Dark Matter Searches and Fine-Tuning in Supersymmetry

Maxim Perelstein, Cornell University

PACIFIC 2011 Symposium

September 9, 2011

Cornell University
Laboratory for Elementary-Particle Physics

Bibliography

- Primary reference:

MP, Shakya, arXiv:1107.5048 [hep-ph]

- Precursors:

Mandic, Pierce, Gondolo, Murayama, hep-ph/0008022

Kitano, Nomura, hep-ph/0606134

Feng, Sanford, arXiv:1009.3934 [hep-ph]

- Independent related recent work:

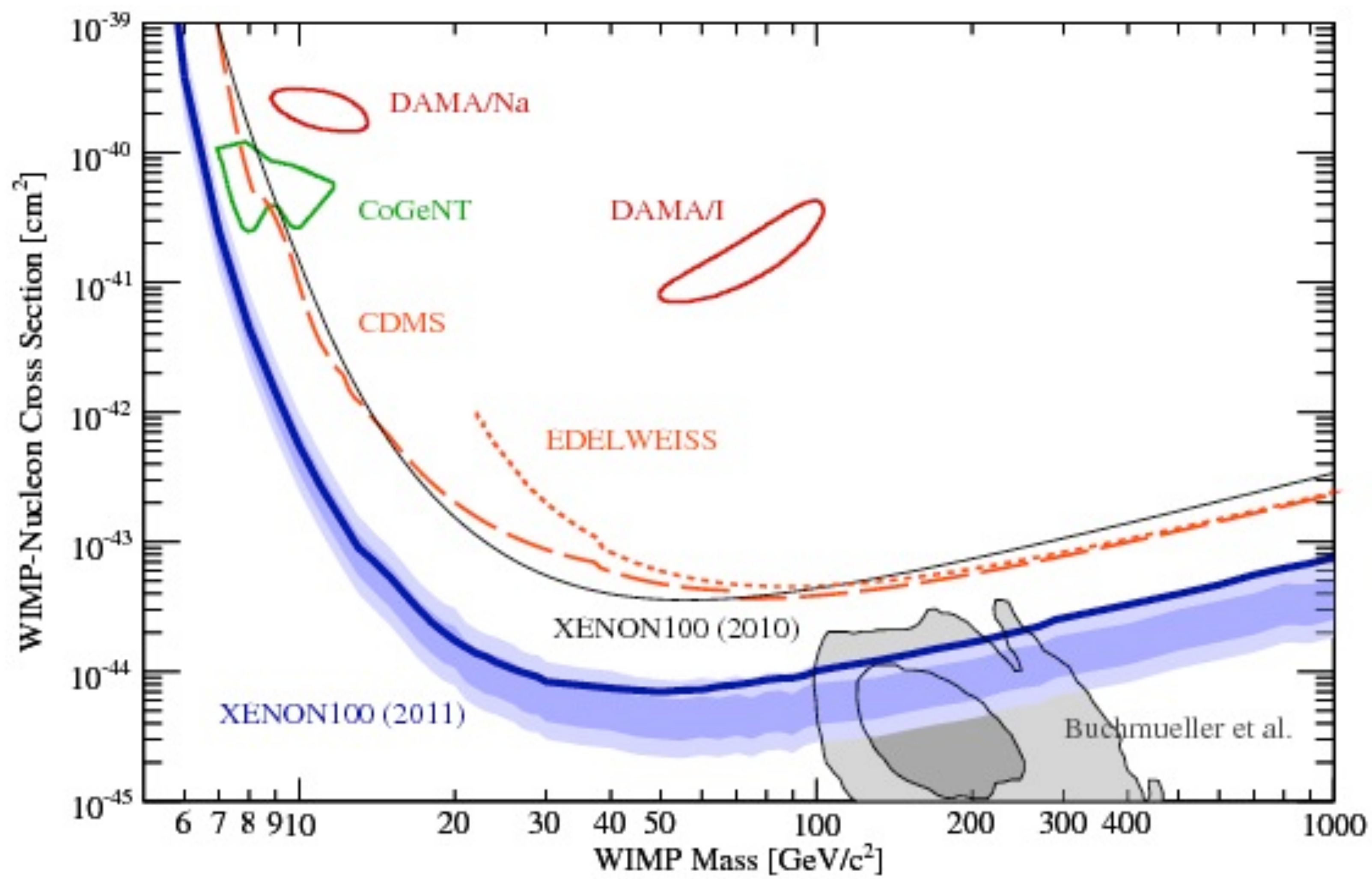
Amsel, Freese, Sandick, arXiv:1108.0448 [hep-ph]

Introduction/Motivation

- Existence of dark matter firmly established through observing its gravitational interaction, on a variety of length scales from galaxy to Hubble
- No SM particle candidate for dark matter ⇒ solid observational evidence for BSM physics!
- Thermal relic with ~weak-scale mass and ~weak annihilation cross section (WIMP) naturally fits the observed dark matter abundance
- BS Models naturally contain WIMPs (e.g. SUSY with R-parity - stable neutralino LSP)

- Direct searches for WIMP dark matter have seen significant improvements recently, e.g. XENON100

[See e.g. B. Sadoulet's talk this morning]

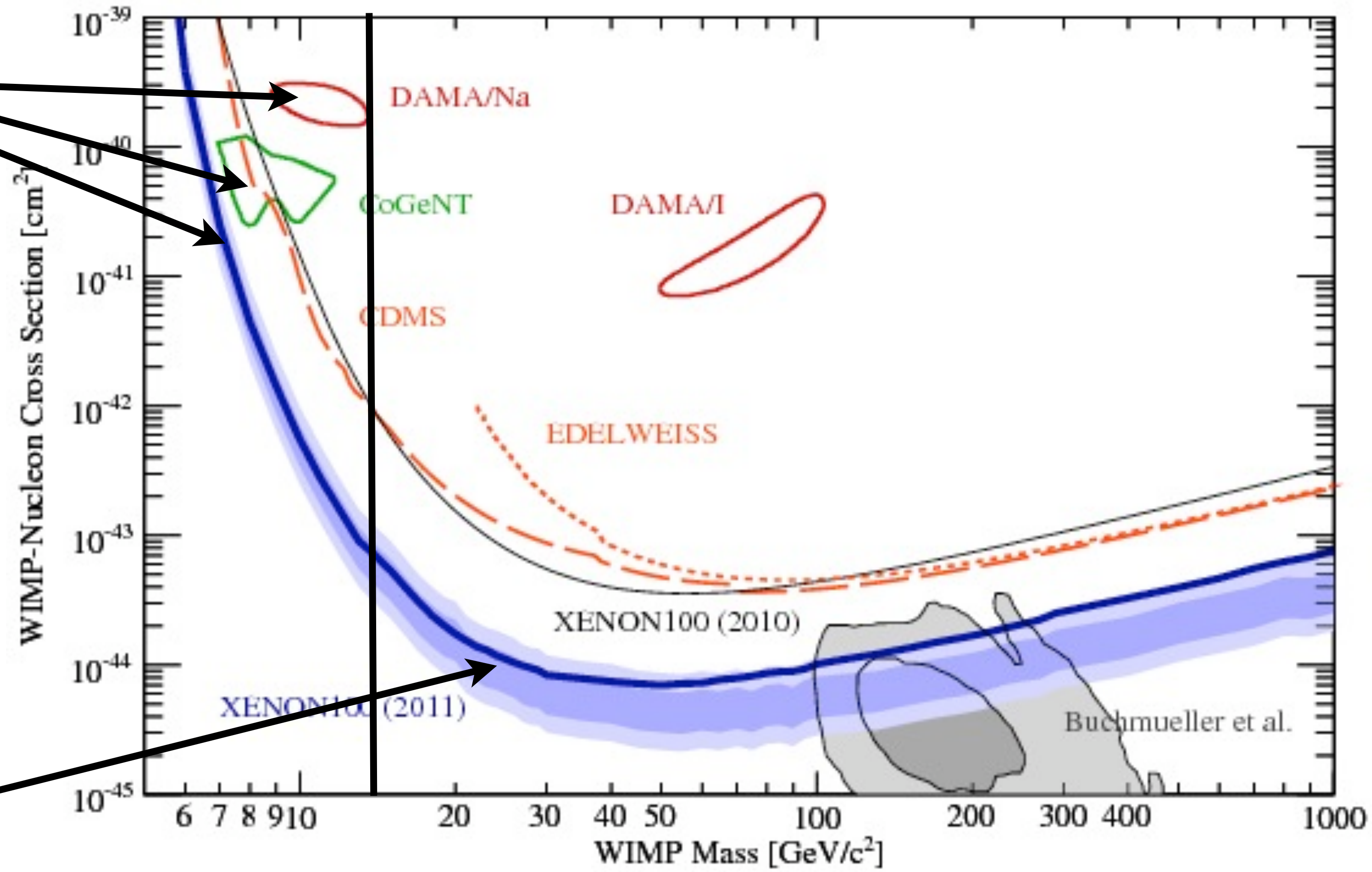


"DD cross section"= spin-independent, elastic DM-proton scattering at zero mom. exchange

XENON-100, arXiv:1104.2549[astro-ph.CO]

- Direct searches for WIMP dark matter have seen significant improvements recently, e.g. XENON100

