have universal charge q

- existence of u- and d–type Yukawas requires that

$$2q + q_{H_u} = 2q_\theta \mod M \quad \text{and} \quad 2q + q_{H_d} = 2q_\theta \mod M$$

R charge of superspace coordinate θ

superpotential has R charge $2q_\theta$

$$\int \mathrm{d}^2\theta\,\mathscr{W} \subset \mathscr{L}$$

SO(10) implies unique symmetry

Lee et al. (2011) ; Chen et al. (2012)

- Consider $\mathbb{Z}_M^R$ symmetry which commutes with SO(10) i.e. quarks and leptons have universal charge q
- existence of u- and d–type Yukawas requires that

$$2q + q_{H_u} = 2q_\theta \mod M \quad \text{and} \quad 2q + q_{H_d} = 2q_\theta \mod M$$

$$\curvearrowright \quad q_{H_u} - q_{H_d} = 0 \mod M$$

SO(10) implies unique symmetry

Lee et al. (2011) ; Chen et al. (2012)

- Consider $\mathbb{Z}_M^R$ symmetry which commutes with SO(10)
i.e. quarks and leptons have universal charge q

- existence of u- and d–type Yukawas requires that

$$2q + q_{H_u} = 2q_\theta \mod M \quad \text{and} \quad 2q + q_{H_d} = 2q_\theta \mod M$$

$$\curvearrowright \quad q_{H_u} - q_{H_d} = 0 \mod M$$

- u–type Yukawa and Weinberg operator requires that

$$2q + q_{H_u} = 2q_\theta \mod M \quad \text{and} \quad 2q + 2q_{H_u} = 2q_\theta \mod M$$

SO(10) implies unique symmetry

Lee et al. (2011) ; Chen et al. (2012)

- Consider $\mathbb{Z}_M^R$ symmetry which commutes with $\mathrm{SO}(10)$ i.e. quarks and leptons have universal charge q

- existence of u- and d–type Yukawas requires that

$$2q + q_{H_u} = 2q_\theta \mod M \quad \text{and} \quad 2q + q_{H_d} = 2q_\theta \mod M$$

$$\curvearrowright \quad q_{H_u} - q_{H_d} = 0 \mod M$$

- u–type Yukawa and Weinberg operator requires that

$$2q + q_{H_u} = 2q_\theta \mod M \quad \text{and} \quad 2q + 2q_{H_u} = 2q_\theta \mod M$$

$$\curvearrowright \quad q_{H_u} = 0 \mod M$$

SO(10) implies unique symmetry

Lee et al. (2011) ; Chen et al. (2012)

- Consider $\mathbb{Z}_M^R$ symmetry which commutes with $\mathrm{SO}(10)$ i.e. quarks and leptons have universal charge q
- existence of u- and d–type Yukawas requires that

$$2q + q_{H_u} = 2q_\theta \mod M \quad \text{and} \quad 2q + q_{H_d} = 2q_\theta \mod M$$

$$\curvearrowright \quad q_{H_u} - q_{H_d} = 0 \mod M$$

- u–type Yukawa and Weinberg operator requires that

$$2q + q_{H_u} = 2q_\theta \mod M \quad \text{and} \quad 2q + 2q_{H_u} = 2q_\theta \mod M$$

$$\curvearrowright \quad q_{H_u} = 0 \mod M$$

bottom–line:

$$q_{H_u} = q_{H_d} = 0 \mod M \;\&\; q = q_\theta \mod M$$

Unique $\mathbb{Z}_4^R$ symmetry

Lee et al. (2011) ; Chen et al. (2012)

☞ We know already that $\begin{cases} \bullet\ q = q_\theta \\ \bullet\ q_{H_u} = q_{H_d} = 0 \mod M \end{cases}$

Unique $\mathbb{Z}_4^R$ symmetry

Lee et al. (2011) ; Chen et al. (2012)

- ☞ We know already that $\begin{cases} \bullet\ q = q_\theta \\ \bullet\ q_{H_u} = q_{H_d} = 0 \mod M \end{cases}$
- ☞ Simplest possibility: $M = 4$ & $q = q_\theta = 1 \curvearrowright \mathbb{Z}_4^R$ symmetry

  $M = 2$ does not work since this is not an R symmetry

Unique $\mathbb{Z}_4^R$ symmetry

Lee et al. (2011) ; Chen et al. (2012)

- ☞ We know already that $\begin{cases} \bullet\, q = q_\theta \\ \bullet\, q_{H_u} = q_{H_d} = 0 \mod M \end{cases}$
- ☞ Simplest possibility: $M = 4$ & $q = q_\theta = 1 \curvearrowright \mathbb{Z}_4^R$ symmetry
- ☞ Alternatives: $\mathbb{Z}_{4m}^R$ symmetry with $q = q_\theta = m$ & $m \in \mathbb{N}$

Unique $\mathbb{Z}_4^R$ symmetry

Lee et al. (2011) ; Chen et al. (2012)

- We know already that $\begin{cases} \bullet\ q = q_\theta \\ \bullet\ q_{H_u} = q_{H_d} = 0 \mod M \end{cases}$
- Simplest possibility: $M = 4$ & $q = q_\theta = 1 \curvearrowright \mathbb{Z}_4^R$ symmetry
- Alternatives: $\mathbb{Z}_{4m}^R$ symmetry with $q = q_\theta = m$ & $m \in \mathbb{N}$
- However: these are only trivial extensions (as far as the MSSM is concerned)

Unique $\mathbb{Z}_4^R$ symmetry

Lee et al. (2011) ; Chen et al. (2012)

- We know already that $\begin{cases} \bullet\ q = q_\theta \\ \bullet\ q_{H_u} = q_{H_d} = 0 \mod M \end{cases}$
- Simplest possibility: $M = 4$ & $q = q_\theta = 1 \curvearrowright \mathbb{Z}_4^R$ symmetry
- Alternatives: $\mathbb{Z}_{4m}^R$ symmetry with $q = q_\theta = m$ & $m \in \mathbb{N}$
- However: these are only trivial extensions (as far as the MSSM is concerned)

bottom–line:

unique symmetry : $\mathbb{Z}_4^R$ w/ $q = q_\theta = 1$ & $q_{H_u} = q_{H_d} = 0$

first discussed in Babu et al. (2003a)

Unique $\mathbb{Z}_4^R$ symmetry & GS anomaly cancellation

☞ Anomaly coefficients

$$\begin{aligned} A_{\mathrm{SU}(3)^2-\mathbb{Z}_4^R} &= 6q - 3q_\theta = 1q_\theta \mod 4/2 \\ A_{\mathrm{SU}(2)^2-\mathbb{Z}_4^R} &= 6q + \frac{1}{2}\left(q_{H_u} + q_{H_d}\right) - 5q_\theta = 1q_\theta \mod 4/2 \end{aligned}$$

➥ Consistent with anomaly universality

bottom-line:

$\mathbb{Z}_4^R$ is anomaly–free via non–trivial GS mechanism

Implication of GS anomaly cancellation

☞ GS axion a contained in superfield S (w/ $S|_{\theta=0} = s + \mathrm{i}\,a$)

Implication of GS anomaly cancellation

- GS axion a contained in superfield S (w/ $S|_{\theta=0} = s + \mathrm{i}\, a$)
- Since $a = \mathrm{Im}\, S|_{\theta=0}$ shifts under the $\mathbb{Z}_M^R$ transformation, non–invariant superpotential terms can be made invariant by multiplying them with $\mathrm{e}^{-b\,S}$

Implication of GS anomaly cancellation

- GS axion a contained in superfield S (w/ $S|_{\theta=0} = s + \mathrm{i}\, a$)
- Since $a = \mathrm{Im}\, S|_{\theta=0}$ shifts under the $\mathbb{Z}_M^R$ transformation, non–invariant superpotential terms can be made invariant by multiplying them with $\mathrm{e}^{-b\,S}$
- Main example

  $\mu\, H_u\, H_d$ forbidden

but

$B\,\mathrm{e}^{-b\,S}\, H_u\, H_d$ allowed (for appropriate b)

R charge 2

R charge 0

Implication of GS anomaly cancellation

- GS axion a contained in superfield S (w/ $S|_{\theta=0} = s + \mathrm{i}\, a$)
- Since $a = \mathrm{Im}\, S|_{\theta=0}$ shifts under the $\mathbb{Z}_M^R$ transformation, non–invariant superpotential terms can be made invariant by multiplying them with $\mathrm{e}^{-b\, S}$
- Main example

  $\mu\, H_u\, H_d$ forbidden

 but

  $B\, \mathrm{e}^{-b\, S}\, H_u\, H_d$ allowed (for appropriate b)

bottom–line:

holomorphic $\mathrm{e}^{-b\, S}$ terms appear to violate $\mathbb{Z}_M^R$ symmetry

Interpretation

☞ GS anomaly cancellation requires coupling

$$\mathscr{L} \supset \int \mathrm{d}^2\theta\, f_S\, S\, W_\alpha W^\alpha$$

Interpretation

- GS anomaly cancellation requires coupling

$$\mathscr{L} \supset \int \mathrm{d}^2\theta \, f_S \, S \, W_\alpha W^\alpha$$

- $s = \mathrm{Re}\, S|_{\theta=0}$ contributes to $1/g^2$

Interpretation

- GS anomaly cancellation requires coupling

$$\mathscr{L} \supset \int \mathrm{d}^2\theta \, f_S \, S \, W_\alpha W^\alpha$$

- $s = \mathrm{Re}\, S|_{\theta=0}$ contributes to $1/g^2$
- holomorphic $B\,\mathrm{e}^{-b\,S}$ terms can be interpreted as non–perturbative effects (e.g. "retrofitting") Dine et al. (2006)

Interpretation

☞ GS anomaly cancellation requires coupling

$$\mathscr{L} \supset \int \mathrm{d}^2\theta\, f_S\, S\, W_\alpha W^\alpha$$

➥ $s = \mathrm{Re}\, S|_{\theta=0}$ contributes to $1/g^2$

➥ holomorphic $B\,\mathrm{e}^{-b\,S}$ terms can be interpreted as non–perturbative effects (e.g. "retrofitting") Dine et al. (2006)

bottom–line:

$$\left.\begin{array}{l}\bullet\ \text{compatibility w/ } \mathrm{SO}(10)\\ \bullet\ \text{anomaly freedom}\end{array}\right\} \curvearrowright \left\{\begin{array}{c}\mu \text{ term appears}\\ \text{non–perturbatively}\end{array}\right.$$

Implications of $\mathbb{Z}_4^R$

- Gauge invariant superpotential terms up to order 4

$$
\begin{aligned}
\mathscr{W} \;=\; & \mu H_d H_u + \kappa_i L_i H_u \\
& + Y_e^{ij} L_i H_d \overline{E}_j + Y_d^{ij} Q_i H_d \overline{D}_j + Y_u^{ij} Q_i H_u \overline{U}_j \\
& + \lambda_{ijk} L_i L_j \overline{E}_k + \lambda'_{ijk} L_i Q_j \overline{D}_k + \lambda''_{ijk} \overline{U}_i \overline{D}_j \overline{D}_k \\
& + \kappa_{ij}^{(0)} H_u L_i H_u L_j + \kappa_{ijk\ell}^{(1)} Q_i Q_j Q_k L_\ell + \kappa_{ijk\ell}^{(2)} \overline{U}_i \overline{U}_j \overline{D}_k \overline{E}_\ell + \ldots
\end{aligned}
$$

