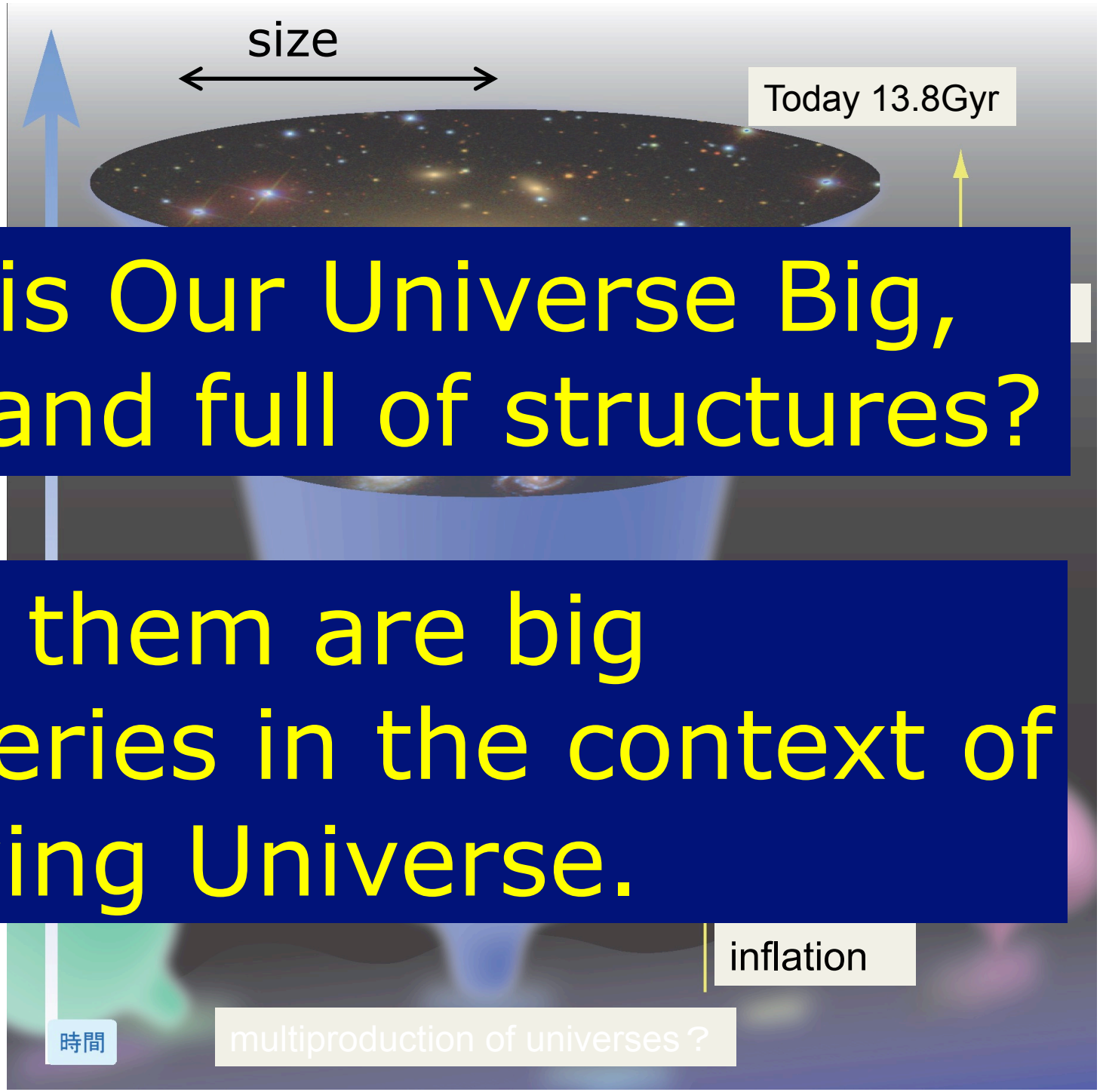
Creation of an Inflationary Universe in the final stage of Black Hole evaporation