Confusion below 15 GeV: contradictory (within minimal WIMP framework) claims



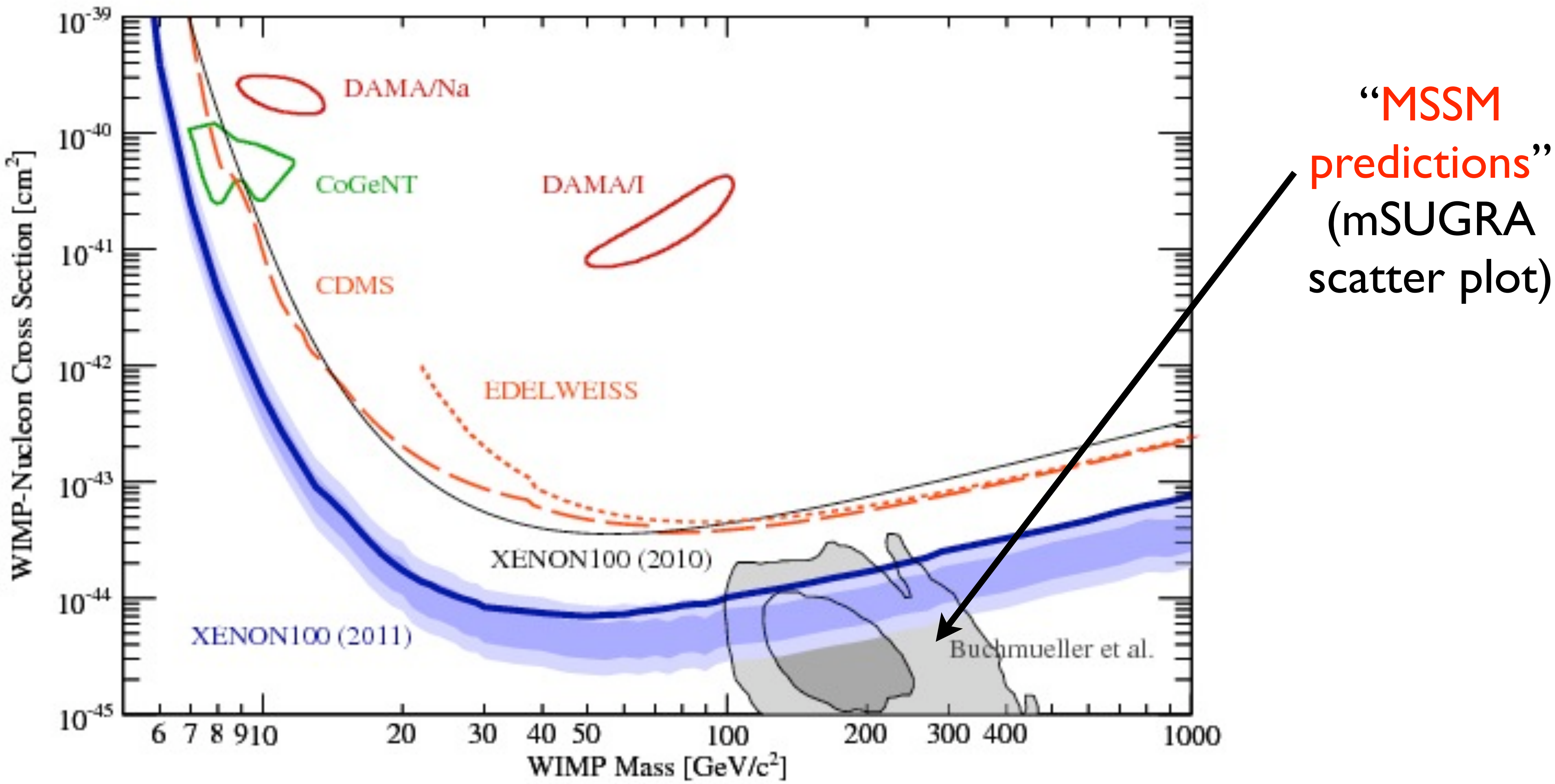
Clean exclusion bound in 15 GeV - 1 TeV range

XENON-100, arXiv:1104.2549[astro-ph.CO]

MSSM Predictions for Direct Detection Cross Section

- Generic problem with MSSM predictions: large number (~100) of free parameters, allowed ranges are highly uncertain (esp. upper bounds on soft masses?)
- Standard approach: reduce parameter space by assuming high-scale unification, specific SUSY breaking model, etc.
- Most studied example, mSUGRA, has 5 parameters (and very serious issues with FCNC constraints!)

- Even then, the predictions span several orders of magnitude



XENON-100, arXiv:1104.2549[astro-ph.CO]

Our Approach

- Work with the MSSM defined in terms of weak-scale parameters, treat all of them as independent (as in pMSSM, "SUSY without prejudice", etc.)
- Reduce # of parameters by assuming absence of accidental cancellations in the DD cross section
- Look for correlations between DD cross section and other physical quantities
- Main question: What's special about points with low DD cross section?

Define "Accidental"?

- Intuitive definition:

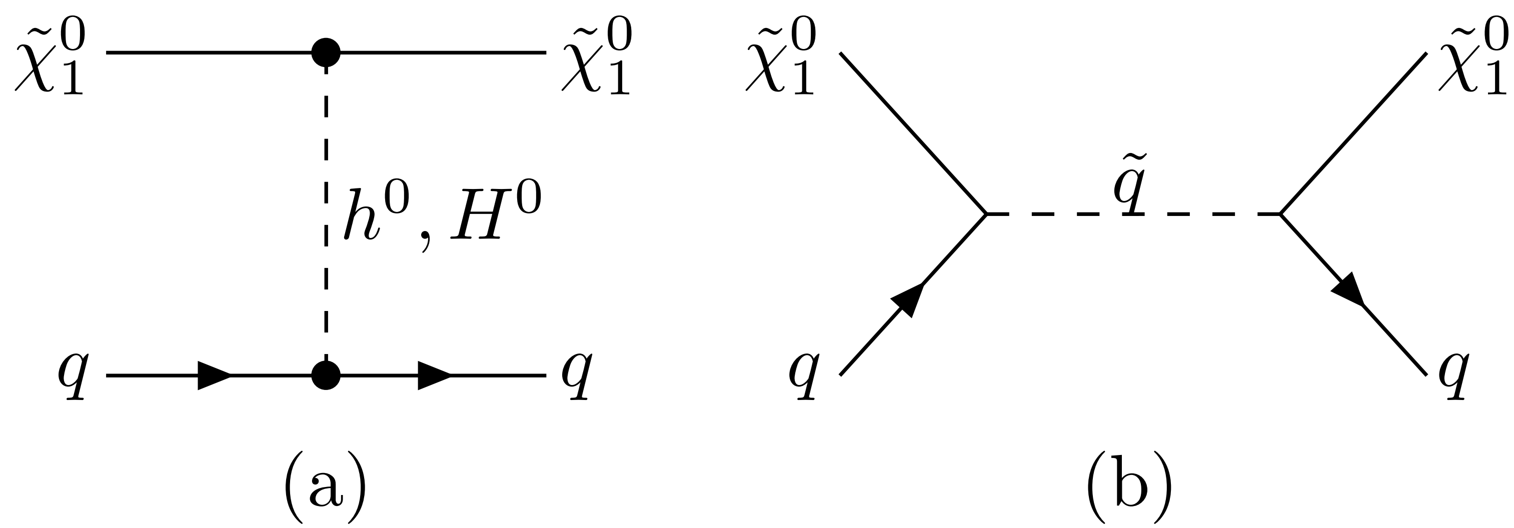
$$\sigma = c|\mathcal{M}_1 + \mathcal{M}_2|^2\,, \qquad \mathcal{M}_1 \approx -\mathcal{M}_2\,, \quad \sigma \ll c|\mathcal{M}_1|^2$$

- More general definition:

$$\sigma = \sigma(\{p_i\}), \quad |\frac{\partial \log \sigma}{\partial \log p_i}| \gg 1$$

- Disclaimer: this definition depends on which parameters are treated as "input". A specific model of SUSY breaking may in fact predict relations that seem accidental from our (low-energy) point of view. It is however VERY hard to imagine this for the particular examples where we use this rule!

Direct Detection in the MSSM



$(M_1, M_2, \mu, \tan\beta, \underline{m_A})$ $(M_1, M_2, \mu, \tan\beta, \underline{m^2_{\tilde{q}_L}, m^2_{\tilde{q}_R}, A_q})$

With no accidental cancellations, sufficient to consider only one diagram class. Choose (a).

EWSB Fine-Tuning in the MSSM

- Z mass at the tree level in the MSSM:

$$m_Z^2 = -m_u^2\left(1-\frac{1}{\cos 2\beta}\right) - m_d^2\left(1+\frac{1}{\cos 2\beta}\right) - 2|\mu|^2$$

- Unless all terms on the r.h.s. are of order 100 GeV, cancellations are required to make it work

- Measure of fine-tuning: sensitivity of mZ to Lagrangian parameters (a la Barbieri-Giudice, but at the weak scale):

$$\delta(\xi) = \left|\frac{\partial \log m_Z^2}{\partial \log \xi}\right|, \qquad \xi = m_u^2, m_d^2, b, \mu \longrightarrow \underline{\mu, \tan\beta, m_A, m_Z}$$

Same pars. as the t-channel direct det. diagram!

- A useful approximation $(\tan\beta >> 1)$:

$$\delta(\mu) \approx \frac{4\mu^2}{m_Z^2}, \quad \delta(b) \approx \frac{4m_A^2}{m_Z^2 \tan\beta}\,.$$

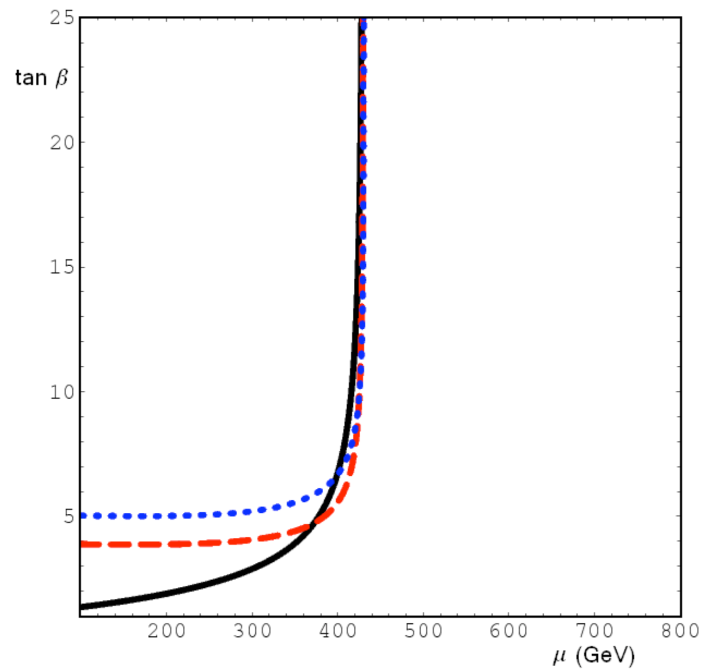


Figure 1: Contours of 1% fine-tuning in the $(\mu, \tan\beta)$ plane. The black (solid) contour corresponds to $m_A = 100$ GeV, but remains essentially unchanged for any value of m_A in the range between 100 and 1000 GeV. The red (dashed) and blue (dotted) contours correspond to $m_A = 1.5$ and 2 TeV, respectively.