Implications of $\mathbb{Z}_4^R$

☞ Gauge invariant superpotential terms up to order 4

$$
\begin{aligned}
\mathscr{W} \;=\; & \mu\, H_d H_u + \kappa_i\, L_i H_u \\
& + Y_e^{ij}\, L_i H_d \overline{E}_j + Y_d^{ij}\, Q_i H_d \overline{D}_j + Y_u^{ij}\, Q_i H_u \overline{U}_j \\
& + \lambda_{ijk}\, L_i L_j \overline{E}_k + \lambda'_{ijk}\, L_i Q_j \overline{D}_k + \lambda''_{ijk}\, \overline{U}_i \overline{D}_j \overline{D}_k \\
& + \kappa_{ij}^{(0)}\, H_u L_i\, H_u L_j + \kappa_{ijk\ell}^{(1)}\, Q_i Q_j Q_k L_\ell + \kappa_{ijk\ell}^{(2)}\, \overline{U}_i \overline{U}_j \overline{D}_k \overline{E}_\ell + \dots
\end{aligned}
$$

forbidden at the perturbative level

Implications of $\mathbb{Z}_4^R$

- Gauge invariant superpotential terms up to order 4

$$\begin{aligned}\mathscr{W} &= \mu\, H_d H_u + \kappa_i\, L_i H_u \\ &\quad + Y_e^{ij}\, L_i H_d \overline{E}_j + Y_d^{ij}\, Q_i H_d \overline{D}_j + Y_u^{ij}\, Q_i H_u \overline{U}_j \\ &\quad + \lambda_{ijk}\, L_i L_j \overline{E}_k + \lambda'_{ijk}\, L_i Q_j \overline{D}_k + \lambda''_{ijk}\, \overline{U}_i \overline{D}_j \overline{D}_k \\ &\quad + \kappa_{ij}^{(0)}\, H_u L_i\, H_u L_j + \kappa_{ijk\ell}^{(1)}\, Q_i Q_j Q_k L_\ell + \kappa_{ijk\ell}^{(2)}\, \overline{U}_i \overline{U}_j \overline{D}_k \overline{E}_\ell + \dots\end{aligned}$$

appear at non–perturbative level

Implications of $\mathbb{Z}_4^R$

- Gauge invariant superpotential terms up to order 4

$$
\begin{aligned}
\mathscr{W} \;=\;& \mu H_d H_u + \kappa_i L_i H_u \\
& + Y_e^{ij} L_i H_d \overline{E}_j + Y_d^{ij} Q_i H_d \overline{D}_j + Y_u^{ij} Q_i H_u \overline{U}_j \\
& + \lambda_{ijk} L_i L_j \overline{E}_k + \lambda'_{ijk} L_i Q_j \overline{D}_k + \lambda''_{ijk} \overline{U}_i \overline{D}_j \overline{D}_k \\
& + \kappa_{ij}^{(0)} H_u L_i H_u L_j + \kappa_{ijk\ell}^{(1)} Q_i Q_j Q_k L_\ell + \kappa_{ijk\ell}^{(2)} \overline{U}_i \overline{U}_j \overline{D}_k \overline{E}_\ell + \dots
\end{aligned}
$$

also forbidden at
non–perturbative level by
"non–anomalous" $\mathbb{Z}_2$ subgroup
which is equivalent
to matter parity

Implications of $\mathbb{Z}_4^R$

- Gauge invariant superpotential terms up to order 4

$$\begin{aligned}\mathscr{W} &= \mu H_d H_u + \kappa_i L_i H_u \\ &\quad + Y_e^{ij} L_i H_d \overline{E}_j + Y_d^{ij} Q_i H_d \overline{D}_j + Y_u^{ij} Q_i H_u \overline{U}_j \\ &\quad + \lambda_{ijk} L_i L_j \overline{E}_k + \lambda'_{ijk} L_i Q_j \overline{D}_k + \lambda''_{ijk} \overline{U}_i \overline{D}_j \overline{D}_k \\ &\quad + \kappa_{ij}^{(0)} H_u L_i H_u L_j + \kappa_{ijk\ell}^{(1)} Q_i Q_j Q_k L_\ell + \kappa_{ijk\ell}^{(2)} \overline{U}_i \overline{U}_j \overline{D}_k \overline{E}_\ell + \dots\end{aligned}$$

μ term from $\left\{\begin{array}{l}\text{Giudice–Masiero mechanism (optional)}\\ \text{holomorphic `non–perturbative' term}\end{array}\right.$

- order parameter for R symmetry breaking: superpotential VEV $\langle\mathscr{W}\rangle$

➥ $\mu \sim m_{3/2} \simeq \langle\mathscr{W}\rangle / M_\mathrm{P}^2$

Implications of $\mathbb{Z}_4^R$

☞ Gauge invariant superpotential terms up to order 4

$$
\begin{aligned}
\mathscr{W} \;=\; & \mu\, H_d H_u + \kappa_i\, L_i H_u \\
& + Y_e^{ij}\, L_i H_d \overline{E}_j + Y_d^{ij}\, Q_i H_d \overline{D}_j + Y_u^{ij}\, Q_i H_u \overline{U}_j \\
& + \lambda_{ijk}\, L_i L_j \overline{E}_k + \lambda'_{ijk}\, L_i Q_j \overline{D}_k + \lambda''_{ijk}\, \overline{U}_i \overline{D}_j \overline{D}_k \\
& + \kappa_{ij}^{(0)}\, H_u L_i\, H_u L_j + \kappa_{ijk\ell}^{(1)}\, Q_i Q_j Q_k L_\ell + \kappa_{ijk\ell}^{(2)}\, \overline{U}_i \overline{U}_j \overline{D}_k \overline{E}_\ell + \dots
\end{aligned}
$$

non–perturbatively
generated terms harmless
$\kappa_{ijk\ell}^{(1,2)} \sim m_{3/2}/M_\mathrm{P}^2 \ll 10^{-8}/M_\mathrm{P}$

R symmetries vs. 4D GUTs

☞ We have seen that only R symmetries can forbid the μ term

$$\left.\begin{array}{l}\bullet\ \text{anomaly freedom}\\ \bullet\ \text{consistency with SU(5)}\end{array}\right\} \curvearrowright \left\{\begin{array}{l}\text{only } R \text{ symmetries}\\ \text{can forbid the } \mu \text{ term}\\ \text{in the MSSM}\end{array}\right.$$

R symmetries vs. 4D GUTs

☞ We have seen that only R symmetries can forbid the μ term

$$\left.\begin{array}{l}\bullet\ \text{anomaly freedom}\\ \bullet\ \text{consistency with SU(5)}\end{array}\right\} \curvearrowright \left\{\begin{array}{l}\text{only } R \text{ symmetries}\\ \text{can forbid the } \mu \text{ term}\\ \text{in the MSSM}\end{array}\right.$$

☞ **However**: R symmetries are not available in 4D SUSY GUTs

Fallbacher et al. (2011)

R symmetries vs. 4D GUTs

☞ We have seen that only R symmetries can forbid the μ term

- anomaly freedom
- consistency with SU(5)

$\Big\} \curvearrowright \Big\{$ only R symmetries can forbid the μ term in the MSSM

☞ **However**: R symmetries are not available in 4D SUSY GUTs

Fallbacher et al. (2011)

☞ Assumptions:

(i) GUT model in four dimensions based on $G \supset \mathrm{SU}(5)$

(ii) GUT symmetry breaking is spontaneous

(iii) Only finite number of fields

R symmetries vs. 4D GUTs

- We have seen that only R symmetries can forbid the μ term

$$\left.\begin{array}{l}\bullet\ \text{anomaly freedom}\\ \bullet\ \text{consistency with SU(5)}\end{array}\right\} \curvearrowright \left\{\begin{array}{l}\text{only } R \text{ symmetries}\\ \text{can forbid the } \mu \text{ term}\\ \text{in the MSSM}\end{array}\right.$$

- **However**: R symmetries are not available in 4D SUSY GUTs

Fallbacher et al. (2011)

- Assumptions:
  - (i) GUT model in four dimensions based on $G \supset \mathrm{SU}(5)$
  - (ii) GUT symmetry breaking is spontaneous
  - (iii) Only finite number of fields

- One can prove that it is impossible to get low–energy effective theory with both:
  1. just the MSSM field content
  2. residual R symmetries

The basic argument

☞ Consider $\mathrm{SU}(5)$ model with an (arbitrary) R symmetry and a $\mathbf{24}$–plet breaking $\mathrm{SU}(5) \to G_{\mathrm{SM}}$

$$\mathbf{24} \;\to\; (\mathbf{8},\mathbf{1})_0 \oplus (\mathbf{1},\mathbf{3})_0 \oplus (\mathbf{3},\mathbf{2})_{-5/6} \oplus (\overline{\mathbf{3}},\mathbf{2})_{5/6} \oplus (\mathbf{1},\mathbf{1})_0$$

The basic argument

- Consider $\mathrm{SU}(5)$ model with an (arbitrary) R symmetry and a $\mathbf{24}$–plet breaking $\mathrm{SU}(5) \to G_{\mathrm{SM}}$

$$\mathbf{24} \;\to\; (\mathbf{8},\mathbf{1})_0 \oplus (\mathbf{1},\mathbf{3})_0 \oplus (\mathbf{3},\mathbf{2})_{-5/6} \oplus (\overline{\mathbf{3}},\mathbf{2})_{5/6} \oplus (\mathbf{1},\mathbf{1})_0$$

R charge 0

get eaten

The basic argument

☞ Consider $\mathrm{SU}(5)$ model with an (arbitrary) R symmetry and a $\mathbf{24}$–plet breaking $\mathrm{SU}(5) \to G_{\mathrm{SM}}$

$$\mathbf{24} \;\to\; (\mathbf{8},\mathbf{1})_0 \oplus (\mathbf{1},\mathbf{3})_0 \oplus (\mathbf{3},\mathbf{2})_{-5/6} \oplus (\overline{\mathbf{3}},\mathbf{2})_{5/6} \oplus (\mathbf{1},\mathbf{1})_0$$

R charge 0

extra massless states

get eaten

The basic argument

- Consider $\mathrm{SU}(5)$ model with an (arbitrary) R symmetry and a $\mathbf{24}$–plet breaking $\mathrm{SU}(5) \to G_{\mathrm{SM}}$

$$\mathbf{24} \;\to\; (\mathbf{8},\mathbf{1})_0 \oplus (\mathbf{1},\mathbf{3})_0 \oplus (\mathbf{3},\mathbf{2})_{-5/6} \oplus (\overline{\mathbf{3}},\mathbf{2})_{5/6} \oplus (\mathbf{1},\mathbf{1})_0$$

R charge 0

extra massless states

get eaten

- Introducing extra $\mathbf{24}$–plets with R charge 2 does not help because this would lead to massless $(\mathbf{3},\mathbf{2})_{-5/6} \oplus (\overline{\mathbf{3}},\mathbf{2})_{5/6}$ representations

The basic argument

- Consider $\mathrm{SU}(5)$ model with an (arbitrary) R symmetry and a $\mathbf{24}$–plet breaking $\mathrm{SU}(5) \to G_{\mathrm{SM}}$

$$\mathbf{24} \;\to\; (\mathbf{8},\mathbf{1})_0 \oplus (\mathbf{1},\mathbf{3})_0 \oplus (\mathbf{3},\mathbf{2})_{-5/6} \oplus (\overline{\mathbf{3}},\mathbf{2})_{5/6} \oplus (\mathbf{1},\mathbf{1})_0$$