Research Center for the Early Universe (RESCEU)

Department of Physics
SCHOOL OF SCIENCE
THE UNIVERSITY OF TOKYO

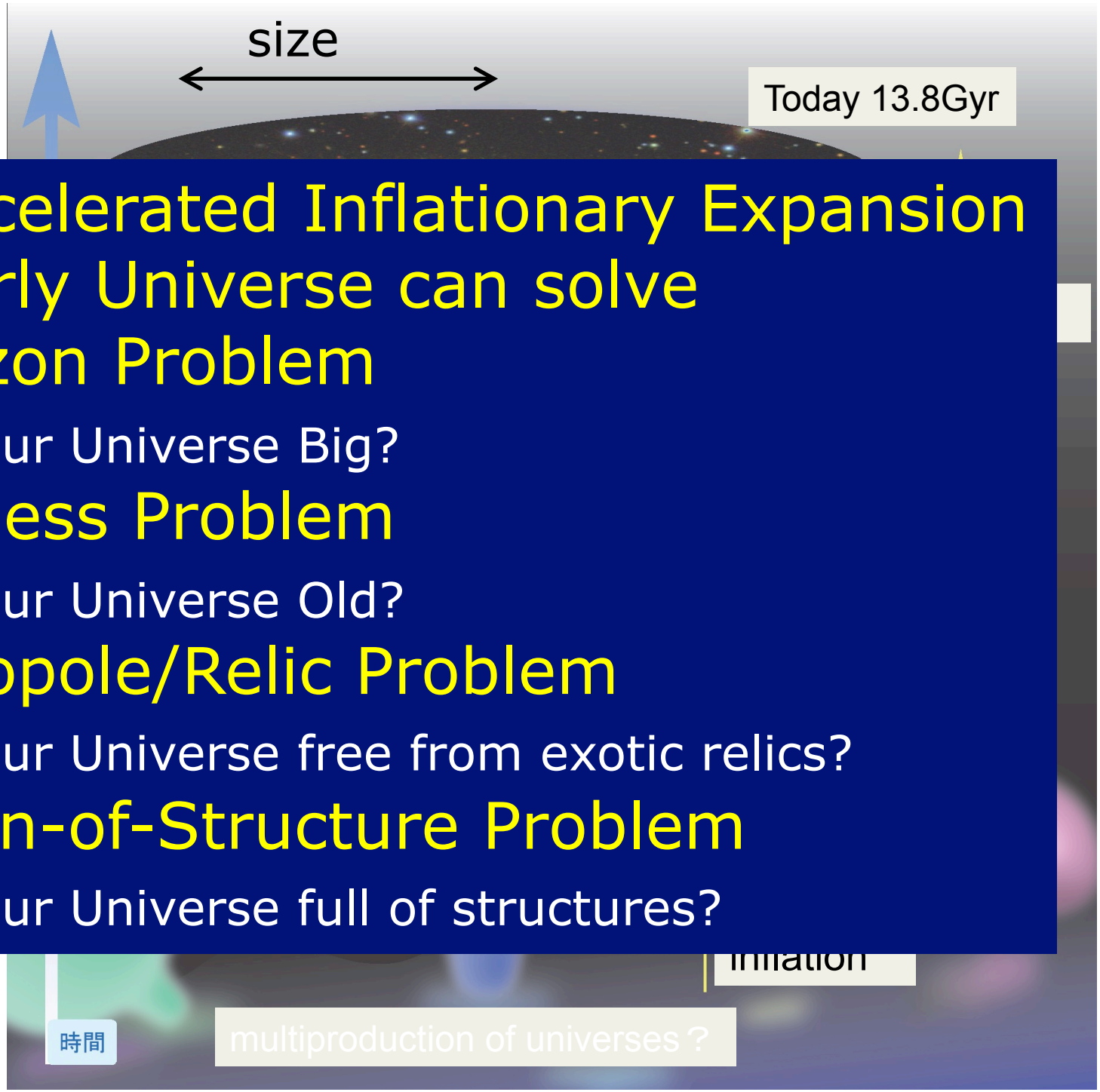
KAVLI IPMU

Jun'ichi Yokoyama
+Naritaka Oshita



Why is Our Universe Big, Old, and full of structures?