[from MP, Spethmann, hep-ph/0702038]

Fine-Tuning Beyond Tree Level

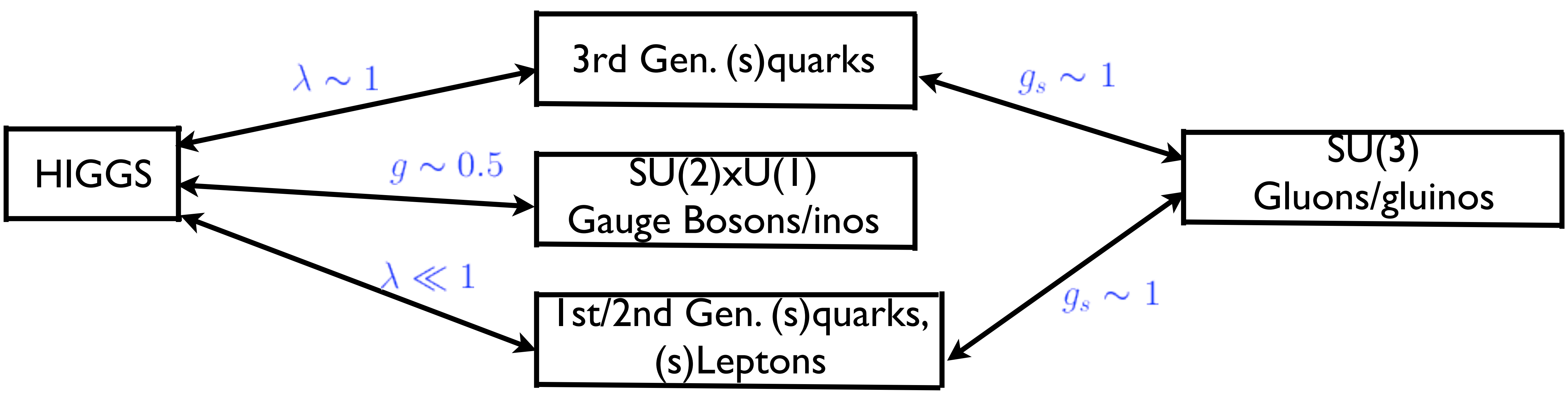
- Consider once again the Z mass formula:

$$m_Z^2 = -m_u^2\left(1-\frac{1}{\cos 2\beta}\right) - m_d^2\left(1+\frac{1}{\cos 2\beta}\right) - 2|\mu|^2$$

- At loop level, the Higgs mass par's. receive quadratically divergent corrections, cut off by superpartner masses ("SUSY solves the hierarchy problem")

$$\delta m_h^2 \sim \frac{g_p^2}{16\pi^2}\Lambda^2 \quad \text{(SM)} \qquad \longrightarrow \qquad \delta m_h^2 \sim \frac{g_p^2}{16\pi^2} m_{\tilde{p}}^2 \log\frac{\Lambda^2}{m_{\tilde{p}}^2} \quad \text{(SUSY)}$$

- While a large number of parameters enter, the "hierarchy of couplings" in the SM/MSSM simplifies the problem:



Fine-Tuning Beyond Tree Level

- So: 3rd gen. squark loops are the most important quantitatively
- Other superpartners may be a factor of 5 or more heavier than the 3rd gen squarks with no effect on fine-tuning
- Gluino first appears at 2 loops - additional suppression of its effect on fine-tuning
- Assuming moderate $\tan\beta$, top/stop dominates over bottom/sbottom, giving

$$\delta m_{H_u}^2 \approx \frac{3}{16\pi^2}\left(y_t^2\left(\tilde{m}_{Q_3}^2 + \tilde{m}_{t^c}^2\right) + y_t^2(A_t \sin\beta - \mu\cos\beta)^2\right)\log\frac{2\Lambda^2}{\tilde{m}_{Q_3}^2 + \tilde{m}_{t^c}^2}$$

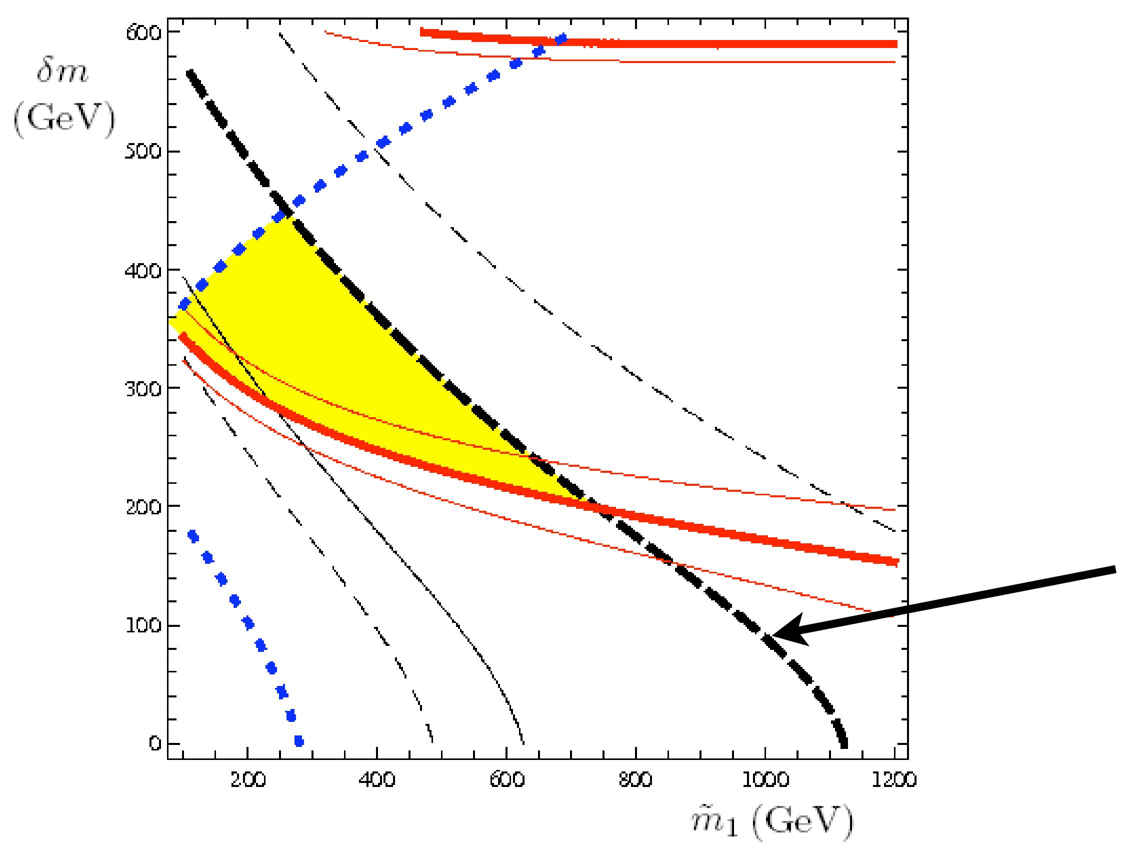
$$\delta_t m_Z^2 \approx -\delta m_{H_u}^2 \left(1 - \frac{1}{\cos 2\beta}\right)$$

- Quantify fine-tuning:

$$\Delta_t = \left|\frac{\delta_t m_Z^2}{m_Z^2}\right|.$$

The “Golden Region”

- At tree level, mh>mZ, while LEP-2 requires mh > 114 GeV
- Solution: large loop corrections ⇒ tension with fine-tuning! (“little hierarchy problem”)



$$\Lambda = 100 \text{ TeV}, \quad \theta_t = \pi/4, \quad \tan\beta = 10$$

[plot from MP, Spethmann, hep-ph/0702038]

What About the LHC?

BBC NEWS

SCIENCE & ENVIRONMENT

27 August 2011 Last updated at 02:41 ET

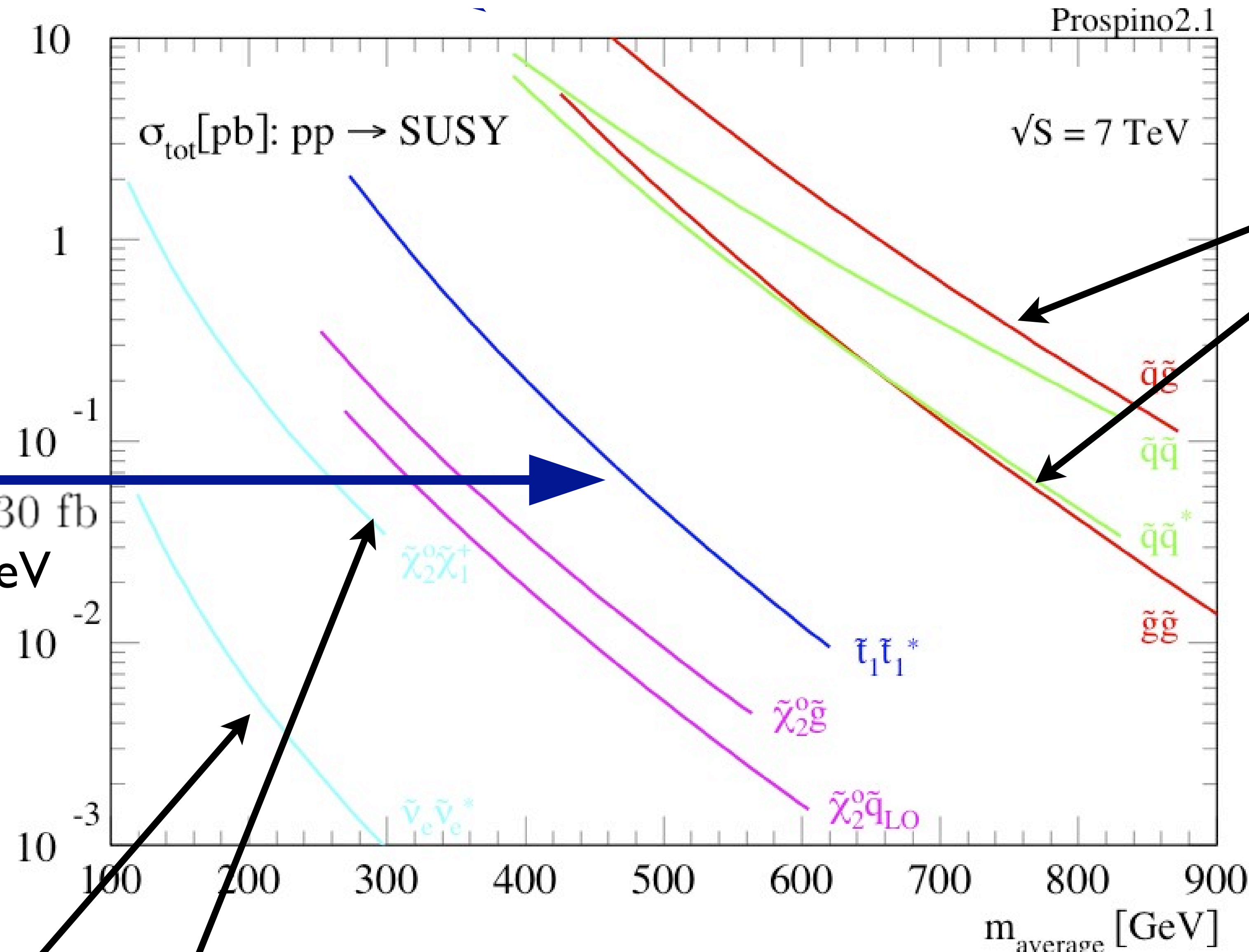
LHC results put supersymmetry theory 'on the spot'

By Pallab Ghosh
Science correspondent, BBC News

Results from the Large Hadron Collider (LHC) have all but killed the simplest version of an enticing theory of sub-atomic physics.