R charge 0

extra massless states

get eaten

- Introducing extra $\mathbf{24}$–plets with R charge 2 does not help because this would lead to massless $(\mathbf{3},\mathbf{2})_{-5/6} \oplus (\overline{\mathbf{3}},\mathbf{2})_{5/6}$ representations

- Iterating this argument shows that with a finite number of $\mathbf{24}$–plets one will always have massless exotics

The basic argument

- Consider $\mathrm{SU}(5)$ model with an (arbitrary) R symmetry and a $\mathbf{24}$–plet breaking $\mathrm{SU}(5) \to G_{\mathrm{SM}}$

$$\mathbf{24} \;\to\; (\mathbf{8},\mathbf{1})_0 \oplus (\mathbf{1},\mathbf{3})_0 \oplus (\mathbf{3},\mathbf{2})_{-5/6} \oplus (\overline{\mathbf{3}},\mathbf{2})_{5/6} \oplus (\mathbf{1},\mathbf{1})_0$$

R charge 0

extra massless states

get eaten

- Introducing extra $\mathbf{24}$–plets with R charge 2 does not help because this would lead to massless $(\mathbf{3},\mathbf{2})_{-5/6} \oplus (\overline{\mathbf{3}},\mathbf{2})_{5/6}$ representations
- Iterating this argument shows that with a finite number of $\mathbf{24}$–plets one will always have massless exotics
- Loophole for infinitely many $\mathbf{24}$–plets

cf. Goodman and Witten (1986)

Generalizing the basic argument

☞ It is possible to generalize the basic argument to

- arbitrary $\mathrm{SU}(5)$ representations
- larger GUT groups $G \supset \mathrm{SU}(5)$
- singlet extensions of the MSSM

for details see Fallbacher et al. (2011)

Discussion

- A 'natural' solution of the μ and/or doublet–triplet splitting problem requires a symmetry that forbids μ

Discussion

- A ‘natural’ solution of the μ and/or doublet–triplet splitting problem requires a symmetry that forbids μ
- We learned that:
  1. only R symmetries can forbid the μ term
  2. anomaly matching requires the existence of split multiplets
  3. R symmetries are not available in 4D GUTs

Discussion

- A 'natural' solution of the μ and/or doublet–triplet splitting problem requires a symmetry that forbids μ
- We learned that:
  1. only R symmetries can forbid the μ term
  2. anomaly matching requires the existence of split multiplets
  3. R symmetries are not available in 4D GUTs

bottom–line:
'Natural' solutions to the μ and/or doublet–triplet splitting problems are not available in four dimensions!

Discussion

- A 'natural' solution of the μ and/or doublet–triplet splitting problem requires a symmetry that forbids μ
- We learned that:
  1. only R symmetries can forbid the μ term
  2. anomaly matching requires the existence of split multiplets
  3. R symmetries are not available in 4D GUTs

bottom–line:

'Natural' solutions to the
μ and/or doublet–triplet splitting problems
are not available in four dimensions!

➥ Need to go to extra dimensions/strings

String model(s)

- evading the no–go theorem
- origin of $\mathbb{Z}_4^R$
- higher–dimensional operators (effective μ term etc.)

Grand unification in higher dimensions

☞ Well known: higher–dimensional GUTs appear more "appealing"

Grand unification in higher dimensions

- ☞ Well known: higher–dimensional GUTs appear more "appealing"
- ☞ New possibilities of symmetry breaking arise

Witten (1985) ; Breit et al. (1985)

Grand unification in higher dimensions

- Well known: higher–dimensional GUTs appear more "appealing"
- New possibilities of symmetry breaking arise

Witten (1985) ; Breit et al. (1985)

- KK towers provide us with infinitely many states and allow us to evade the no–go theorem

Grand unification in higher dimensions

- Well known: higher–dimensional GUTs appear more "appealing"
- New possibilities of symmetry breaking arise

Witten (1985) ; Breit et al. (1985)

- KK towers provide us with infinitely many states and allow us to evade the no–go theorem
- Even more, R symmetries have a clear geometric interpretation in terms of the Lorentz symmetry of compact dimensions

Discrete R symmetries from orbifolds

- R symmetries **are** available in higher–dimensional/stringy GUTs

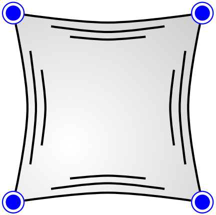
Discrete R symmetries from orbifolds

- R symmetries **are** available in higher–dimensional/stringy GUTs
- Discrete R symmetries arise as remnants of the Lorentz symmetry of compact dimensions
  and are arguably on the same footing as the fundamental symmetries C, P and T

Discrete R symmetries from orbifolds

- R symmetries **are** available in higher–dimensional/stringy GUTs
- Discrete R symmetries arise as remnants of the Lorentz symmetry of compact dimensions
- Example: order four discrete R symmetry $\mathbb{Z}_4^R$ from $\mathbb{Z}_2$ orbifold plane

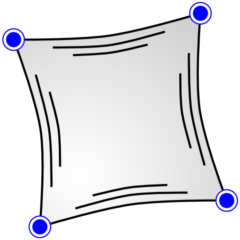
back



Discrete R symmetries from orbifolds

- R symmetries **are** available in higher–dimensional/stringy GUTs
- Discrete R symmetries arise as remnants of the Lorentz symmetry of compact dimensions
- Example: order four discrete R symmetry $\mathbb{Z}_4^R$
  from $\mathbb{Z}_2$ orbifold plane

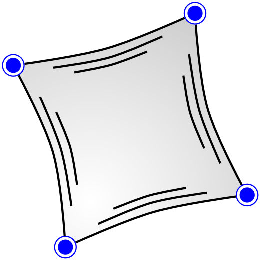
back



Discrete R symmetries from orbifolds

- R symmetries **are** available in higher–dimensional/stringy GUTs
- Discrete R symmetries arise as remnants of the Lorentz symmetry of compact dimensions
- Example: order four discrete R symmetry $\mathbb{Z}_4^R$ from $\mathbb{Z}_2$ orbifold plane

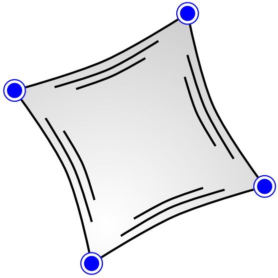
back



Discrete R symmetries from orbifolds

- R symmetries **are** available in higher–dimensional/stringy GUTs
- Discrete R symmetries arise as remnants of the Lorentz symmetry of compact dimensions
- Example: order four discrete R symmetry $\mathbb{Z}_4^R$ from $\mathbb{Z}_2$ orbifold plane

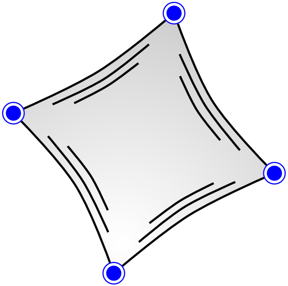
back



Discrete R symmetries from orbifolds

- R symmetries **are** available in higher–dimensional/stringy GUTs
- Discrete R symmetries arise as remnants of the Lorentz symmetry of compact dimensions
- Example: order four discrete R symmetry $\mathbb{Z}_4^R$ from $\mathbb{Z}_2$ orbifold plane

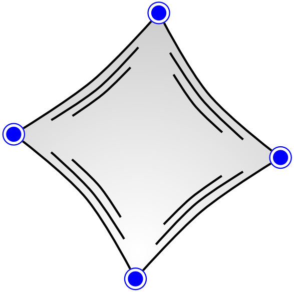
back



Discrete R symmetries from orbifolds

- R symmetries **are** available in higher–dimensional/stringy GUTs
- Discrete R symmetries arise as remnants of the Lorentz symmetry of compact dimensions
- Example: order four discrete R symmetry $\mathbb{Z}_4^R$ from $\mathbb{Z}_2$ orbifold plane

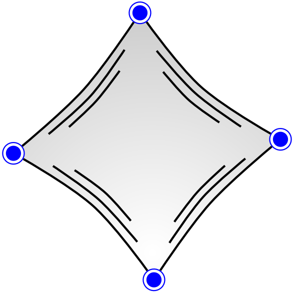
back



Discrete R symmetries from orbifolds

- R symmetries **are** available in higher–dimensional/stringy GUTs
- Discrete R symmetries arise as remnants of the Lorentz symmetry of compact dimensions
- Example: order four discrete R symmetry $\mathbb{Z}_4^R$ from $\mathbb{Z}_2$ orbifold plane

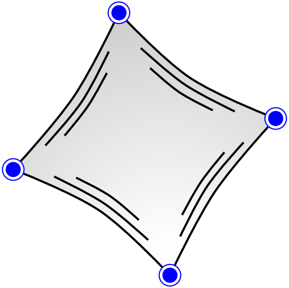
back



Discrete R symmetries from orbifolds

- R symmetries **are** available in higher–dimensional/stringy GUTs
- Discrete R symmetries arise as remnants of the Lorentz symmetry of compact dimensions
- Example: order four discrete R symmetry $\mathbb{Z}_4^R$ from $\mathbb{Z}_2$ orbifold plane

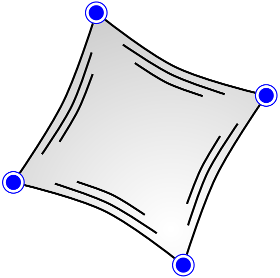
back



Discrete R symmetries from orbifolds

- R symmetries **are** available in higher–dimensional/stringy GUTs
- Discrete R symmetries arise as remnants of the Lorentz symmetry of compact dimensions
- Example: order four discrete R symmetry $\mathbb{Z}_4^R$ from $\mathbb{Z}_2$ orbifold plane

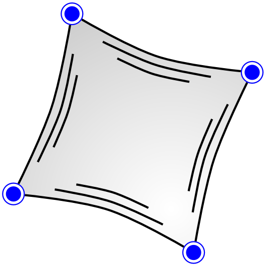
back



Discrete R symmetries from orbifolds

- R symmetries **are** available in higher–dimensional/stringy GUTs
- Discrete R symmetries arise as remnants of the Lorentz symmetry of compact dimensions
- Example: order four discrete R symmetry $\mathbb{Z}_4^R$ from $\mathbb{Z}_2$ orbifold plane

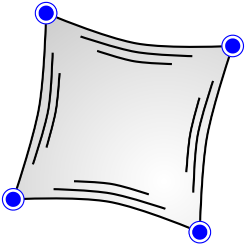
back



Discrete R symmetries from orbifolds

- R symmetries **are** available in higher–dimensional/stringy GUTs
- Discrete R symmetries arise as remnants of the Lorentz symmetry of compact dimensions
- Example: order four discrete R symmetry $\mathbb{Z}_4^R$ from $\mathbb{Z}_2$ orbifold plane

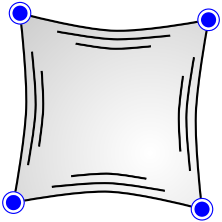
back



Discrete R symmetries from orbifolds

- R symmetries **are** available in higher–dimensional/stringy GUTs
- Discrete R symmetries arise as remnants of the Lorentz symmetry of compact dimensions
- Example: order four discrete R symmetry $\mathbb{Z}_4^R$ from $\mathbb{Z}_2$ orbifold plane