All of them are big mysteries in the context of evolving Universe.



Rapid Accelerated Inflationary Expansion in the early Universe can solve

The Horizon Problem

Why is our Universe Big?

The Flatness Problem

Why is our Universe Old?

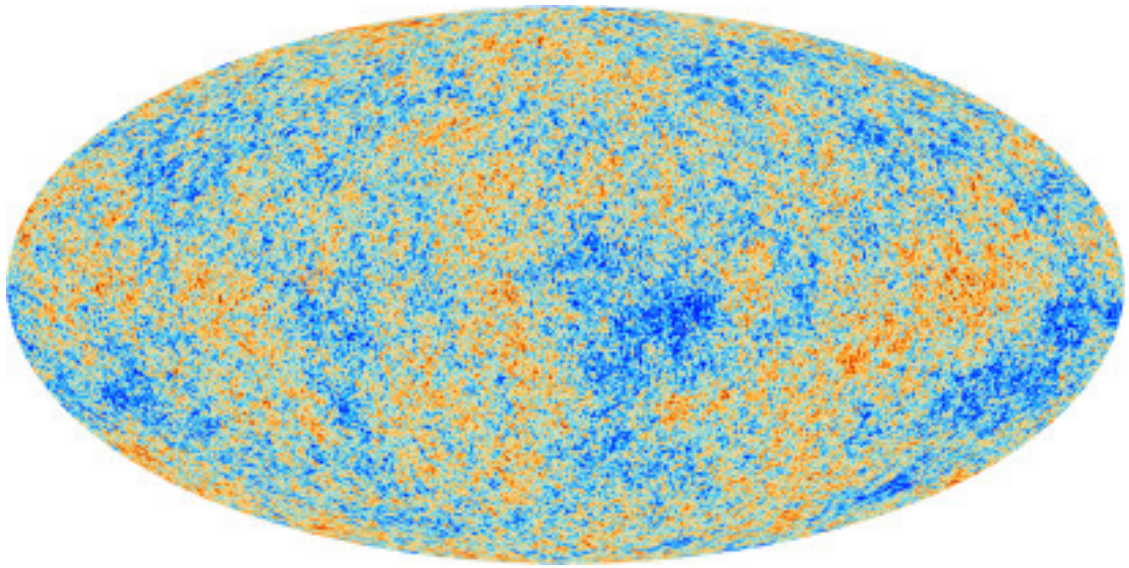
The Monopole/Relic Problem

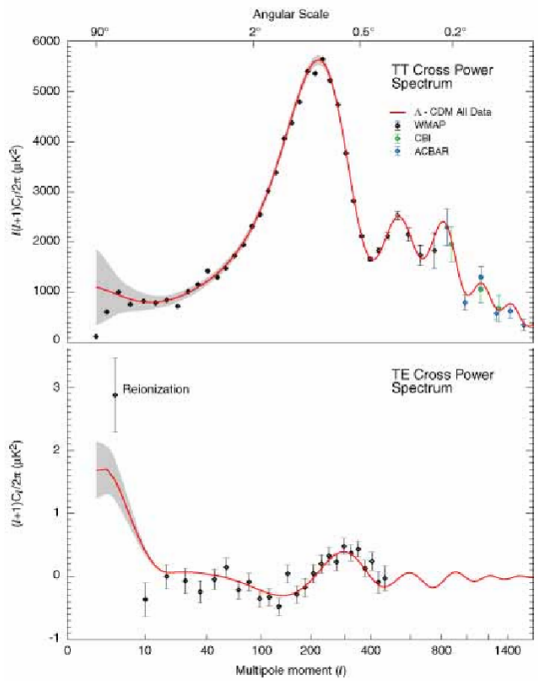
Why is our Universe free from exotic relics?