Researchers failed to find evidence of so-called "supersymmetric" particles, which many physicists had hoped would plug holes in the current theory.

What About the LHC?



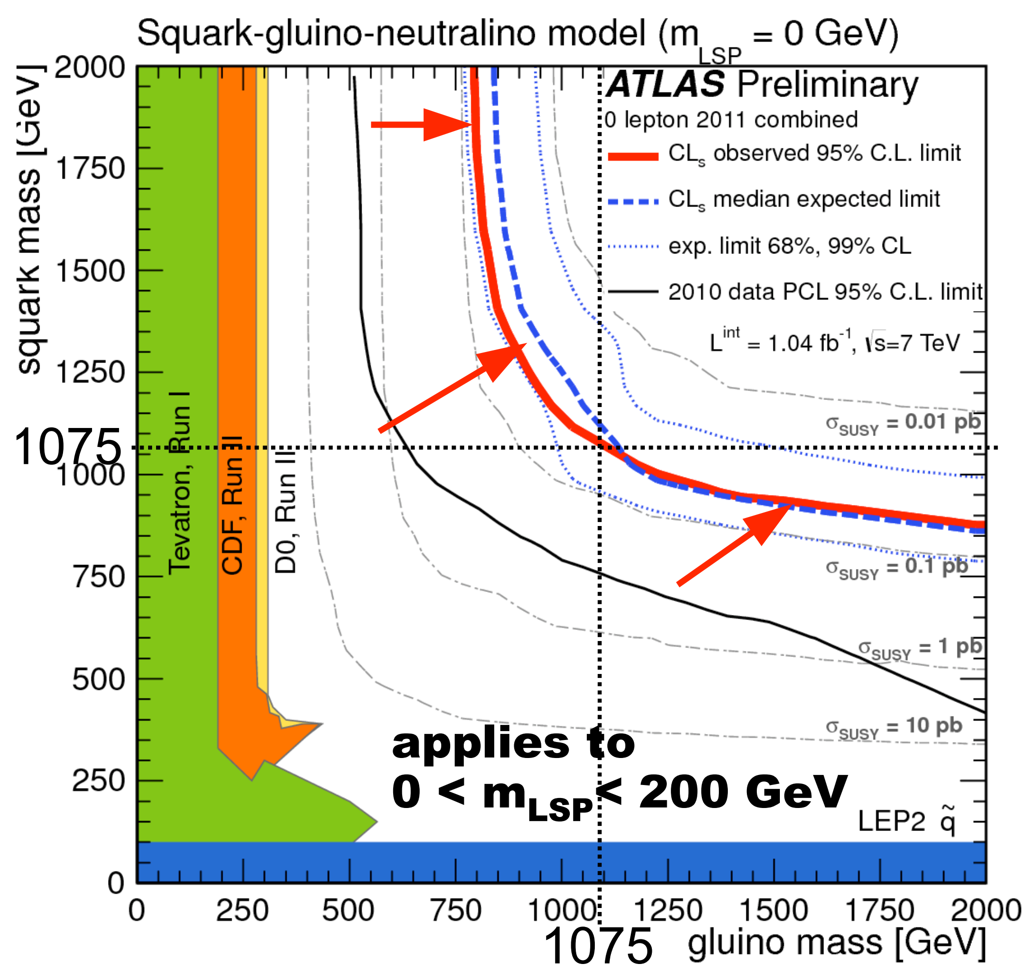
$\sigma(\tilde{t}\tilde{t}^*) \approx 30$ fb
at 500 GeV

All searches so far rely on producing gluinos and/or 1st, 2nd gen. squarks, different decay channels

Chargino/neutralino (e.g. higgsino) cross sections are also small

Plot credit: H. Bachacou talk at LP-11

What About the LHC?



Plot credit:
H. Bachacou
talk at LP-11

BOTTOM LINE: So far the LHC has had NO* impact on fine-tuning in the MSSM
[Not so in specific SUSY breaking models, e.g. where three gen. of squarks have common mass term at some scale]

Now, Back to Dark Matter

Procedure: Parameter Scans

- Generate ~100000 random points in the 5-dim. parameter space $(M_1, M_2, \mu, m_A, \tan\beta)$ [all real, >0!]
- Uniformly distributed in $\log M_1, \log M_2, \log\mu, \log m_A, \tan\beta$
- Scan boundaries:

$$M_1 \in [10, 10^4]\ \mathrm{GeV}; \qquad M_2 \in [80, 10^4]\ \mathrm{GeV};$$
$$\mu \in [80, 10^4]\ \mathrm{GeV}; \qquad m_A \in [100, 10^4]\ \mathrm{GeV};$$
$$\tan\beta \in [2, 50]\,.$$

- Require: neutral LSP, no charginos below 100 GeV. Note: we do not require correct relic density!
- Fix mh=120 GeV (assume top/stop loops fix it)

DD Cross Section

$$\sigma = \frac{4m_r^2 f_p^2}{\pi} \qquad \frac{f_p}{m_p} = \sum_{q=u,d,s} f_{T_q}^{(p)} A_q + \frac{2}{27} f_{TG}^{(p)} \sum_{q=c,b,t} A_q. \qquad f_{TG}^{(p)} = 1 - \sum_{q=u,d,s} f_{T_q}^{(p)}.$$

$$f_{Tu}^{(p)} = 0.08, \quad f_{Td}^{(p)} = 0.037, \quad \underline{f_{Ts}^{(p)} = 0.34.}$$

$$\begin{aligned} A_i &= -\frac{g}{4m_W B_i}\Big[\left(\frac{D_i^2}{m_h^2} + \frac{C_i^2}{m_H^2}\right) \mathrm{Re}\left[\delta_{2i}(gZ_{\chi 2} - g' Z_{\chi 1})\right] \\ &\quad + C_i D_i \left(\frac{1}{m_h^2} - \frac{1}{m_H^2}\right) \mathrm{Re}\left[\delta_{1i}(gZ_{\chi 2} - g' Z_{\chi 1})\right]\Big], \end{aligned} \tag{6}$$

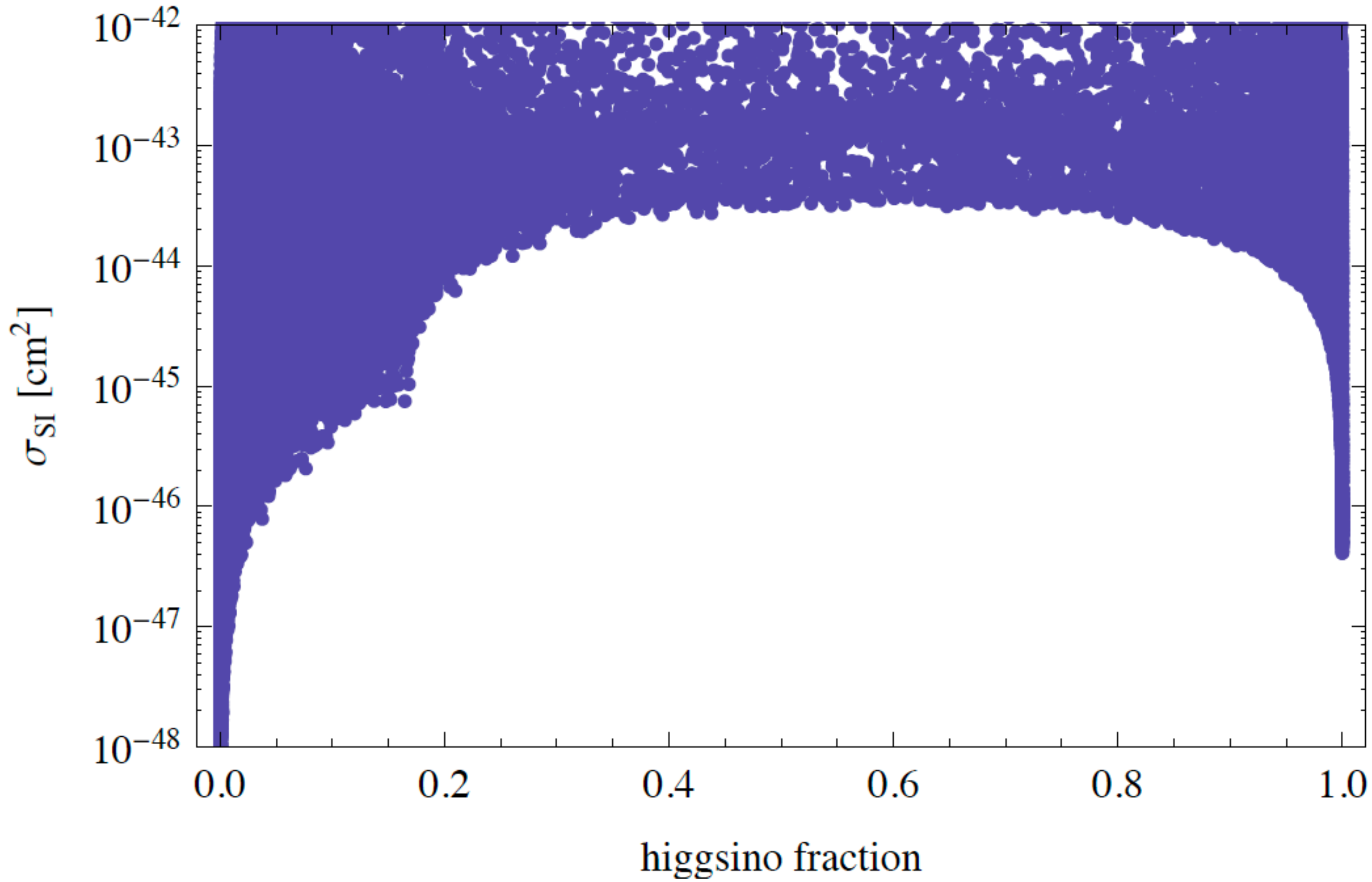
where for up-type quarks

$$B_u = \sin\beta, \quad C_u = \sin\alpha, \quad D_u = \cos\alpha, \quad \delta_{1u} = Z_{\chi 3}, \quad \delta_{2u} = Z_{\chi 4}; \tag{7}$$

while for down-type quarks

$$B_d = \cos\beta, \quad C_d = \cos\alpha, \quad D_d = -\sin\alpha, \quad \delta_{1d} = Z_{\chi 4}, \quad \delta_{2d} = -Z_{\chi 3}. \tag{8}$$