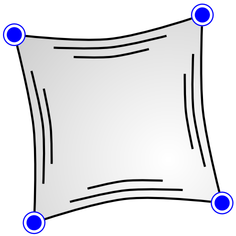
back



Discrete R symmetries from orbifolds

- R symmetries **are** available in higher–dimensional/stringy GUTs
- Discrete R symmetries arise as remnants of the Lorentz symmetry of compact dimensions
- Example: order four discrete R symmetry $\mathbb{Z}_4^R$ from $\mathbb{Z}_2$ orbifold plane

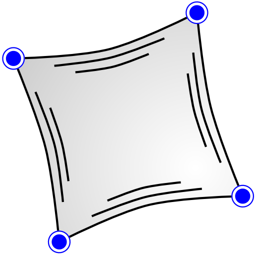
back



Discrete R symmetries from orbifolds

- R symmetries **are** available in higher–dimensional/stringy GUTs
- Discrete R symmetries arise as remnants of the Lorentz symmetry of compact dimensions
- Example: order four discrete R symmetry $\mathbb{Z}_4^R$
  from $\mathbb{Z}_2$ orbifold plane

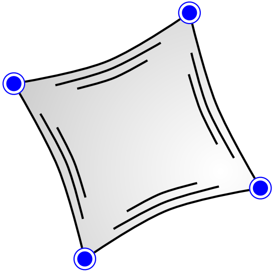
back



Discrete R symmetries from orbifolds

- R symmetries **are** available in higher–dimensional/stringy GUTs
- Discrete R symmetries arise as remnants of the Lorentz symmetry of compact dimensions
- Example: order four discrete R symmetry $\mathbb{Z}_4^R$ from $\mathbb{Z}_2$ orbifold plane

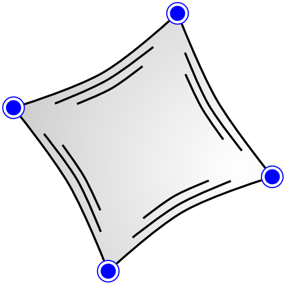
back



Discrete R symmetries from orbifolds

- R symmetries **are** available in higher–dimensional/stringy GUTs
- Discrete R symmetries arise as remnants of the Lorentz symmetry of compact dimensions
- Example: order four discrete $\boldsymbol{R}$ symmetry $\mathbb{Z}_4^R$ from $\mathbb{Z}_2$ orbifold plane

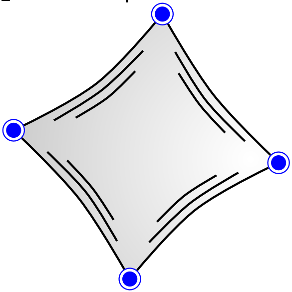
back



Discrete R symmetries from orbifolds

- R symmetries **are** available in higher–dimensional/stringy GUTs
- Discrete R symmetries arise as remnants of the Lorentz symmetry of compact dimensions
- Example: order four discrete R symmetry $\mathbb{Z}_4^R$ from $\mathbb{Z}_2$ orbifold plane

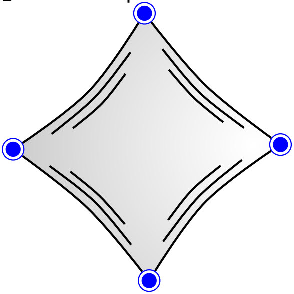
back



Discrete R symmetries from orbifolds

- R symmetries **are** available in higher–dimensional/stringy GUTs
- Discrete R symmetries arise as remnants of the Lorentz symmetry of compact dimensions
- Example: order four discrete R symmetry $\mathbb{Z}_4^R$ from $\mathbb{Z}_2$ orbifold plane

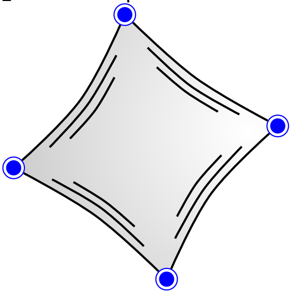
back



Discrete R symmetries from orbifolds

- R symmetries **are** available in higher–dimensional/stringy GUTs
- Discrete R symmetries arise as remnants of the Lorentz symmetry of compact dimensions
- Example: order four discrete R symmetry $\mathbb{Z}_4^R$ from $\mathbb{Z}_2$ orbifold plane

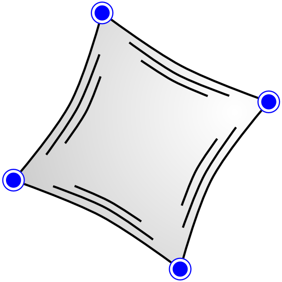
back



Discrete R symmetries from orbifolds

- R symmetries **are** available in higher–dimensional/stringy GUTs
- Discrete R symmetries arise as remnants of the Lorentz symmetry of compact dimensions
- Example: order four discrete R symmetry $\mathbb{Z}_4^R$ from $\mathbb{Z}_2$ orbifold plane

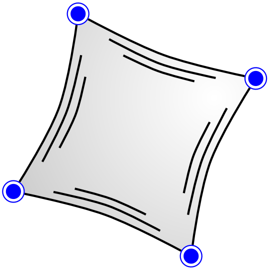
back



Discrete R symmetries from orbifolds

- R symmetries **are** available in higher–dimensional/stringy GUTs
- Discrete R symmetries arise as remnants of the Lorentz symmetry of compact dimensions
- Example: order four discrete R symmetry $\mathbb{Z}_4^R$ from $\mathbb{Z}_2$ orbifold plane

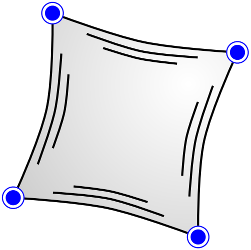
back



Discrete R symmetries from orbifolds

- R symmetries **are** available in higher–dimensional/stringy GUTs
- Discrete R symmetries arise as remnants of the Lorentz symmetry of compact dimensions
- Example: order four discrete R symmetry $\mathbb{Z}_4^R$ from $\mathbb{Z}_2$ orbifold plane

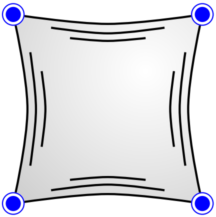
back



Discrete R symmetries from orbifolds

- R symmetries **are** available in higher–dimensional/stringy GUTs
- Discrete R symmetries arise as remnants of the Lorentz symmetry of compact dimensions
- Example: order four discrete R symmetry $\mathbb{Z}_4^R$ from $\mathbb{Z}_2$ orbifold plane

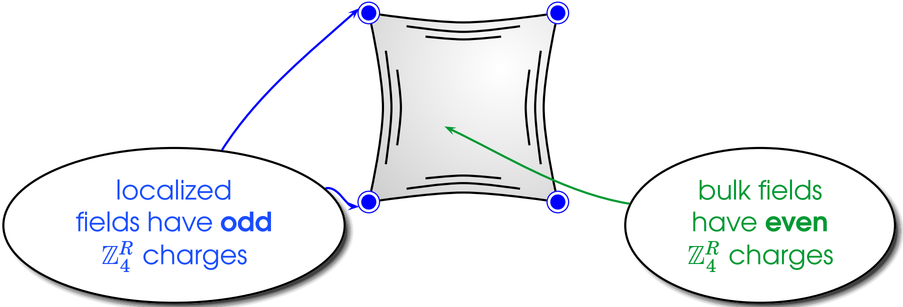
back



Discrete R symmetries from orbifolds

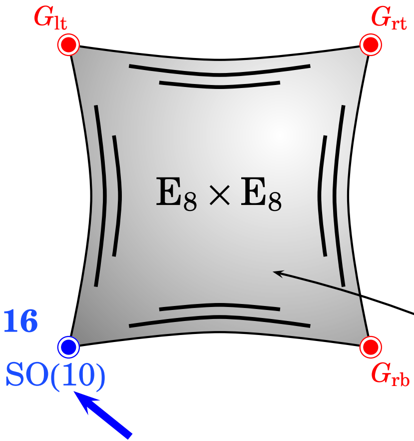
- R symmetries **are** available in higher–dimensional/stringy GUTs
- Discrete R symmetries arise as remnants of the Lorentz symmetry of compact dimensions
- Example: order four discrete R symmetry $\mathbb{Z}_4^R$ from $\mathbb{Z}_2$ orbifold plane

back



Local grand unification & $\mathbb{Z}_4^R$

Buchmuller et al. (2005)



$\mathbb{Z}_4^R$ (bottom–up)

	$\mathbb{Z}_4^R$ charge
matter	1
Higgs	0

SM generation(s):

localized in region with SO(10) symmetry

Higgs doublets:

live in the 'bulk'

$\mathbb{Z}_4^R$ from a $\mathbb{Z}_2 \times \mathbb{Z}_2$ orbifold model

Blaszczyk et al. (2010) ; Kappl et al. (2011)

- We constructed models with the exact MSSM spectrum based on $\mathbb{Z}_2 \times \mathbb{Z}_2$ orbifolds

details

$\mathbb{Z}_4^R$ from a $\mathbb{Z}_2 \times \mathbb{Z}_2$ orbifold model

Blaszczyk et al. (2010) ; Kappl et al. (2011)

- We constructed models with the exact MSSM spectrum based on $\mathbb{Z}_2 \times \mathbb{Z}_2$ orbifolds

details

- We succeeded in finding vacua with the $\mathbb{Z}_4^R$ symmetry

'vacuum' means a field configuration that preserves supersymmetry perturbatively

$\mathbb{Z}_4^R$ from a $\mathbb{Z}_2 \times \mathbb{Z}_2$ orbifold model

Blaszczyk et al. (2010) ; Kappl et al. (2011)

- We constructed models with the exact MSSM spectrum based on $\mathbb{Z}_2 \times \mathbb{Z}_2$ orbifolds

details

- We succeeded in finding vacua with the $\mathbb{Z}_4^R$ symmetry

- Various good features
  - ✓ non–local GUT breaking

$\mathbb{Z}_4^R$ from a $\mathbb{Z}_2 \times \mathbb{Z}_2$ orbifold model

Blaszczyk et al. (2010) ; Kappl et al. (2011)

- We constructed models with the exact MSSM spectrum based on $\mathbb{Z}_2 \times \mathbb{Z}_2$ orbifolds

▸ details

- We succeeded in finding vacua with the $\mathbb{Z}_4^R$ symmetry
- ☺ Various good features
  - ✓ non–local GUT breaking
  - ✓ no ‘fractionally charged exotics’

$\mathbb{Z}_4^R$ from a $\mathbb{Z}_2 \times \mathbb{Z}_2$ orbifold model

Blaszczyk et al. (2010) ; Kappl et al. (2011)

- We constructed models with the exact MSSM spectrum based on $\mathbb{Z}_2 \times \mathbb{Z}_2$ orbifolds

▸ details

- We succeeded in finding vacua with the $\mathbb{Z}_4^R$ symmetry

- ☺ Various good features
  - ✓ non–local GUT breaking
  - ✓ no 'fractionally charged exotics'
  - ✓ (most) exotics decouple at the linear level in SM singlets, i.e. just MSSM below GUT scale with masslessness of Higgs fields ensured by $\mathbb{Z}_4^R$

$\mathbb{Z}_4^R$ from a $\mathbb{Z}_2 \times \mathbb{Z}_2$ orbifold model

Blaszczyk et al. (2010) ; Kappl et al. (2011)