The Origin-of-Structure Problem

Why is our Universe full of structures?

The last 50∼60 e-folds of inflationary expansion has been probed by high precision observations such as those conducted by WMAP, Planck etc., and they are highly consistent with the theory.





But it is totally unknown how inflation started in our Universe.

Out of quantum gravity regime?

Likely, but not necessarily.

Many inflation n
multiproduction

Physics Lette

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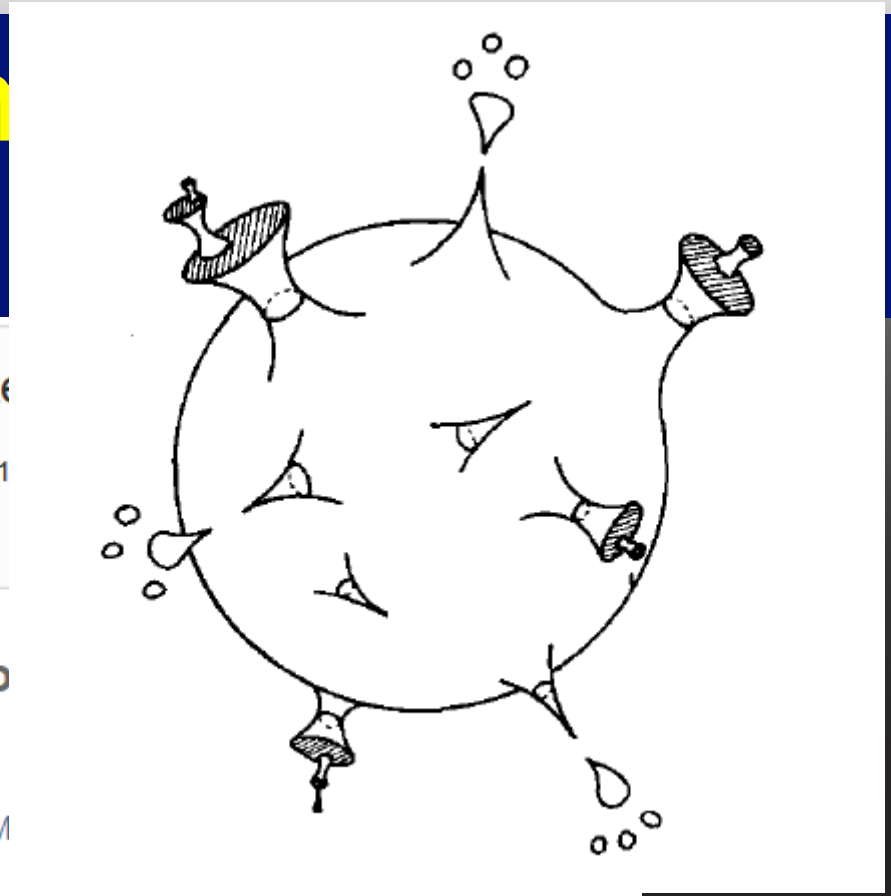
Multi-production of universes by first-o vacuum

Katsuhiko Sato [1], Hideo Kodama [1], Misao Sasaki [a], Kei-ichi M

[1] Department of Physics, Kyoto University, Kyoto 606, Japan

[a] Research Institute for Fundamental Physics, Kyoto University, Kyoto 606, Japan