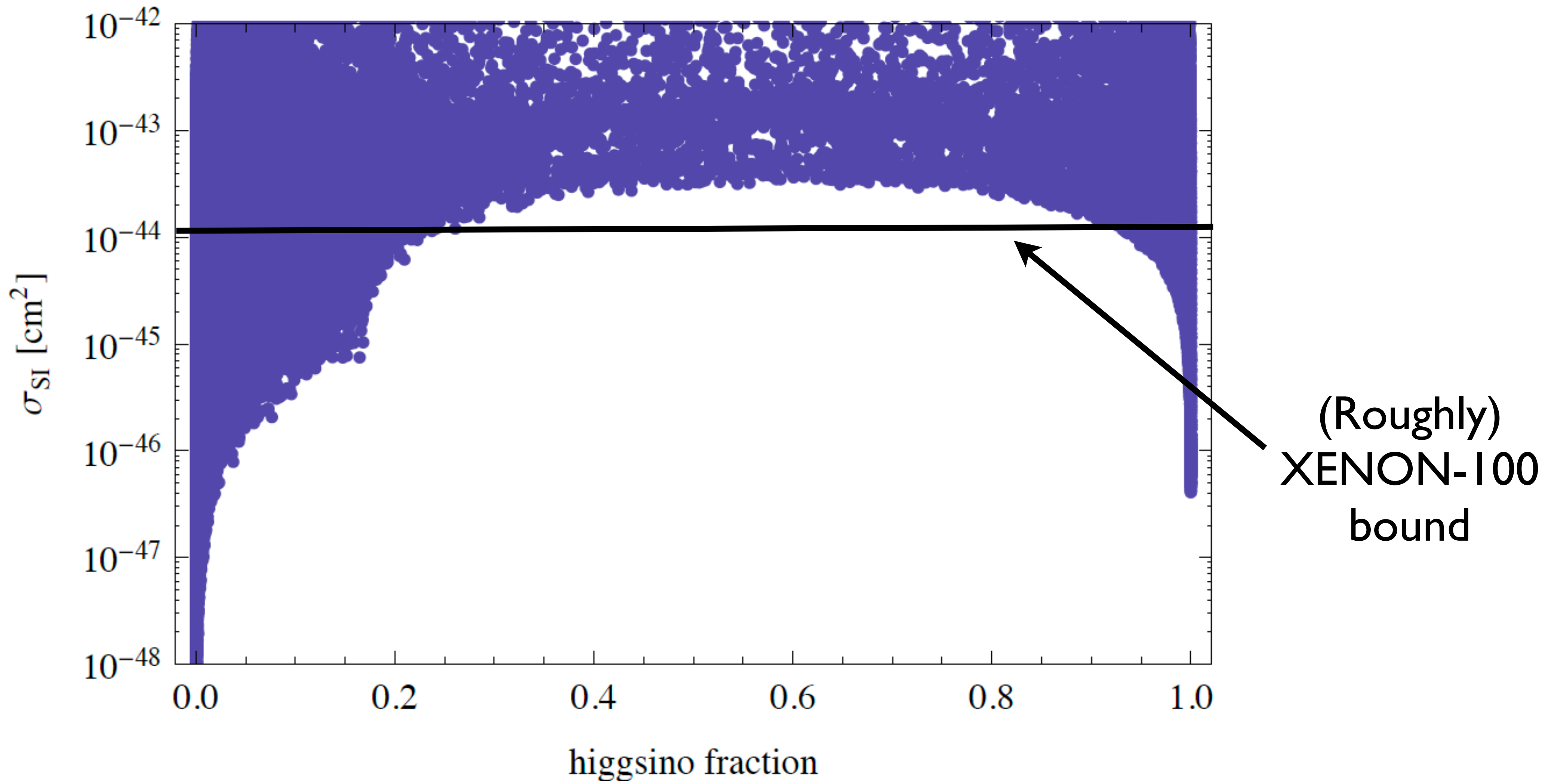
Results: Higgsino Fraction



$$F_H = |Z_{\chi 3}|^2 + |Z_{\chi 4}|^2 \, .$$

$$\tilde{\chi}_1^0 = Z_{\chi 1}\tilde{B} + Z_{\chi 2}\tilde{W}^3 + Z_{\chi 3}\tilde{H}_d^0 + Z_{\chi 4}\tilde{H}_u^0$$

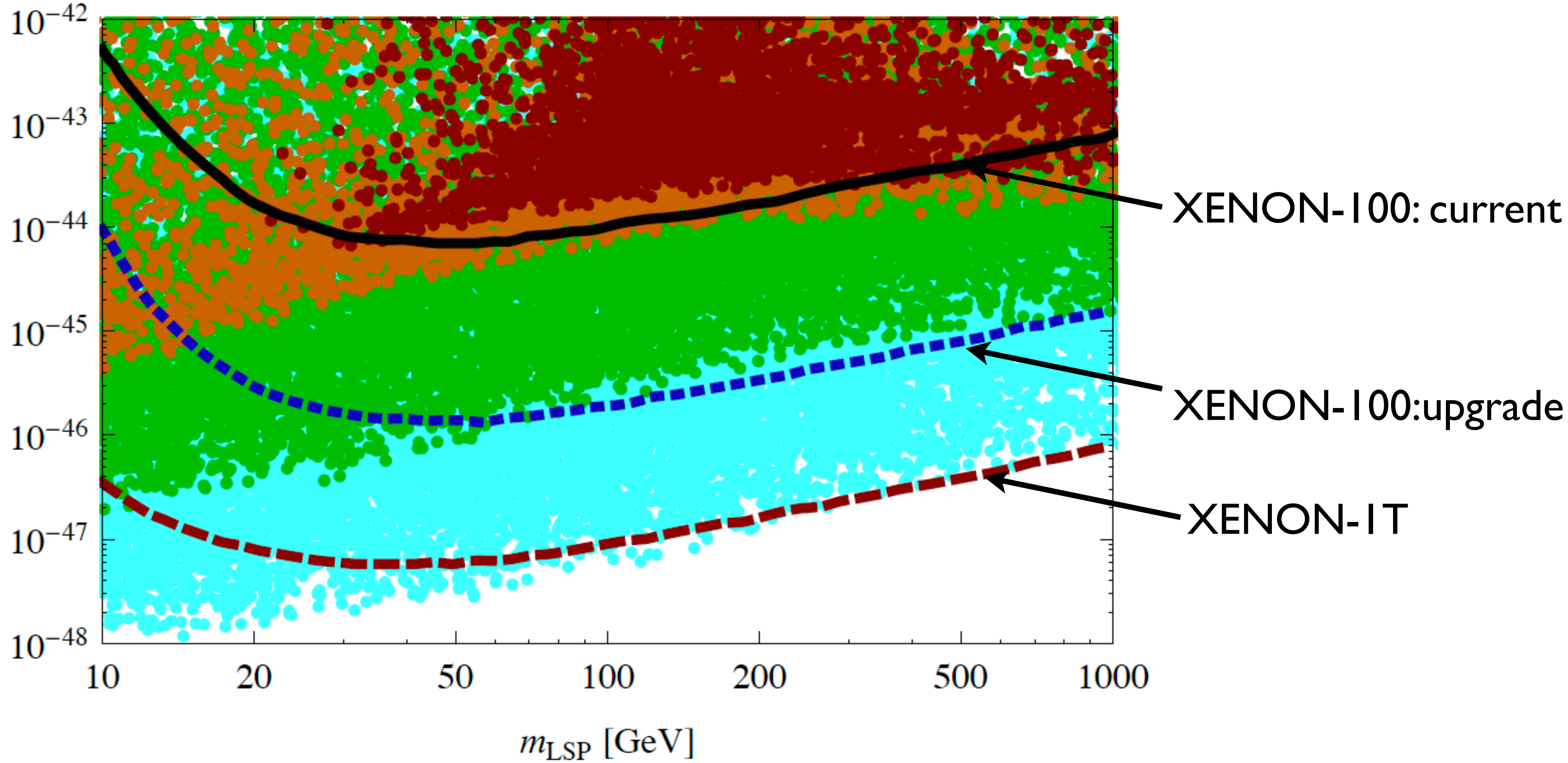
Results: Higgsino Fraction



$$F_H = |Z_{\chi 3}|^2 + |Z_{\chi 4}|^2 .$$

Low DD cross section ⟺ pure gaugino OR pure Higgsino LSP

Results: Higgsino Fraction

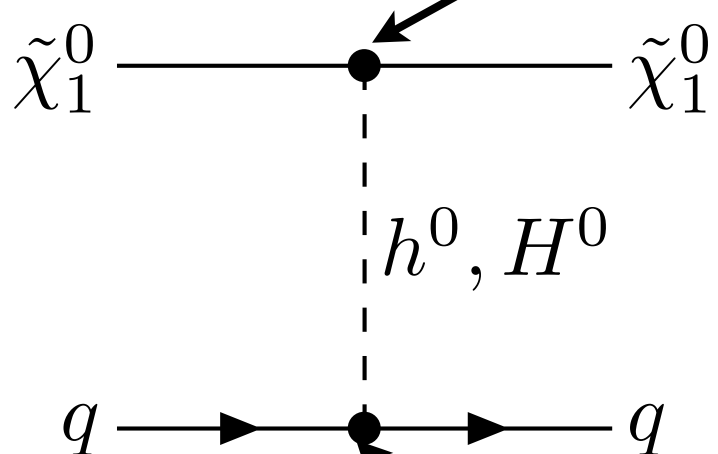


Color-code: "purity" $p = \min(F_H, 1 - F_H).$

Red	$p > 0.2$
Orange	$0.1 < p < 0.2$
Green	$0.01 < p < 0.1$
Cyan	$0.001 < p < 0.01$

Understanding H-Fraction Bound

$$\tilde{\chi}^0\tilde{\chi}^0 h: \quad (gZ_{\chi 2} - g' Z_{\chi 1})(\cos\alpha Z_{\chi 4} + \sin\alpha Z_{\chi 3})\,,$$
$$\tilde{\chi}^0\tilde{\chi}^0 H: \quad (gZ_{\chi 2} - g' Z_{\chi 1})(\sin\alpha Z_{\chi 4} - \cos\alpha Z_{\chi 3})\,.$$



"Generic" mixings: $Z_{\chi i} \sim 1$

"Natural" cross section (h exchange only):

$$\sigma \sim (\text{a few}) \times 10^{-44}\ \text{cm}^2$$

➡ $Z_{\chi i} \ll 1$ required to get lower x-section!