- We constructed models with the exact MSSM spectrum based on $\mathbb{Z}_2 \times \mathbb{Z}_2$ orbifolds

details

- We succeeded in finding vacua with the $\mathbb{Z}_4^R$ symmetry

- Various good features
  - ✓ non–local GUT breaking
  - ✓ no ‘fractionally charged exotics’
  - ✓ (most) exotics decouple at the linear level in SM singlets, i.e. just MSSM below GUT scale with masslessness of Higgs fields ensured by $\mathbb{Z}_4^R$
  - ✓ non–trivial full–rank Yukawa couplings

$\mathbb{Z}_4^R$ from a $\mathbb{Z}_2 \times \mathbb{Z}_2$ orbifold model

Blaszczyk et al. (2010) ; Kappl et al. (2011)

- We constructed models with the exact MSSM spectrum based on $\mathbb{Z}_2 \times \mathbb{Z}_2$ orbifolds

details

- We succeeded in finding vacua with the $\mathbb{Z}_4^R$ symmetry

- Various good features
  - ✓ non–local GUT breaking
  - ✓ no ‘fractionally charged exotics’
  - ✓ (most) exotics decouple at the linear level in SM singlets, i.e. just MSSM below GUT scale with masslessness of Higgs fields ensured by $\mathbb{Z}_4^R$
  - ✓ non–trivial full–rank Yukawa couplings
  - ✓ gauge–top unification

$\mathbb{Z}_4^R$ from a $\mathbb{Z}_2 \times \mathbb{Z}_2$ orbifold model

Blaszczyk et al. (2010) ; Kappl et al. (2011)

- We constructed models with the exact MSSM spectrum based on $\mathbb{Z}_2 \times \mathbb{Z}_2$ orbifolds

details

- We succeeded in finding vacua with the $\mathbb{Z}_4^R$ symmetry

- Various good features
  - ✓ non–local GUT breaking
  - ✓ no 'fractionally charged exotics'
  - ✓ (most) exotics decouple at the linear level in SM singlets, i.e. just MSSM below GUT scale with masslessness of Higgs fields ensured by $\mathbb{Z}_4^R$
  - ✓ non–trivial full–rank Yukawa couplings
  - ✓ gauge–top unification
  - ✓ SU(5) relation $y_\tau \simeq y_b$

$\mathbb{Z}_4^R$ from a $\mathbb{Z}_2 \times \mathbb{Z}_2$ orbifold model

Blaszczyk et al. (2010) ; Kappl et al. (2011)

☞ We constructed models with the exact MSSM spectrum based on $\mathbb{Z}_2 \times \mathbb{Z}_2$ orbifolds

▸ details

☞ We succeeded in finding vacua with the $\mathbb{Z}_4^R$ symmetry

☺ Various good features

- ✓ non–local GUT breaking
- ✓ no 'fractionally charged exotics'
- ✓ (most) exotics decouple at the linear level in SM singlets, i.e. just MSSM below GUT scale with masslessness of Higgs fields ensured by $\mathbb{Z}_4^R$
- ✓ non–trivial full–rank Yukawa couplings
- ✓ gauge–top unification
- ✓ SU(5) relation $y_\tau \simeq y_b$

☹ However:

- $\mathrm{SU}(5)$ Yukawa relations also for light generations
- hidden sector gauge group only $\mathrm{SU}(3)$

$\mathbb{Z}_4^R$ from a $\mathbb{Z}_2 \times \mathbb{Z}_2$ orbifold model

Blaszczyk et al. (2010) ; Kappl et al. (2011)

- We constructed models with the exact MSSM spectrum based on $\mathbb{Z}_2 \times \mathbb{Z}_2$ orbifolds

details

- We succeeded in finding vacua with the $\mathbb{Z}_4^R$ symmetry
- Various good features
  - ✓ non–local GUT breaking
  - ✓ no 'fractionally charged exotics'
  - ✓ (most) exotics decouple at the linear level in SM singlets, i.e. just MSSM below GUT scale with masslessness of Higgs fields ensured by $\mathbb{Z}_4^R$
  - ✓ non–trivial full–rank Yukawa couplings
  - ✓ gauge–top unification
  - ✓ SU(5) relation $y_\tau \simeq y_b$

bottom–line:

Successful string embedding of $\mathbb{Z}_4^R$ possible!

SUSY vacua with $\mathbb{Z}_4^R$

- Recall: situation for gauge theories with generic superpotential

e.g. Luty and Taylor (1996)

solutions of D–equations $\cap$ solutions of F–equations $=$ non–trivial

SUSY vacua with $\mathbb{Z}_4^R$

☞ Recall: situation for gauge theories with generic superpotential e.g. Luty and Taylor (1996)

solutions of D–equations $\cap$ solutions of F–equations = non–trivial

☞ However: $\langle \mathscr{W} \rangle \neq 0$ generically

SUSY vacua with $\mathbb{Z}_4^R$

- Recall: situation for gauge theories with generic superpotential e.g. Luty and Taylor (1996)

solutions of D–equations $\cap$ solutions of F–equations $=$ non–trivial

- However: $\langle \mathscr{W} \rangle \neq 0$ generically
- Vacua with residual $\mathbb{Z}_4^R$ are slightly different

SUSY vacua with $\mathbb{Z}_4^R$

- Recall: situation for gauge theories with generic superpotential e.g. Luty and Taylor (1996)

  solutions of D–equations $\cap$ solutions of F–equations $=$ non–trivial

- However: $\langle \mathscr{W} \rangle \neq 0$ generically
- Vacua with residual $\mathbb{Z}_4^R$ are slightly different
- Example: consider one field ϕ_0 with R-charge 0 and one field ϕ_2 with R-charge 2

$$\mathscr{W} = \phi_2 \cdot f(\phi_0) + \mathcal{O}({\phi_2}^3) \qquad \text{with} \quad \langle \mathscr{W} \rangle = 0 \text{ automatic}$$

SUSY vacua with $\mathbb{Z}_4^R$

- Recall: situation for gauge theories with generic superpotential e.g. Luty and Taylor (1996)

  solutions of D–equations $\cap$ solutions of F–equations = non–trivial

- However: $\langle \mathscr{W} \rangle \neq 0$ generically
- Vacua with residual $\mathbb{Z}_4^R$ are slightly different
- Example: consider one field ϕ_0 with R-charge 0 and one field ϕ_2 with R-charge 2

$$\mathscr{W} \;=\; \phi_2 \cdot f(\phi_0) + \mathcal{O}({\phi_2}^3) \qquad \text{with} \quad \langle \mathscr{W} \rangle \;=\; 0 \text{ automatic}$$

$$F_{\phi_0} \;=\; \frac{\partial \mathscr{W}}{\partial \phi_0} \;=\; \phi_2 \cdot f'(\phi_0) + \mathcal{O}({\phi_2}^3) \;=\; 0 \quad @\ \phi_2 \;=\; 0$$

SUSY vacua with $\mathbb{Z}_4^R$

- Recall: situation for gauge theories with generic superpotential

e.g. Luty and Taylor (1996)

solutions of D–equations $\cap$ solutions of F–equations $=$ non–trivial

- However: $\langle \mathscr{W} \rangle \neq 0$ generically
- Vacua with residual $\mathbb{Z}_4^R$ are slightly different
- Example: consider one field ϕ_0 with R-charge 0 and one field ϕ_2 with R-charge 2

$$\mathscr{W} = \phi_2 \cdot f(\phi_0) + \mathcal{O}({\phi_2}^3) \qquad \text{with} \quad \langle \mathscr{W} \rangle = 0 \text{ automatic}$$

$$F_{\phi_0} = \frac{\partial \mathscr{W}}{\partial \phi_0} = \phi_2 \cdot f'(\phi_0) + \mathcal{O}({\phi_2}^3) = 0 \quad @\ \phi_2 = 0$$

$$F_{\phi_2} = \frac{\partial \mathscr{W}}{\partial \phi_2} = f(\phi_0) \stackrel{!}{=} 0 \quad \text{fixes } \phi_0$$

SUSY vacua with $\mathbb{Z}_4^R$ (cont'd)

- Generalization: consider N fields $\phi_0^{(i)}$ with R-charge 0 and M fields $\phi_2^{(j)}$ with R-charge 2

SUSY vacua with $\mathbb{Z}_4^R$ (cont'd)

☞ Generalization: consider N fields $\phi_0^{(i)}$ with R-charge 0 and M fields $\phi_2^{(j)}$ with R-charge 2

$$\mathscr{W} = \sum_j \phi_2^{(j)} \cdot f^{(j)}(\phi_0^{(1)}, \dots) + \dots$$

SUSY vacua with $\mathbb{Z}_4^R$ (cont'd)

- Generalization: consider N fields $\phi_0^{(i)}$ with R-charge 0 and M fields $\phi_2^{(j)}$ with R-charge 2

$$\mathscr{W} = \sum_j \phi_2^{(j)} \cdot f^{(j)}(\phi_0^{(1)}, \dots) + \dots$$

$$F_{\phi_0^{(i)}} = 0 \quad \text{automatically}$$

SUSY vacua with $\mathbb{Z}_4^R$ (cont'd)

- Generalization: consider N fields $\phi_0^{(i)}$ with R-charge 0 and M fields $\phi_2^{(j)}$ with R-charge 2

$$\mathscr{W} = \sum_j \phi_2^{(j)} \cdot f^{(j)}(\phi_0^{(1)}, \dots) + \dots$$

$$F_{\phi_0^{(i)}} = 0 \quad \text{automatically}$$

$$F_{\phi_2^{(j)}} = 0 \quad \curvearrowright \quad f^{(j)}(\phi_0^{(1)}, \dots) \stackrel{!}{=} 0 \quad \curvearrowright \text{ M constraints on N fields}$$

SUSY vacua with $\mathbb{Z}_4^R$ (cont'd)

- Generalization: consider N fields $\phi_0^{(i)}$ with R-charge 0 and M fields $\phi_2^{(j)}$ with R-charge 2

$$\mathscr{W} = \sum_j \phi_2^{(j)} \cdot f^{(j)}(\phi_0^{(1)}, \dots) + \dots$$

$$F_{\phi_0^{(i)}} = 0 \quad \text{automatically}$$

$$F_{\phi_2^{(j)}} = 0 \quad \curvearrowright \quad f^{(j)}(\phi_0^{(1)}, \dots) \stackrel{!}{=} 0 \quad \curvearrowright \quad M \text{ constraints on } N \text{ fields}$$

- ➥ expect solutions for $N \geq M$

SUSY vacua with $\mathbb{Z}_4^R$ (cont'd)

- Generalization: consider N fields $\phi_0^{(i)}$ with R-charge 0 and M fields $\phi_2^{(j)}$ with R-charge 2

$$\mathscr{W} = \sum_j \phi_2^{(j)} \cdot f^{(j)}(\phi_0^{(1)}, \dots) + \dots$$

$$F_{\phi_0^{(i)}} = 0 \quad \text{automatically}$$

$$F_{\phi_2^{(j)}} = 0 \quad \curvearrowright \quad f^{(j)}(\phi_0^{(1)}, \dots) \stackrel{!}{=} 0 \quad \curvearrowright \quad M \text{ constraints on } N \text{ fields}$$

- ↪ expect solutions for $N \geq M$
- ↪ M non–trivial mass terms (also for T- and Z–moduli!)