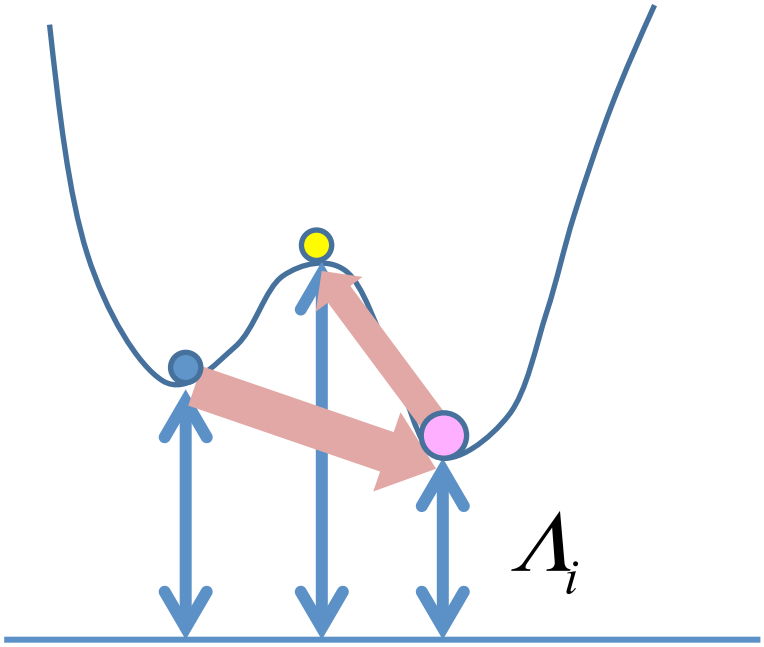
Then a question arises:

Is our Universe of 1st generation?

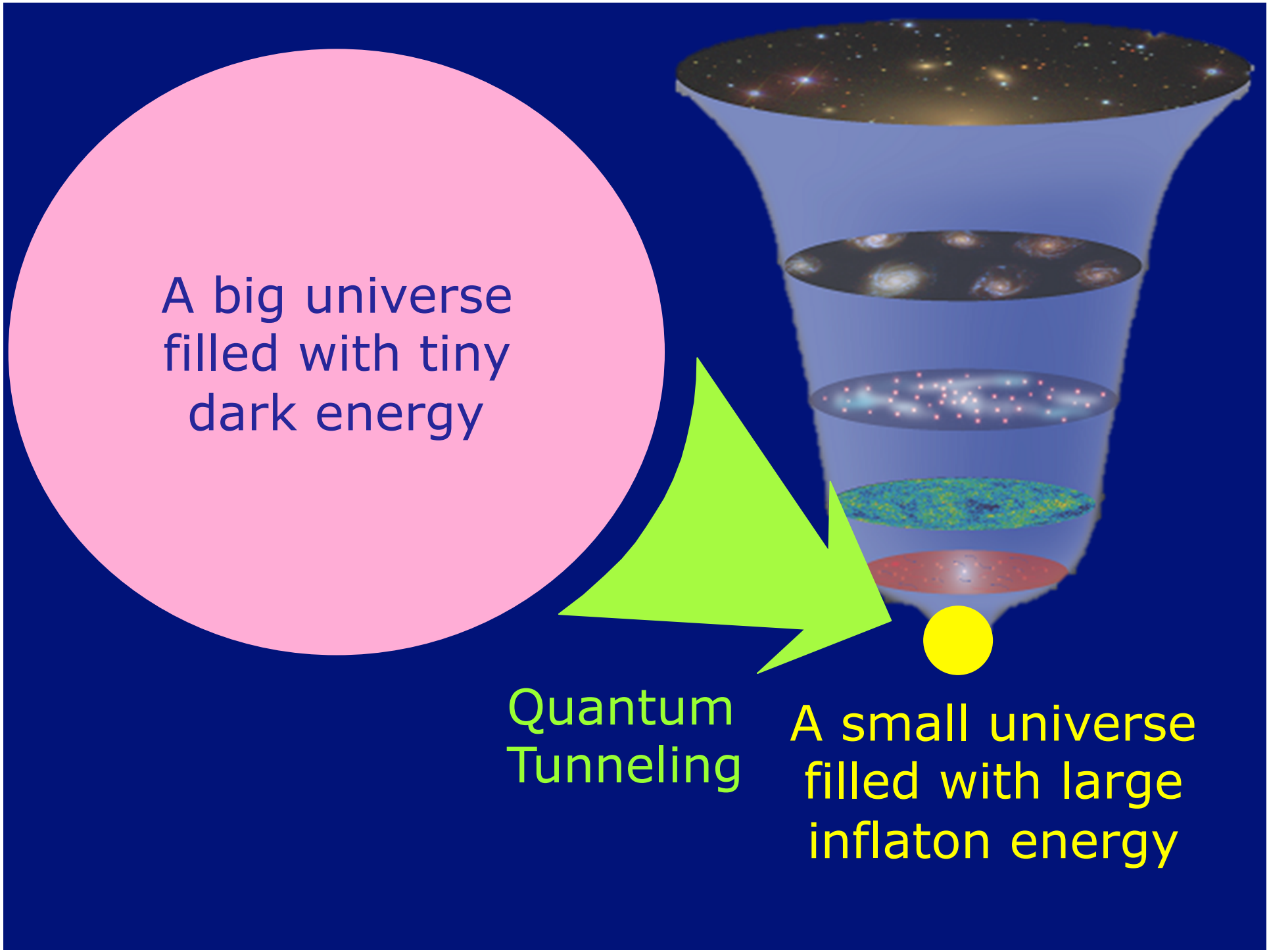
If dark energy is Λ,our Universe will be asymptotically de Sitter.