$$u\bar{u}h: \frac{\sqrt{2}m_u}{v\cos\beta}$$

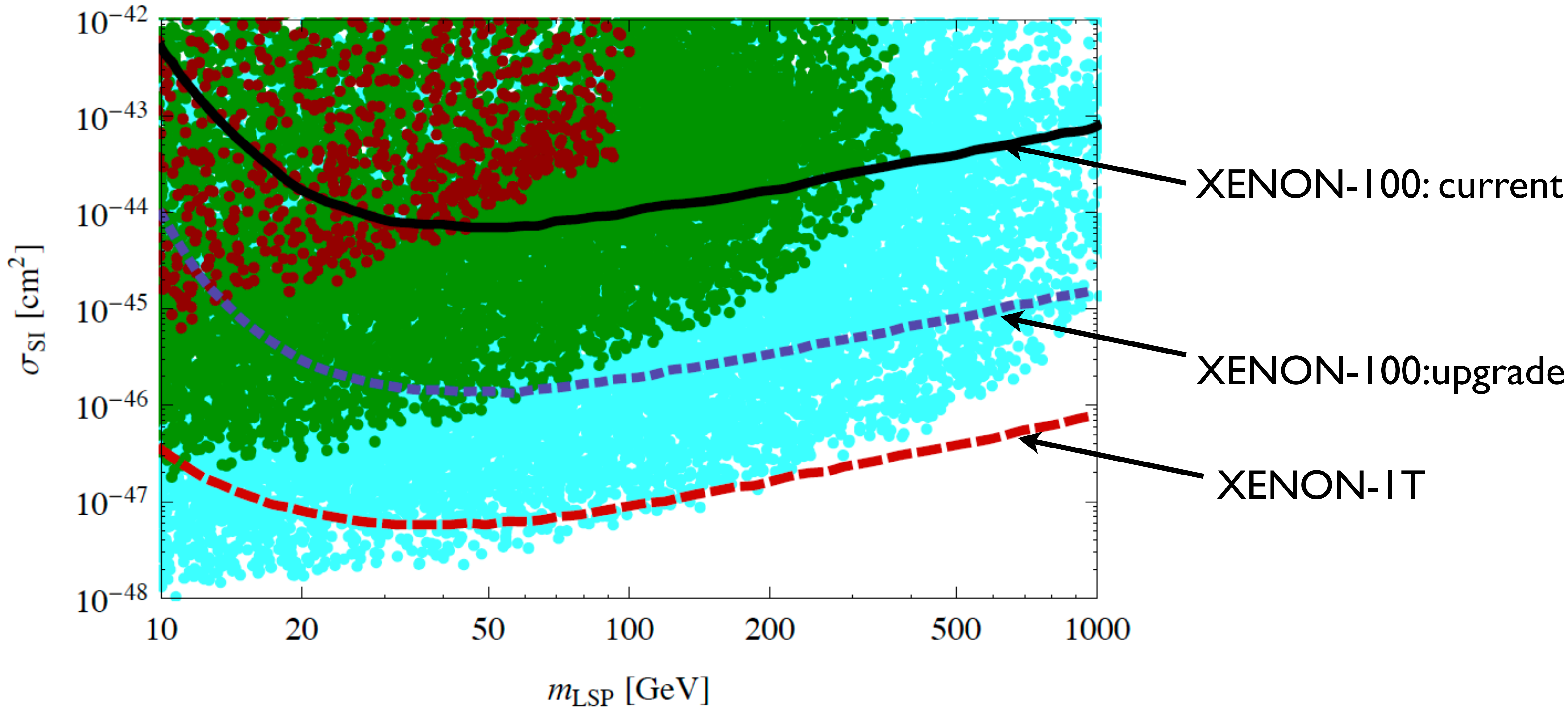
$$d\bar{d}h: \frac{\sqrt{2}m_d}{v\sin\beta}$$

(basically fixed)

Higgsino Fraction Summary

- Generic DD cross section from light Higgs exchange is $\sigma \sim (\text{a few}) \times 10^{-44}\ \text{cm}^2$, already being explored by XENON-100 and others
- Pure-gaugino or pure-higgsino LSP is required to suppress the cross section below this level
- Current bounds: p>0.2 for any mLSP, and p>0.1 for mLSP>50 GeV, is ruled out
- In the rest of the talk, we consider the "gaugino LSP sample", $M_1 < \mu$ and/or $M_2 < \mu$, and "Higgsino LSP sample", $M_1 > \mu$ and $M_2 > \mu$, separately

EWSB Fine-Tuning: Gaugino LSP



Color-code: EWSB fine-tuning

Red	$\Delta < 10$
Green	$10 < \Delta < 100$
Cyan	$100 < \Delta < 1000$

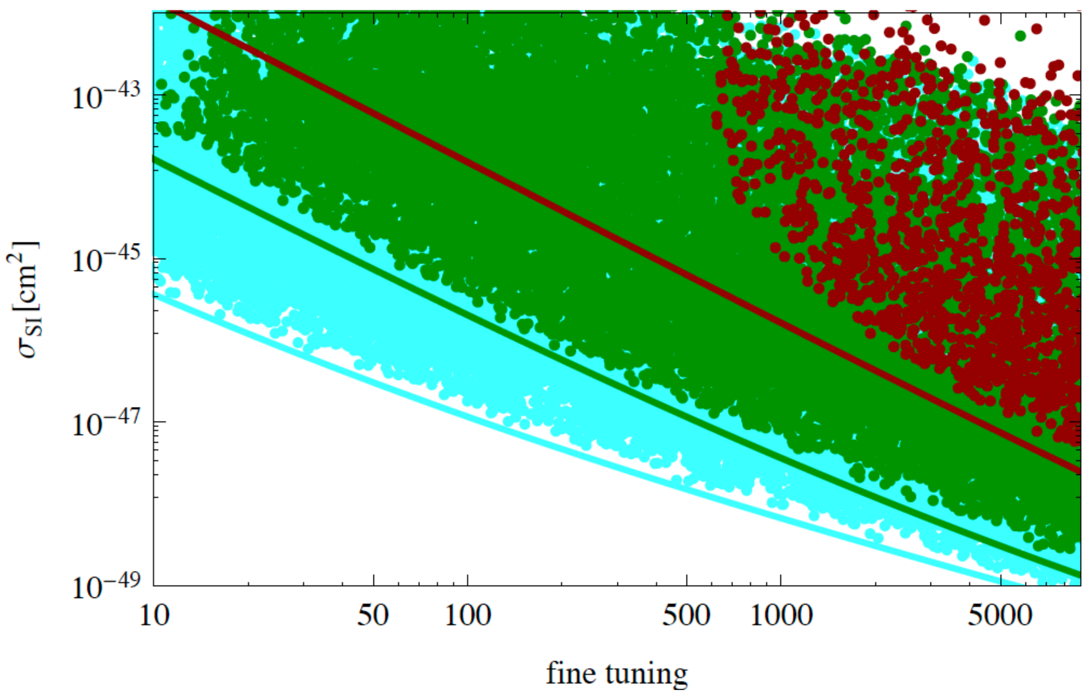
Lower DD cross section means MORE FINE-TUNING!

EWSB Fine-Tuning: Gaugino LSP

- This correlation is easy to understand: Smaller DD cross section ⇒ purer gaugino LSP ⇒ larger μ ⇒ more severe EWSB fine-tuning

- Minimal DD cross section consistent with given fine-tuning*:

$$\sigma_{\min} = (1.2 \times 10^{-42}\ \mathrm{cm}^2) \left(\frac{120\ \mathrm{GeV}}{m_h}\right)^4 \frac{1}{\Delta} \left(\frac{1}{\tan\beta} + \frac{1}{\sqrt{\Delta}} \frac{M_{\mathrm{LSP}}}{m_Z}\right)^2$$



Color-code: LSP mass

Red $M_{\mathrm{LSP}} \in [10, 100]$ GeV
Green $M_{\mathrm{LSP}} \in (100, 1000]$ GeV
Cyan $M_{\mathrm{LSP}} \in (1, 10]$ TeV

$$(\tan\beta)_{\max} = 50$$

✳ Positive, real parameters are required! (more general case - discuss later)

Gaugino LSP Summary

- Observed an inverse correlation between DD cross section and EWSB fine-tuning
- Obtained a simple analytic formula for minimal DD cross section consistent with given FT
- Current bounds already relevant: XENON-100 implies FT worse that 1/10 for mLSP>70 GeV
- XENON-1T will probe down to FT~1%!

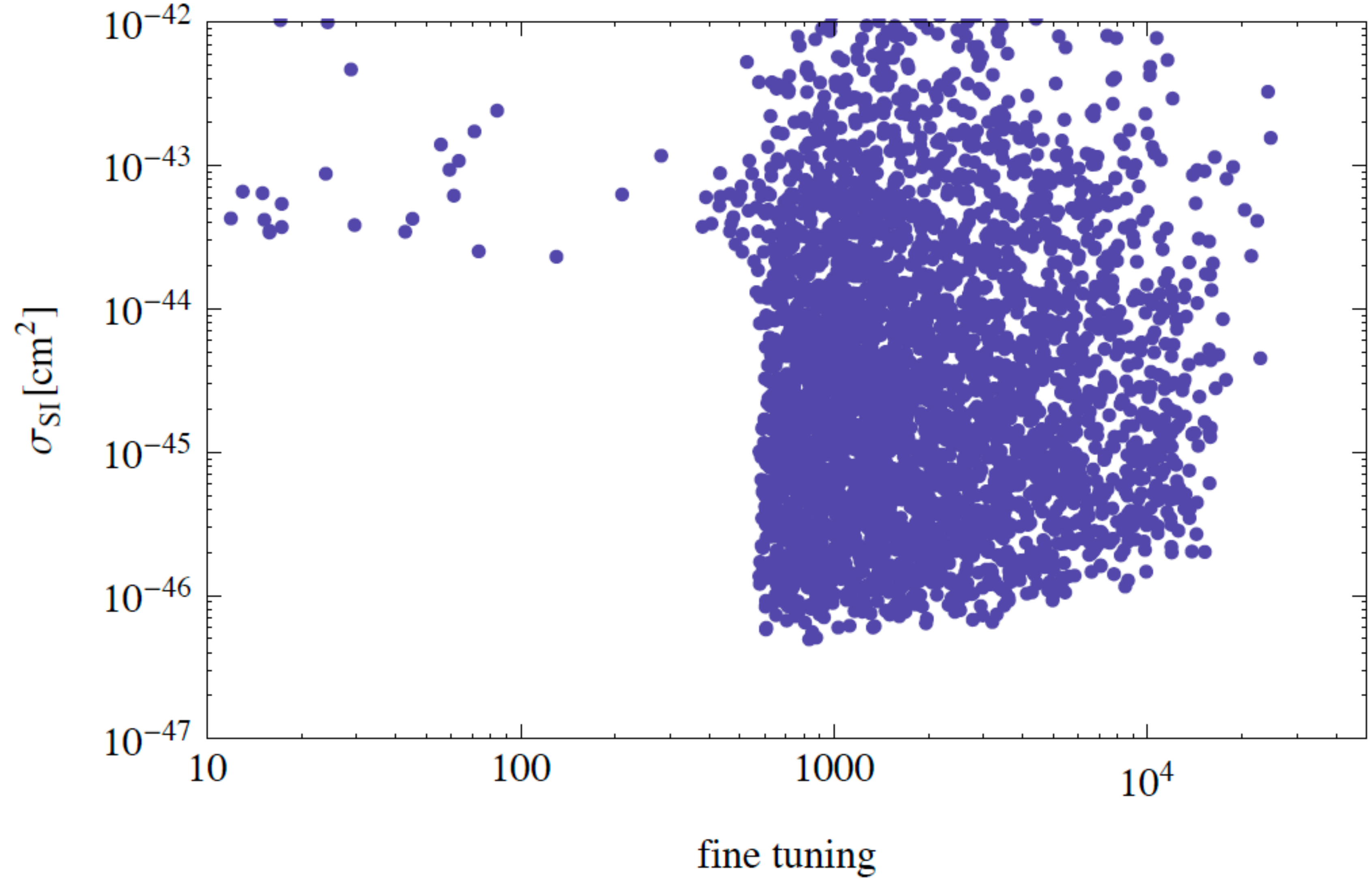
Higgsino LSP

- No correlation between DD cross section and FT in this case: μ fixed, $M_{1,2} \to \infty$ ➡ $\sigma \to 0$
- Add a relic density constraint: $\Omega_{\rm dm} h^2 = 0.110 \pm 0.006$
- For fixed MSSM parameters, relic density uniquely predicted (assuming standard FRW cosmology up to $T \sim M_{\rm LSP}$):