SUSY vacua with $\mathbb{Z}_4^R$ (cont'd)

- Generalization: consider N fields $\phi_0^{(i)}$ with R-charge 0 and M fields $\phi_2^{(j)}$ with R-charge 2

$$\mathscr{W} = \sum_j \phi_2^{(j)} \cdot f^{(j)}(\phi_0^{(1)}, \dots) + \dots$$

$$F_{\phi_0^{(i)}} = 0 \quad \text{automatically}$$

$$F_{\phi_2^{(j)}} = 0 \quad \curvearrowright \quad f^{(j)}(\phi_0^{(1)}, \dots) \stackrel{!}{=} 0 \quad \curvearrowright \ M \text{ constraints on } N \text{ fields}$$

- ➥ expect solutions for $N \geq M$
- ➥ M non–trivial mass terms (also for T- and Z–moduli!)

- Have identified configurations with $N \geq M$ in our $\mathbb{Z}_2 \times \mathbb{Z}_2$ model(s)

Summary

Summary

- Assumptions:
  (i) anomaly freedom (allow for GS anomaly cancellation)
  (ii) μ term forbidden at perturbative level
  (iii) Yukawa couplings and Weinberg neutrino mass operator allowed
  (iv) $\mathrm{SU}(5)$ or $\mathrm{SO}(10)$ GUT relations for quarks and leptons

Summary

- Assumptions:
  - (i) anomaly freedom (allow for GS anomaly cancellation)
  - (ii) μ term forbidden at perturbative level
  - (iii) Yukawa couplings and Weinberg neutrino mass operator allowed
  - (iv) $\mathrm{SU}(5)$ or $\mathrm{SO}(10)$ GUT relations for quarks and leptons

- Have shown:
  1. assuming (i) & $\mathrm{SU}(5)$ relations:
     $\curvearrowright$ only R symmetries can forbid the μ term
  2. assuming (i)–(iii) & $\mathrm{SO}(10)$ relations:
     $\curvearrowright$ unique $\mathbb{Z}_4^R$ symmetry
  3. R symmetries are not available in 4D GUTs
     $\curvearrowright$ no 'natural' solution to doublet–triplet splitting in 4D!

Summary

☞ A simple anomaly–free $\mathbb{Z}_4^R$ symmetry can

- provide a solution to the μ problem
- suppress proton decay operators

Summary

☞ A simple anomaly–free $\mathbb{Z}_4^R$ symmetry can

- provide a solution to the μ problem
- suppress proton decay operators

$$\left.\begin{array}{r}\text{universal anomaly coefficients}\\ \text{universal charges for matter}\\ \text{forbid } \mu \text{ @ tree–level}\\ \text{allow Yukawa couplings}\\ \text{allow Weinberg operator}\end{array}\right\} \curvearrowright \text{ unique } \mathbb{Z}_4^R$$

Summary

☞ A simple anomaly–free $\mathbb{Z}_4^R$ symmetry can

- provide a solution to the μ problem
- suppress proton decay operators

$$\left.\begin{array}{r}\text{universal anomaly coefficients}\\ \text{universal charges for matter}\\ \text{forbid } \mu \text{ @ tree–level}\\ \text{allow Yukawa couplings}\\ \text{allow Weinberg operator}\end{array}\right\} \curvearrowright \text{ unique } \mathbb{Z}_4^R$$

$$\mathbb{Z}_4^R \curvearrowright \left\{\begin{array}{l}\text{dim. 4 proton decay operators completely forbidden}\\ \text{dim. 5 proton decay operators highly suppressed}\\ \mu \text{ appears non–perturbatively}\end{array}\right.$$

Summary

☞ Embedding into string theory allows us to understand where the $\mathbb{Z}_4^R$ symmetry comes from: it may arise as a discrete remnant of Lorentz symmetry in extra dimensions

Summary

- Embedding into string theory allows us to understand where the $\mathbb{Z}_4^R$ symmetry comes from: it may arise as a discrete remnant of Lorentz symmetry in extra dimensions
- Guided by the (unique) $\mathbb{Z}_4^R$ symmetry we have constructed a globally consistent string model with:
  - exact MSSM spectrum
  - non–local/Wilson line GUT breaking
  - non–trivial full–rank Yukawa couplings
  - exact matter parity
  - $\mu \sim m_{3/2}$
  - dimension five proton decay operators sufficiently suppressed

Summary

- Embedding into string theory allows us to understand where the $\mathbb{Z}_4^R$ symmetry comes from: it may arise as a discrete remnant of Lorentz symmetry in extra dimensions

- Guided by the (unique) $\mathbb{Z}_4^R$ symmetry we have constructed a globally consistent string model with:
  - exact MSSM spectrum
  - non–local/Wilson line GUT breaking
  - non–trivial full–rank Yukawa couplings
  - exact matter parity
  - $\mu \sim m_{3/2}$
  - dimension five proton decay operators sufficiently suppressed

- Arguments for supersymmetric Minkowski vacua (@ perturbative level) where most moduli attain large supersymmetric masses

Thank you very much!

Backup slides

- (Discrete) Green–Schwarz anomaly cancellation
- Anomaly universality
- Blaszczyk model

Green–Schwarz anomaly cancellation

- Under 'anomalous' $\mathrm{U}(1)$ symmetry transformation of the fermions $\psi^{(f)} \to \mathrm{e}^{\mathrm{i}\,\alpha\, Q^{(f)}_{\mathrm{anom}}}\,\psi^{(f)}$ the path integral measure exhibits non–trivial transformation

Fujikawa (1979) ; Fujikawa (1980)

$$\mathcal{D}\Psi\,\mathcal{D}\overline{\Psi} \;\to\; J(\alpha)\,\mathcal{D}\Psi\,\mathcal{D}\overline{\Psi} \qquad \text{with non–trivial } J(\alpha)$$

path integral measure | Jacobian | (infinitesimal) parameter

Green–Schwarz anomaly cancellation

- Under 'anomalous' $\mathrm{U}(1)$ symmetry transformation of the fermions $\psi^{(f)} \to \mathrm{e}^{\mathrm{i}\,\alpha\, Q^{(f)}_{\mathrm{anom}}}\,\psi^{(f)}$ the path integral measure exhibits non–trivial transformation Fujikawa (1979) ; Fujikawa (1980)
- One can absorb the change of the path integral measure in a change of Lagrangean

$$\Delta\mathscr{L}_{\mathsf{anomaly}} = \frac{\alpha}{32\pi^2} F^a \widetilde{F}^a A_{G\text{–}G\text{–}\mathrm{U}(1)_{\mathrm{anom}}}$$

anomaly coefficients

e.g. $\mathrm{SU}(3) \subset G_{\mathrm{SM}}$

'anomalous' $\mathrm{U}(1)$

Green–Schwarz anomaly cancellation

- Under 'anomalous' $\mathrm{U}(1)$ symmetry transformation of the fermions $\psi^{(f)} \to \mathrm{e}^{\mathrm{i}\,\alpha\, Q^{(f)}_\mathrm{anom}}\,\psi^{(f)}$ the path integral measure exhibits non–trivial transformation Fujikawa (1979) ; Fujikawa (1980)

- One can absorb the change of the path integral measure in a change of Lagrangean

$$\Delta\mathscr{L}_\text{anomaly} = \frac{\alpha}{32\pi^2} F^a \widetilde{F}^a \, A_{G\text{–}G\text{–}\mathrm{U}(1)_\mathrm{anom}}$$

- Provided the Lagrangean also includes axion couplings

$$\mathscr{L} \supset -\frac{a}{8} F^a \widetilde{F}^a$$

$\Delta\mathscr{L}_\text{anomaly}$ can be compensated by a shift of the axion a

Green and Schwarz (1984)

Discrete GS anomaly cancellation in SUSY

- Analysis applies also for discrete symmetries
- Specifically for a $\mathbb{Z}_N$ transformation

$$\Phi^{(f)} \;\to\; \mathrm{e}^{-\mathrm{i}\,\frac{2\pi}{N}\,q^{(f)}}\,\Phi^{(f)}$$

the dilaton (containing the axion) has to transform as

$$S \;\to\; S + \frac{\mathrm{i}}{2}\Delta_{\mathrm{GS}}$$

where

$$\pi N\,\Delta_{\mathrm{GS}} \;\equiv\; A_{G-G-\mathbb{Z}_N} \mod \eta \qquad \text{where } \eta = \begin{cases} N & \text{if } N \text{ odd} \\ N/2 & \text{if } N \text{ even} \end{cases}$$

- If $\mathrm{SU}(3)\times\mathrm{SU}(2)\times\mathrm{U}(1)\subset\mathrm{SU}(5)$ the anomaly coefficients need to be universal

Comments on discrete GS mechanism

1. Although the GS mechanism plays a prominent role in string theory, it does not rely on strings.
2. Unlike in the continuous case, for discrete symmetries the transformation of the axion is only fixed modulo η.
3. In the continuous case, the axion has to be massless for the shift symmetry to be a symmetry of the Lagrangean. That is, the axion potential needs to be flat. By contrast, in the discrete case the potential is only required to be periodic, i.e. invariant under the discrete shift. Therefore the axion may have a non–trivial mass prior to the breakdown of the symmetry.

Anomaly universality

back

- Universality condition $A_{G_i-G_i-\mathrm{U}(1)_{\mathrm{anom}}} = \rho$
- Obviously: even in the presence of multiple axions, there is only one unique linear combination a that shifts under a given $\mathrm{U}(1)_{\mathrm{anom}}$, $\mathbb{Z}_N$ or $\mathbb{Z}_M^R$ transformation
- However, a may have different couplings c_i to different field strengths of the SM gauge group

$$\mathscr{L}_{\mathrm{axion}} \supset \sum_i c_i \frac{a}{8} F_i^b \widetilde{F}_i^b$$

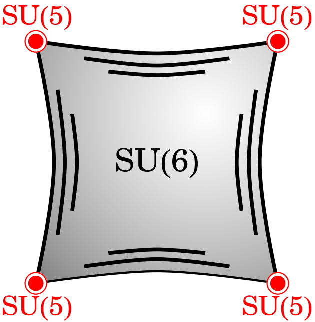
↪ no anomaly universality in general

however:

- different c_i are inconsistent with an underlying GUT symmetry
- a non–trivial VEV of the scalar partner of a will destroy gauge coupling unification

$\mathbb{Z}_2 \times \mathbb{Z}_2$ orbifold example

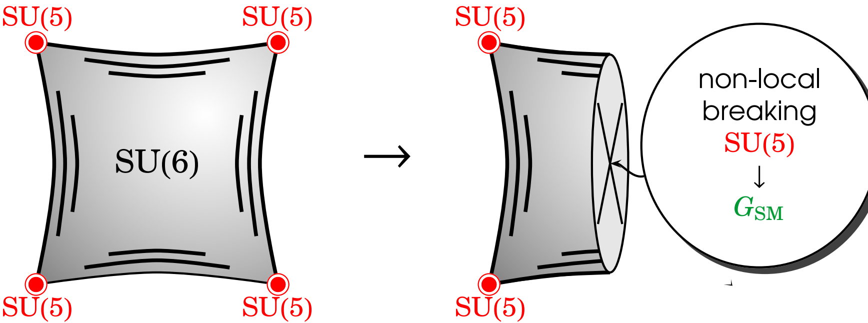
Blaszczyk et al. (2010) ; Kappl et al. (2011)



1. step: 6 generation $\mathbb{Z}_2 \times \mathbb{Z}_2$ model with $\mathrm{SU}(5)$ symmetry

$\mathbb{Z}_2 \times \mathbb{Z}_2$ orbifold example

Blaszczyk et al. (2010) ; Kappl et al. (2011)