Inflation ≅ de Sitter expansion

De Sitter space can tunnel to another de Sitter space with a different Λ.



Hawking and Moss (1983)
Lee and Weinberg (1987)
…
multiverse
string landscape…

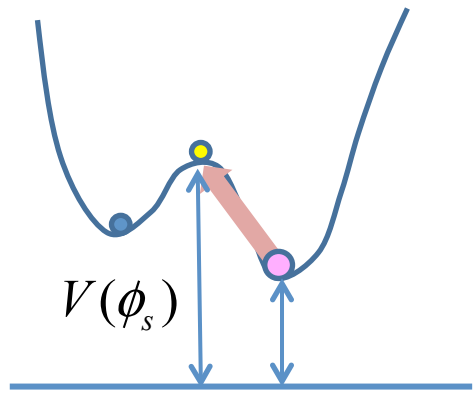

A big universe filled with tiny dark energy
Quantum Tunneling
A small universe filled with large inflaton energy

The Hawking Moss Instanton

An O(4) symmetric bounce solution

$$ds^2 = d\xi^2 + \rho(\xi)^2 d\Omega_{\rm III}^2$$

unit 3 sphere



Euclidean action of the Einstein scalar theory

$$I_{\rm E} = 2\pi^2 \int d\xi \left[\rho^3 \left(\frac{1}{2}\dot{\phi}^2 + V(\phi) \right) + \frac{3}{8\pi G} \left(\rho^2 \ddot{\rho} + \rho \dot{\rho}^2 - \rho \right) \right]$$

$$\dot{\rho} = \frac{d\rho}{d\xi}$$

$$\ddot{\phi} + \frac{3\dot{\rho}}{\rho}\dot{\phi} = \frac{dV}{d\phi},$$

$$\dot{\rho}^2 = 1 + \frac{8\pi G}{3}\rho^2 \left(\frac{1}{2}\dot{\phi}^2 - V \right)$$