$$\Omega h^2 \approx 10^9 \frac{(n+1) x_f^{n+1}\ {\rm GeV}^{-1}}{(g_{*S}/g_*^{1/2}) m_{\rm Pl} \sigma_0}$$

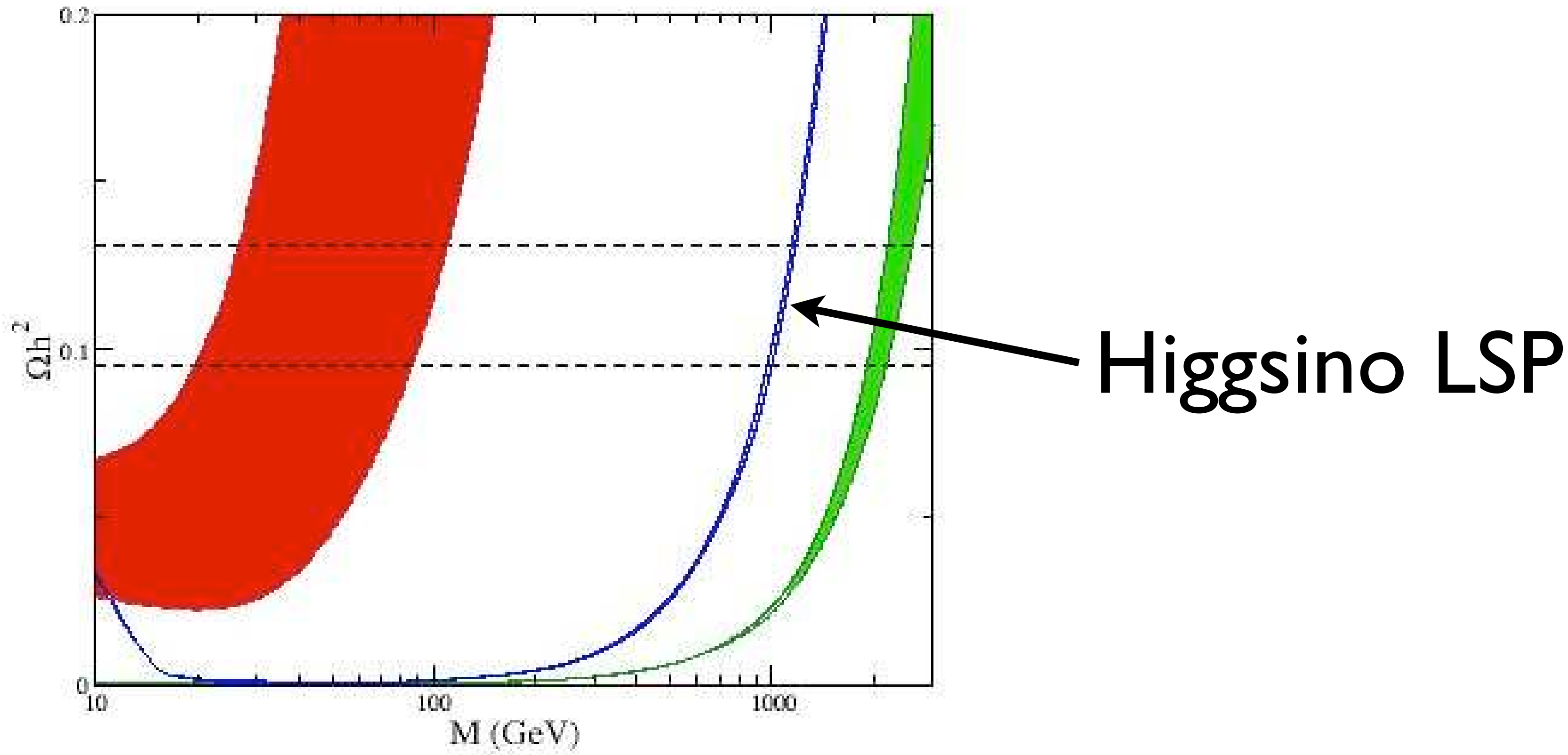
- Higgsino (co-)annihilation channels: W/Zs, quarks (predominantly 3rd gen.)
- Our approach: ignore quark final states, impose a "one-sided" RD constraint $\Omega_{\rm pred} \geq \Omega_{\rm obs}$

Higgsino LSP



Conclusion: XENON-100 already implies FT>500, if the (one-sided) relic density constraint is assumed!

Higgsino LSP Relic Density



Observed (or higher) relic density requires $\mu \geq 1$ TeV ⇒ EWSB FINE-TUNING!

[Plot from Arkani-Hamed, Delgado, Giudice, hep-ph/0601041]

Including Signs/Phases

- While the Higgs/gauge sector of the MSSM has 5 parameters, $(M_1, M_2, \mu, m_A, \tan\beta)$, only 2 phases are physical: $\varphi_1 = \arg(\mu M_1 \sin 2\beta), \quad \varphi_2 = \arg(\mu M_2 \sin 2\beta)$
- Repeat the scan, allowing both signs of $M_1,\ M_2$
- New feature: accidental cancellations within the t-channel Higgs exchange amplitude are possible, lowering the cross section
- Quantify these cancellations: $\Delta_{\rm acc} \equiv \sqrt{\sum_{i=1}^{5} \left(\frac{\partial \log\sigma}{\partial \log p_i} \right)^2}$

Accidental Cancellations

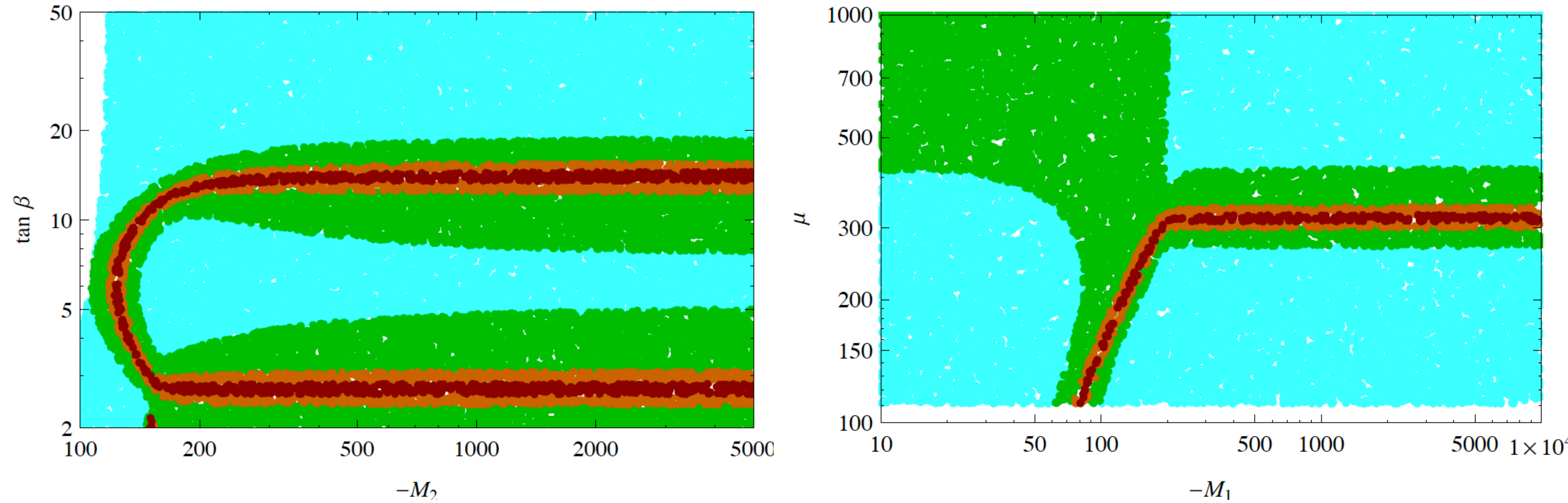
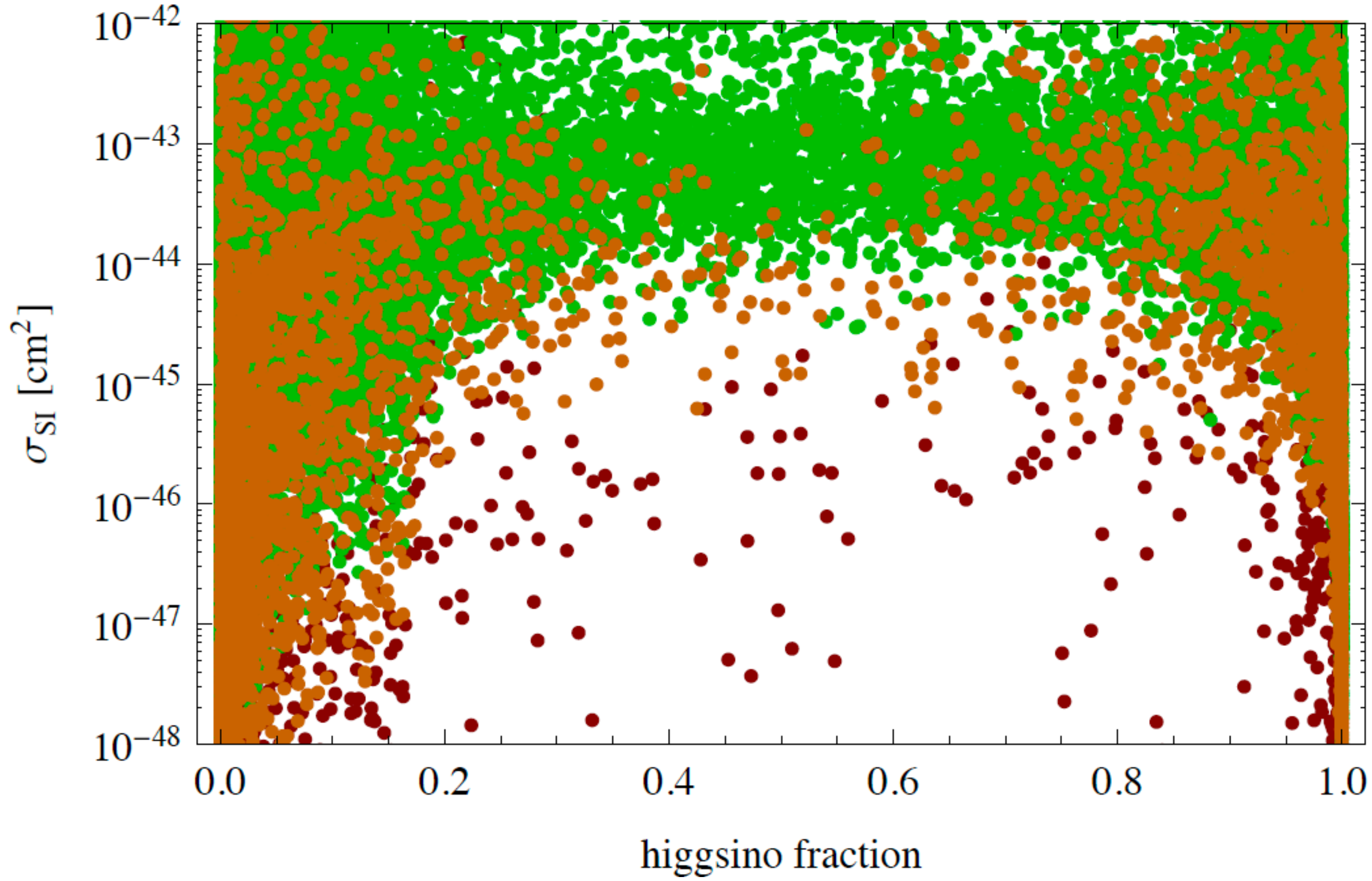