1. step: 6 generation $\mathbb{Z}_2 \times \mathbb{Z}_2$ model with $\mathrm{SU}(5)$ symmetry
2. step: mod out a freely acting $\mathbb{Z}_2$ symmetry which:
   - breaks $\mathrm{SU}(5) \to \mathrm{SU}(3)_C \times \mathrm{SU}(2)_\mathrm{L} \times \mathrm{U}(1)_Y$
   - reduces the number of generations to 3

analogous mechanism in CY MSSMs Bouchard and Donagi (2006)
Braun et al. (2005)

Main features

1. GUT symmetry breaking non-local
$\curvearrowright$ (almost) no 'logarithmic running above the GUT scale'

Hebecker and Trapletti (2005) ; Anandakrishnan and Raby (2012)

Main features

1. GUT symmetry breaking non-local
2. No localized flux in hypercharge direction
$\curvearrowright$ complete blow–up without breaking SM gauge symmetry in principle possible

Main features

1. GUT symmetry breaking non-local
2. No localized flux in hypercharge direction
3. 4D gauge group:
   $\mathsf{SU}(3)_C \times \mathsf{SU}(2)_\mathrm{L} \times \mathsf{U}(1)_Y \times [\mathsf{SU}(3) \times \mathsf{SU}(2)^2 \times \mathsf{U}(1)^8]$

Main features

1. GUT symmetry breaking non-local
2. No localized flux in hypercharge direction
3. 4D gauge group:
   $\mathrm{SU}(3)_C \times \mathrm{SU}(2)_\mathrm{L} \times \mathrm{U}(1)_Y \times [\mathrm{SU}(3) \times \mathrm{SU}(2)^2 \times \mathrm{U}(1)^8]$
4. massless spectrum

#	representation	label
3	$(\mathbf{3},\mathbf{2};\mathbf{1},\mathbf{1},\mathbf{1})_{1/6}$	Q
3	$(\overline{\mathbf{3}},\mathbf{1};\mathbf{1},\mathbf{1},\mathbf{1})_{1/3}$	$\overline{D}$
3	$(\mathbf{1},\mathbf{1};\mathbf{1},\mathbf{1},\mathbf{1})_{1}$	$\overline{E}$
6	$(\mathbf{1},\mathbf{2};\mathbf{1},\mathbf{1},\mathbf{1})_{-1/2}$	h
3	$(\overline{\mathbf{3}},\mathbf{1};\mathbf{1},\mathbf{1},\mathbf{1})_{1/3}$	$\overline{\delta}$
3	$(\mathbf{1},\mathbf{1};\mathbf{3},\mathbf{1},\mathbf{1})_{0}$	x
6	$(\mathbf{1},\mathbf{1};\mathbf{1},\mathbf{1},\mathbf{2})_{0}$	y

#	representation	label
3	$(\overline{\mathbf{3}},\mathbf{1};\mathbf{1},\mathbf{1},\mathbf{1})_{-\frac{2}{3}}$	$\overline{U}$
3	$(\mathbf{1},\mathbf{2};\mathbf{1},\mathbf{1},\mathbf{1})_{-\frac{1}{2}}$	L
37	$(\mathbf{1},\mathbf{1};\mathbf{1},\mathbf{1},\mathbf{1})_{0}$	s
6	$(\mathbf{1},\mathbf{2};\mathbf{1},\mathbf{1},\mathbf{1})_{1/2}$	$\overline{h}$
3	$(\mathbf{3},\mathbf{1};\mathbf{1},\mathbf{1},\mathbf{1})_{-1/3}$	δ
5	$(\mathbf{1},\mathbf{1};\overline{\mathbf{3}},\mathbf{1},\mathbf{1})_{0}$	$\overline{x}$
6	$(\mathbf{1},\mathbf{1};\mathbf{1},\mathbf{2},\mathbf{1})_{0}$	z

Main features

1. GUT symmetry breaking non-local
2. No localized flux in hypercharge direction
3. 4D gauge group:
   $\mathsf{SU}(3)_C \times \mathsf{SU}(2)_\mathrm{L} \times \mathsf{U}(1)_Y \times [\mathsf{SU}(3) \times \mathsf{SU}(2)^2 \times \mathsf{U}(1)^8]$
4. massless spectrum

   spectrum $=$ $3 \times$ generation + vector-like

Spectrum and $\mathbb{Z}_4^R$

#	representation	label
3	$(\mathbf{3},\mathbf{2};\mathbf{1},\mathbf{1},\mathbf{1})_{1/6}$	Q
3	$(\overline{\mathbf{3}},\mathbf{1};\mathbf{1},\mathbf{1},\mathbf{1})_{1/3}$	$\overline{D}$
3	$(\mathbf{1},\mathbf{1};\mathbf{1},\mathbf{1},\mathbf{1})_{1}$	$\overline{E}$
6	$(\mathbf{1},\mathbf{2};\mathbf{1},\mathbf{1},\mathbf{1})_{-1/2}$	h
3	$(\overline{\mathbf{3}},\mathbf{1};\mathbf{1},\mathbf{1},\mathbf{1})_{1/3}$	$\overline{\delta}$
5	$(\mathbf{1},\mathbf{1};\mathbf{3},\mathbf{1},\mathbf{1})_{0}$	x
6	$(\mathbf{1},\mathbf{1};\mathbf{1},\mathbf{1},\mathbf{2})_{0}$	y

#	representation	label
3	$(\overline{\mathbf{3}},\mathbf{1};\mathbf{1},\mathbf{1},\mathbf{1})_{-2/3}$	$\overline{U}$
3	$(\mathbf{1},\mathbf{2};\mathbf{1},\mathbf{1},\mathbf{1})_{-1/2}$	L
37	$(\mathbf{1},\mathbf{1};\mathbf{1},\mathbf{1},\mathbf{1})_{0}$	s
6	$(\mathbf{1},\mathbf{2};\mathbf{1},\mathbf{1},\mathbf{1})_{1/2}$	$\overline{h}$
3	$(\mathbf{3},\mathbf{1};\mathbf{1},\mathbf{1},\mathbf{1})_{-1/3}$	δ
5	$(\mathbf{1},\mathbf{1};\overline{\mathbf{3}},\mathbf{1},\mathbf{1})_{0}$	$\overline{x}$
6	$(\mathbf{1},\mathbf{1};\mathbf{1},\mathbf{2},\mathbf{1})_{0}$	z

$\mathbb{Z}_4^R$: discriminate between

matter with $\mathbb{Z}_4^R$ charge 1

and

Higgs/exotics with $\mathbb{Z}_4^R$ charge 0 or 2

Spectrum and $\mathbb{Z}_4^R$

#	representation	label
3	$(\mathbf{3},\mathbf{2};\mathbf{1},\mathbf{1},\mathbf{1})_{1/6}$	Q
3	$(\overline{\mathbf{3}},\mathbf{1};\mathbf{1},\mathbf{1},\mathbf{1})_{1/3}$	$\overline{D}$
3	$(\mathbf{1},\mathbf{1};\mathbf{1},\mathbf{1},\mathbf{1})_{1}$	$\overline{E}$
6	$(\mathbf{1},\mathbf{2};\mathbf{1},\mathbf{1},\mathbf{1})_{-1/2}$	h
3	$(\overline{\mathbf{3}},\mathbf{1};\mathbf{1},\mathbf{1},\mathbf{1})_{1/3}$	$\overline{\delta}$
5	$(\mathbf{1},\mathbf{1};\mathbf{3},\mathbf{1},\mathbf{1})_{0}$	x
6	$(\mathbf{1},\mathbf{1};\mathbf{1},\mathbf{1},\mathbf{2})_{0}$	y

#	representation	label
3	$(\overline{\mathbf{3}},\mathbf{1};\mathbf{1},\mathbf{1},\mathbf{1})_{-2/3}$	$\overline{U}$
3	$(\mathbf{1},\mathbf{2};\mathbf{1},\mathbf{1},\mathbf{1})_{-1/2}$	L
37	$(\mathbf{1},\mathbf{1};\mathbf{1},\mathbf{1},\mathbf{1})_{0}$	s
6	$(\mathbf{1},\mathbf{2};\mathbf{1},\mathbf{1},\mathbf{1})_{1/2}$	$\overline{h}$
3	$(\mathbf{3},\mathbf{1};\mathbf{1},\mathbf{1},\mathbf{1})_{-1/3}$	δ
5	$(\mathbf{1},\mathbf{1};\overline{\mathbf{3}},\mathbf{1},\mathbf{1})_{0}$	$\overline{x}$
6	$(\mathbf{1},\mathbf{1};\mathbf{1},\mathbf{2},\mathbf{1})_{0}$	z

☞ Many other good features:

- no fractionally charged exotics (all SM charged fields come in SU(5) multiplets)
- non–trivial full–rank Yukawa couplings
- gauge–top unification
- SU(5) relation $y_\tau \simeq y_b$ (but also for light generations)

$\mathbb{Z}_4^R$ charges

- SM fields

	Q_i	$\overline{U}_i$	$\overline{D}_i$	L_i	$\overline{E}_i$
$\mathbb{Z}_4^R$	1	1	1	1	1

- Exotics

	h_1	h_2	h_3	h_4	h_5	h_6	$\overline{h}_1$	$\overline{h}_2$	$\overline{h}_3$	$\overline{h}_4$	$\overline{h}_5$	$\overline{h}_6$
$\mathbb{Z}_4^R$	0	2	0	2	0	0	0	2	0	0	2	2

	δ_1	δ_2	δ_3	$\overline{\delta}_1$	$\overline{\delta}_2$	$\overline{\delta}_3$
$\mathbb{Z}_4^R$	0	2	2	2	0	0

Higgs candidate mass matrix

	h_1	h_2	h_3	h_4	h_5	h_6	$\overline{h}_1$	$\overline{h}_2$	$\overline{h}_3$	$\overline{h}_4$	$\overline{h}_5$	$\overline{h}_6$
$\mathbb{Z}_4^R$	0	2	0	2	0	0	0	2	0	0	2	2

- Mass matrix for exotic doublets h_i and $\overline{h}_j$

$$\mathcal{M}_{h\overline{h}} = \begin{pmatrix} 0 & \phi_6 & 0 & \phi_4 & 0 & 0 \\ \phi_7 & 0 & \phi_2 & 0 & \phi_{13} & \phi_{14} \\ 0 & \phi_1 & 0 & \widetilde{\phi}^3 & 0 & 0 \\ 0 & \widetilde{\phi}^3 & 0 & \widetilde{\phi}^5 & 0 & 0 \\ \widetilde{\phi}^3 & 0 & \phi_{11} & 0 & \phi_8 & \widetilde{\phi}^3 \\ \widetilde{\phi}^3 & 0 & \phi_{12} & 0 & \widetilde{\phi}^3 & \phi_8 \end{pmatrix}$$

SM singlets with zero $\mathbb{Z}_4^R$ charge

Higgs candidate mass matrix

	h_1	h_2	h_3	h_4	h_5	h_6	$\overline{h}_1$	$\overline{h}_2$	$\overline{h}_3$	$\overline{h}_4$	$\overline{h}_5$	$\overline{h}_6$
$\mathbb{Z}_4^R$	0	2	0	2	0	0	0	2	0	0	2	2