Static scalar field configuration

$$\dot{\phi} = \ddot{\phi} = \frac{dV}{d\phi} = 0$$

$$\dot{\rho}^2 = 1 - \frac{8\pi G}{3}\rho^2 V$$

Solution

$$\rho(\xi) = H_{\rm s}^{-1} \sin(H_{\rm s}\xi),$$

$$H_{\rm s}^2 \equiv \frac{8\pi G}{3} V(\phi_{\rm s}),$$

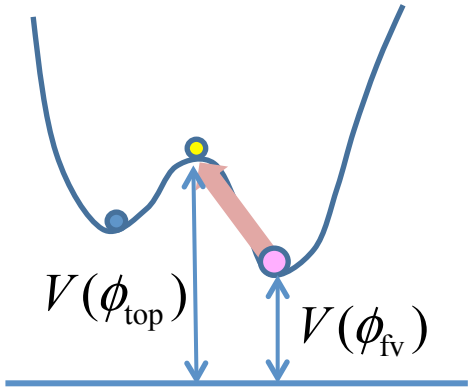
Euclidean action

$$I_{\rm E}(\phi_{\rm s}) = -\frac{3}{8G^2 V(\phi_{\rm s})}$$

$$\Gamma_{\mathrm{fv}\to\mathrm{top}} = Ae^{-B_{\mathrm{HM}}} = A\exp\left[-I_{\mathrm{E}}(\phi_{\mathrm{top}}) + I_{\mathrm{E}}(\phi_{\mathrm{fv}})\right]$$
$$= A\exp\left[\frac{3}{8G^2}\left(\frac{1}{V(\phi_{\mathrm{top}})} - \frac{1}{V(\phi_{\mathrm{fv}})}\right)\right]$$

If $\Delta V \equiv V(\phi_{top}) - V(\phi_{fv}) \ll V(\phi_{top})$, so that the change of the geometry is negligible, we find

$$B_{HM} \cong \frac{3\Delta V}{8G^2V^2(\phi_{fv})} = \frac{8\pi^2}{3}H_{fv}^{-4}\Delta V = \frac{4\pi}{3}H_{fv}^{-3}\frac{\Delta V}{T_H}$$



with $\boxed{T_H = \frac{H(\phi_{fv})}{2\pi}}$ Hawking Temperature of De Sitter space

$$H_{fv}^2 = \frac{8\pi G}{3}V(\phi_{fv})$$

Thus this process can be interpreted as a thermal transition with the Hawking temperature of de Sitter space, but only approximately.

Weinberg (2007)

In fact, the Hawking-Moss bounce can be interpreted solely by gravitational (Bekenstein, Horizon) entropy.

Horizon area

$$A = 4\pi H^{-2} = 4\pi \left(\frac{8\pi G V(\phi_s)}{3} \right)^{-1} = \frac{3}{2GV(\phi_s)}$$

Horizon entropy

$$S = \frac{A}{4G} = \frac{3}{8G^2 V(\phi_s)} \quad \Longleftrightarrow \quad I_{\mathrm{E}}(\phi_{\mathrm{s}}) = -\frac{3}{8G^2 V(\phi_{\mathrm{s}})}$$

This result can be obtained by calculating the Euclidean action using static coordinate with which "bulk Lagrangian" vanishes due to Hamiltonian constraint, so that only horizon contribution remains.

Schematically: $\mathcal{L} = p\dot{q} - \mathcal{H} = 0$

=0 due to the Hamiltonian constraint

=0 due to static nature of the metric

The horizon contribution arises because…
Euclidean De Sitter space in static representation $\tilde{t}$: Euclidean time