Figure 10: Left panel: Scatter plot of direct detection cross section as a function of $\tan\beta$ and $-M_2$, with $M_1 = -150$ GeV, $\mu = 200$ GeV, and $m_A = 500$ GeV. Cyan, green, orange and red points have $\log_{10}\sigma_{\rm cm^2} > -45, \log_{10}\sigma_{\rm cm^2} \in (-46, -45), \log_{10}\sigma_{\rm cm^2} \in (-46, -47)$ and $\log_{10}\sigma_{\rm cm^2} < -47$, respectively. Right panel: Same, as a function of μ and $-M_1$, with $M_2 = -200$ GeV, $\tan\beta = 10$, and $m_A = 500$ GeV.

Results: Higgsino Fraction



Color-code: "Accidentality"

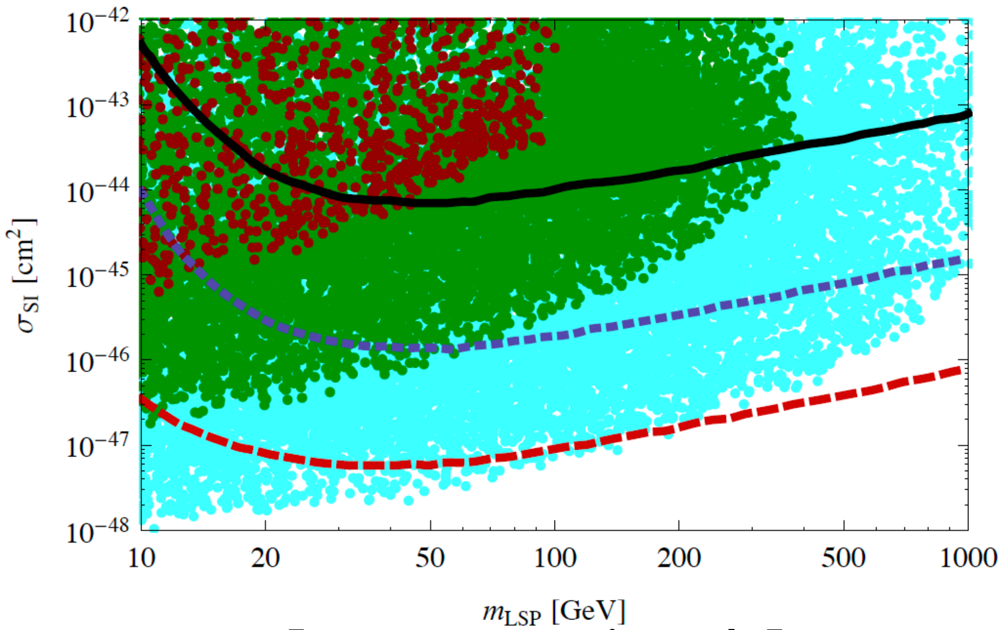
Red $\Delta_{acc} > 30$

Orange $10 < \Delta_{acc} < 30$

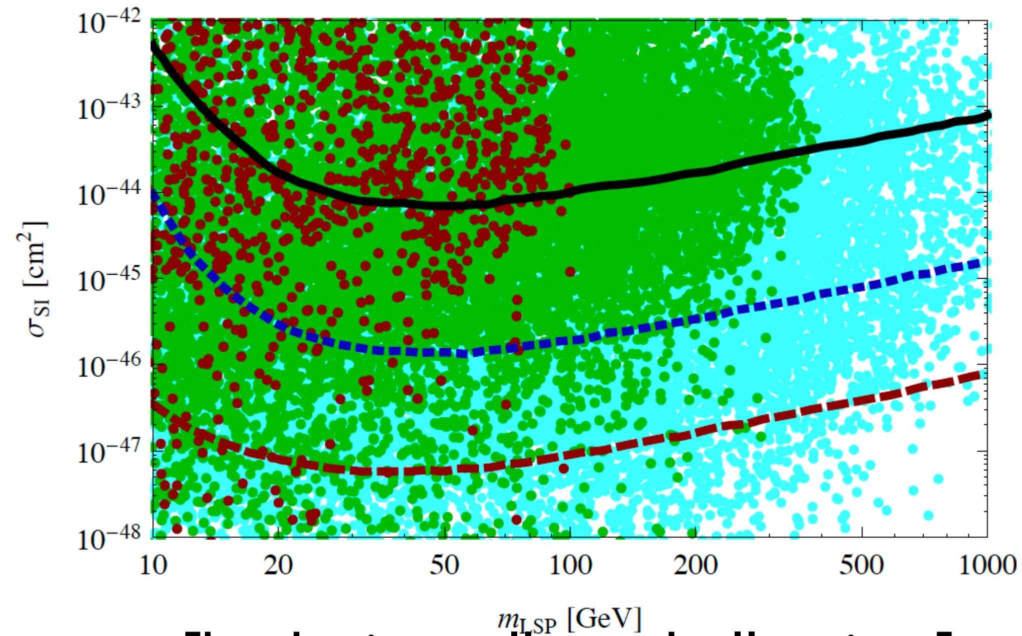
Green $\Delta_{acc} < 10$

Low DD cross section ⟷ pure gaugino OR pure Higgsino LSP OR accidental cancellations

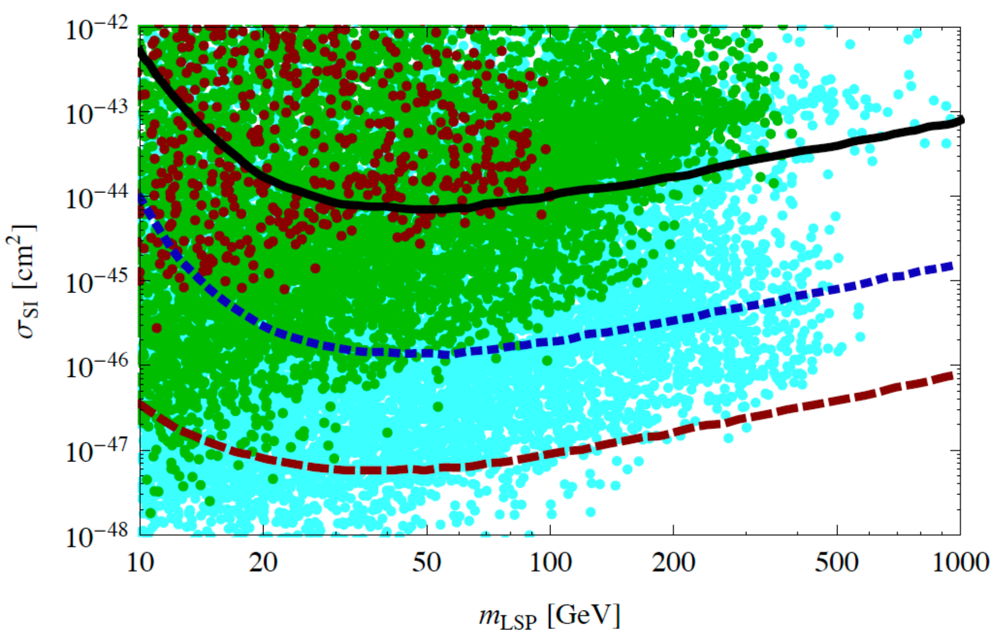
EWSB Fine-Tuning for Gaugino LSP



[positive par's only]



[both signs allowed, all points]



[both signs allowed, $\Delta_{\rm acc} < 5$]

Conclusion: the x-sec/FT correlation still holds, except for points with significant accidental cancellations

Conclusions

- "Generic" direct detection cross section (for order-one Higgsino/gaugino mixture LSP) is $\sim 2 \times 10^{-44}\ \mathrm{cm}^2$, ruled out by XENON-100 for most LSP masses
- Cross sections below $10^{-44}\ \mathrm{cm}^2$ require the LSP to be pure gaugino or pure Higgsino, with lower cross sections corresponding to higher LSP purity
- In the gaugino LSP case, this implies that lower cross sections correspond to stronger fine-tuning in the EWSB sector
- Current XENON-100 bound implies FT of 10% or worse for mLSP>70 GeV
- XENON-1T will probe down to FT~1% for all LSP masses
- All these statements are true in the most general MSSM, assuming only the absence of accidental cancellations in the cross section, do not need to fix thermal relic abundance (e.g. non-standard cosmology is OK)
- In the higgsino LSP case, XENON-100 bound implies FT of 1/500 or worse IF one-sided relic density constraint is imposed