☞ Mass matrix for exotic doublets h_i and $\overline{h}_j$

$$\mathcal{M}_{h\overline{h}} = \begin{pmatrix} 0 & \phi_6 & 0 & \phi_4 & 0 & 0 \\ \phi_7 & 0 & \phi_2 & 0 & \phi_{13} & \phi_{14} \\ 0 & \phi_1 & 0 & \widetilde{\phi}^3 & 0 & 0 \\ 0 & \widetilde{\phi}^3 & 0 & \widetilde{\phi}^5 & 0 & 0 \\ \widetilde{\phi}^3 & 0 & \phi_{11} & 0 & \phi_8 & \widetilde{\phi}^3 \\ \widetilde{\phi}^3 & 0 & \phi_{12} & 0 & \widetilde{\phi}^3 & \phi_8 \end{pmatrix}$$

☞ One massless linear combination (= Higgs pair)

$$H_u = a_1\,\overline{h}_1 + a_2\,\overline{h}_3 + a_3\,\overline{h}_4$$
$$H_d = b_1\,h_1 + b_2\,h_3 + b_3\,h_5 + b_4\,h_6$$

Triplet mass matrix

	δ_1	δ_2	δ_3	$\overline{\delta}_1$	$\overline{\delta}_2$	$\overline{\delta}_3$
$\mathbb{Z}_4^R$	0	2	2	2	0	0

☞ Mass matrix for exotic color triplet

$$\mathcal{M}_\delta = \begin{pmatrix} \widetilde{\phi}^5 & 0 & 0 \\ 0 & \phi_8 & \widetilde{\phi}^3 \\ 0 & \widetilde{\phi}^3 & \phi_8 \end{pmatrix}$$

SM singlets with zero $\mathbb{Z}_4^R$ charge

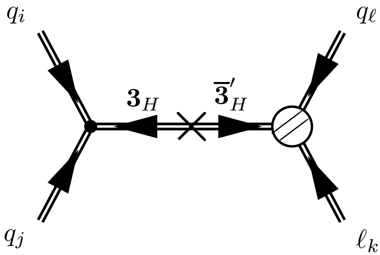
Triplet mass matrix

	δ_1	δ_2	δ_3	$\overline{\delta}_1$	$\overline{\delta}_2$	$\overline{\delta}_3$
$\mathbb{Z}_4^R$	0	2	2	2	0	0

- Mass matrix for exotic color triplet

$$\mathcal{M}_\delta = \begin{pmatrix} \widetilde{\phi}^5 & 0 & 0 \\ 0 & \phi_8 & \widetilde{\phi}^3 \\ 0 & \widetilde{\phi}^3 & \phi_8 \end{pmatrix}$$

- **Note**: exotic triplets cannot mediate proton decay



Triplet mass matrix

$$\begin{array}{c|cccccc} & \delta_1 & \delta_2 & \delta_3 & \overline{\delta}_1 & \overline{\delta}_2 & \overline{\delta}_3 \\ \mathbb{Z}_4^R & 0 & 2 & 2 & 2 & 0 & 0 \end{array}$$

- Mass matrix for exotic color triplet

$$\mathcal{M}_\delta = \begin{pmatrix} \widetilde{\phi}^5 & 0 & 0 \\ 0 & \phi_8 & \widetilde{\phi}^3 \\ 0 & \widetilde{\phi}^3 & \phi_8 \end{pmatrix}$$

- **Note**: exotic triplets cannot mediate proton decay
- The fact that the numbers of massless doublet and triplet pairs differ is not an accident but already follows from anomaly matching

Effective Yukawa couplings

☞ Effective superpotential

$$\begin{aligned}
\mathscr{W}_{\mathrm{Y}} &= \sum_{i=1,3,4} \left[\left(Y_u^{(i)}\right)^{fg} Q_f \, \overline{U}_g \, \overline{h}_i \right] \\
&+ \sum_{i=1,3,5,6} \left[\left(Y_d^{(i)}\right)^{fg} Q_f \, \overline{D}_g \, h_i + \left(Y_e^{(i)}\right)^{fg} L_f \, \overline{E}_g \, h_i \right]
\end{aligned}$$

Effective Yukawa couplings

- Effective superpotential

$$\mathscr{W}_Y = \sum_{i=1,3,4} \left[\left(Y_u^{(i)}\right)^{fg} Q_f \overline{U}_g \overline{h}_i \right] + \sum_{i=1,3,5,6} \left[\left(Y_d^{(i)}\right)^{fg} Q_f \overline{D}_g h_i + \left(Y_e^{(i)}\right)^{fg} L_f \overline{E}_g h_i \right]$$

- Effective Yukawa matrices (examples)

$$Y_u^{(1)} = \begin{pmatrix} \widetilde{\phi}^2 & \widetilde{\phi}^4 & \widetilde{\phi}^6 \\ \widetilde{\phi}^4 & \widetilde{\phi}^2 & \widetilde{\phi}^6 \\ \widetilde{\phi}^6 & \widetilde{\phi}^6 & 1 \end{pmatrix}$$

$$Y_u^{(3)} = \begin{pmatrix} 1 & \widetilde{\phi}^6 & \widetilde{\phi}^4 \\ \widetilde{\phi}^6 & 1 & \widetilde{\phi}^4 \\ \widetilde{\phi}^4 & \widetilde{\phi}^4 & \widetilde{\phi}^2 \end{pmatrix}$$

Effective Yukawa couplings

- Effective superpotential

$$\mathscr{W}_Y = \sum_{i=1,3,4} \left[\left(Y_u^{(i)}\right)^{fg} Q_f \overline{U}_g \bar{h}_i \right] + \sum_{i=1,3,5,6} \left[\left(Y_d^{(i)}\right)^{fg} Q_f \overline{D}_g h_i + \left(Y_e^{(i)}\right)^{fg} L_f \overline{E}_g h_i \right]$$

- Effective Yukawa matrices (examples)

$$Y_e^{(5)} = \left(Y_d^{(5)}\right)^T = \begin{pmatrix} \widetilde{\phi}^6 & \widetilde{\phi}^6 & \widetilde{\phi}^6 \\ \widetilde{\phi}^6 & \widetilde{\phi}^6 & 1 \\ \widetilde{\phi}^6 & 1 & \widetilde{\phi}^4 \end{pmatrix}$$

$$Y_e^{(6)} = \left(Y_d^{(6)}\right)^T = \begin{pmatrix} \widetilde{\phi}^6 & \widetilde{\phi}^6 & 1 \\ \widetilde{\phi}^6 & \widetilde{\phi}^6 & \widetilde{\phi}^6 \\ 1 & \widetilde{\phi}^6 & \widetilde{\phi}^4 \end{pmatrix}$$

References

References I

Archana Anandakrishnan and Stuart Raby. SU(6) GUT Breaking on a Projective Plane. 2012.

K. S. Babu, Ilia Gogoladze, and Kai Wang. Natural r-parity, mu-term, and fermion mass hierarchy from discrete gauge symmetries. *Nucl. Phys.*, B660:322–342, 2003a.

K. S. Babu, Ilia Gogoladze, and Kai Wang. Gauged baryon parity and nucleon stability. *Phys. Lett.*, B570:32–38, 2003b. doi: 10.1016/j.physletb.2003.07.036.

Tom Banks and Michael Dine. Note on discrete gauge anomalies. *Phys. Rev.*, D45:1424–1427, 1992.

Michael Blaszczyk, Stefan Groot Nibbelink, Michael Ratz, Fabian Ruehle, Michele Trapletti, et al. A Z2xZ2 standard model. *Phys.Lett.*, B683:340–348, 2010. doi: 10.1016/j.physletb.2009.12.036.

References II

Vincent Bouchard and Ron Donagi. An SU(5) heterotic standard model. *Phys. Lett.*, B633:783–791, 2006.

Volker Braun, Yang-Hui He, Burt A. Ovrut, and Tony Pantev. A heterotic standard model. *Phys. Lett.*, B618:252–258, 2005.

J. D. Breit, Burt A. Ovrut, and Gino C. Segre. E(6) symmetry breaking in the superstring theory. *Phys. Lett.*, B158:33, 1985.

Wilfried Buchmuller, Koichi Hamaguchi, Oleg Lebedev, and Michael Ratz. Local grand unification. pages 143–156, 2005.

Ali H. Chamseddine and Herbert K. Dreiner. Anomaly free gauged R symmetry in local supersymmetry. *Nucl.Phys.*, B458:65–89, 1996. doi: 10.1016/0550-3213(95)00583-8.

Mu-Chun Chen, Michael Ratz, Christian Staudt, and Patrick K.S. Vaudrevange. The mu term and neutrino masses. 2012.

References III

Savas Dimopoulos, Stuart Raby, and Frank Wilczek. Proton decay in supersymmetric models. *Phys. Lett.*, B112:133, 1982.

Michael Dine, Jonathan L. Feng, and Eva Silverstein. Retrofitting O'Raifeartaigh models with dynamical scales. *Phys. Rev.*, D74:095012, 2006.

Herbi K. Dreiner, Christoph Luhn, and Marc Thormeier. What is the discrete gauge symmetry of the MSSM? *Phys. Rev.*, D73: 075007, 2006.

Maximilian Fallbacher, Michael Ratz, and Patrick K.S. Vaudrevange. No-go theorems for R symmetries in four-dimensional GUTs. *Phys.Lett.*, B705:503–506, 2011. doi: 10.1016/j.physletb.2011.10.063.

References IV

Glennys R. Farrar and Pierre Fayet. Phenomenology of the Production, Decay, and Detection of New Hadronic States Associated with Supersymmetry. *Phys. Lett.*, B76:575–579, 1978. doi: 10.1016/0370-2693(78)90858-4.

Kazuo Fujikawa. Path integral measure for gauge invariant fermion theories. *Phys. Rev. Lett.*, 42:1195, 1979.

Kazuo Fujikawa. Path integral for gauge theories with fermions. *Phys. Rev.*, D21:2848, 1980.

Mark W. Goodman and Edward Witten. GLOBAL SYMMETRIES IN FOUR-DIMENSIONS AND HIGHER DIMENSIONS. *Nucl.Phys.*, B271:21, 1986.

Michael B. Green and John H. Schwarz. Anomaly Cancellation in Supersymmetric $D = 10$ Gauge Theory and Superstring Theory. *Phys. Lett.*, B149:117–122, 1984.

References V

A. Hebecker and M. Trapletti. Gauge unification in highly anisotropic string compactifications. *Nucl. Phys.*, B713: 173–203, 2005.

Luis E. Ibáñez and Graham G. Ross. Discrete gauge symmetry anomalies. *Phys. Lett.*, B260:291–295, 1991.

Luis E. Ibáñez and Graham G. Ross. Discrete gauge symmetries and the origin of baryon and lepton number conservation in supersymmetric versions of the standard model. *Nucl. Phys.*, B368:3–37, 1992.

Rolf Kappl, Bjoern Petersen, Stuart Raby, Michael Ratz, Roland Schieren, and Patrick K.S. Vaudrevange. String-derived MSSM vacua with residual R symmetries. *Nucl.Phys.*, B847: 325–349, 2011. doi: 10.1016/j.nuclphysb.2011.01.032.

References VI

Hyun Min Lee, Stuart Raby, Michael Ratz, Graham G. Ross, Roland Schieren, Kai Schmidt-Hoberg, and Patrick K.S. Vaudrevange. A unique Z_4^R symmetry for the MSSM. *Phys.Lett.*, B694:491–495, 2011. doi: 10.1016/j.physletb.2010.10.038.

Markus A. Luty and Washington Taylor. Varieties of vacua in classical supersymmetric gauge theories. *Phys. Rev.*, D53: 3399–3405, 1996.

Edward Witten. Symmetry breaking patterns in superstring models. *Nucl. Phys.*, B258:75, 1985.