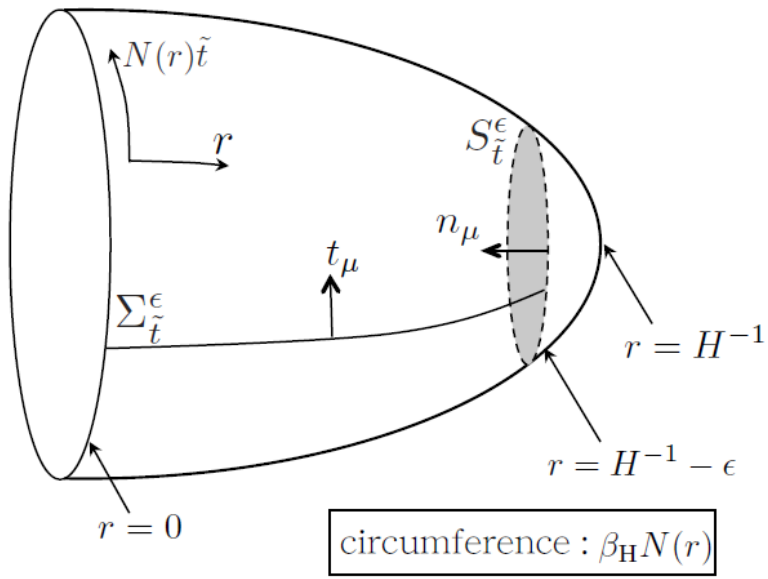
$$d\tilde{s}^2 = \tilde{g}_{\mu\nu} dx^\mu dx^\nu = \left(1 - H^2 r^2\right) d\tilde{t}^2 + \frac{dr^2}{1 - H^2 r^2} + r^2 d\Omega_{\rm II}^2$$

unit 2 sphere

Lapse function $N = \sqrt{1 - H^2 r^2}$.

In this Euclidean description, the horizon $r = H^{-1}$ is a coordinate singularity similar to the origin in the 2D polar coordinate where $\tilde{t}$ behaves like an angular variable θ.

We regularize the coordinate singularity at the horizon introducing a hypothetical surface at $r = H^{-1} - \epsilon$.



The surface integral

$$I^{(\rm tot)}_{\rm E\ static} = -\int_{\rm S} d\tilde{t} d^2x \sqrt{\sigma} \left(\frac{n^i \partial_i N}{8\pi G} \right)$$

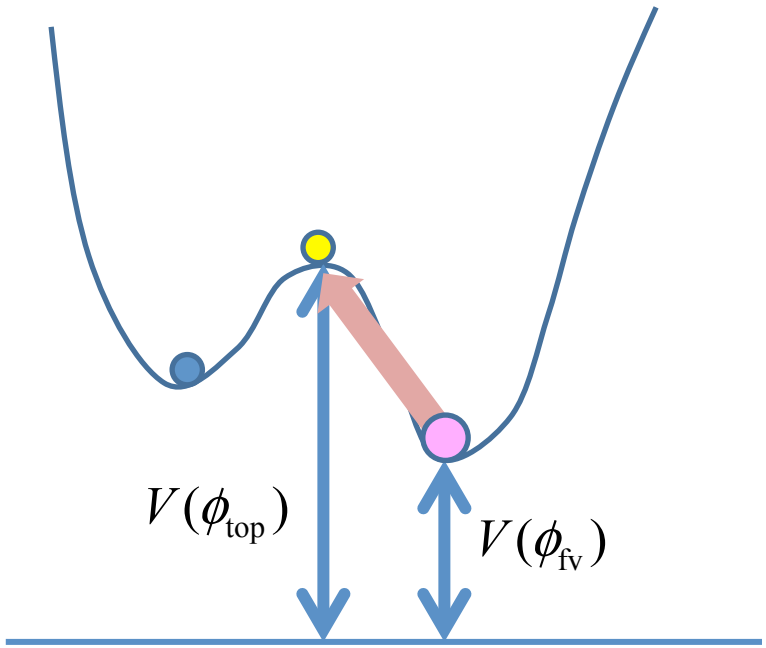
gives rise to the desired result.

$$\Gamma_{\mathrm{fv}\to\mathrm{top}} = Ae^{-B_{\mathrm{HM}}} = A\exp\left[-I_{\mathrm{E}}(\phi_{\mathrm{top}}) + I_{\mathrm{E}}(\phi_{\mathrm{fv}})\right]$$

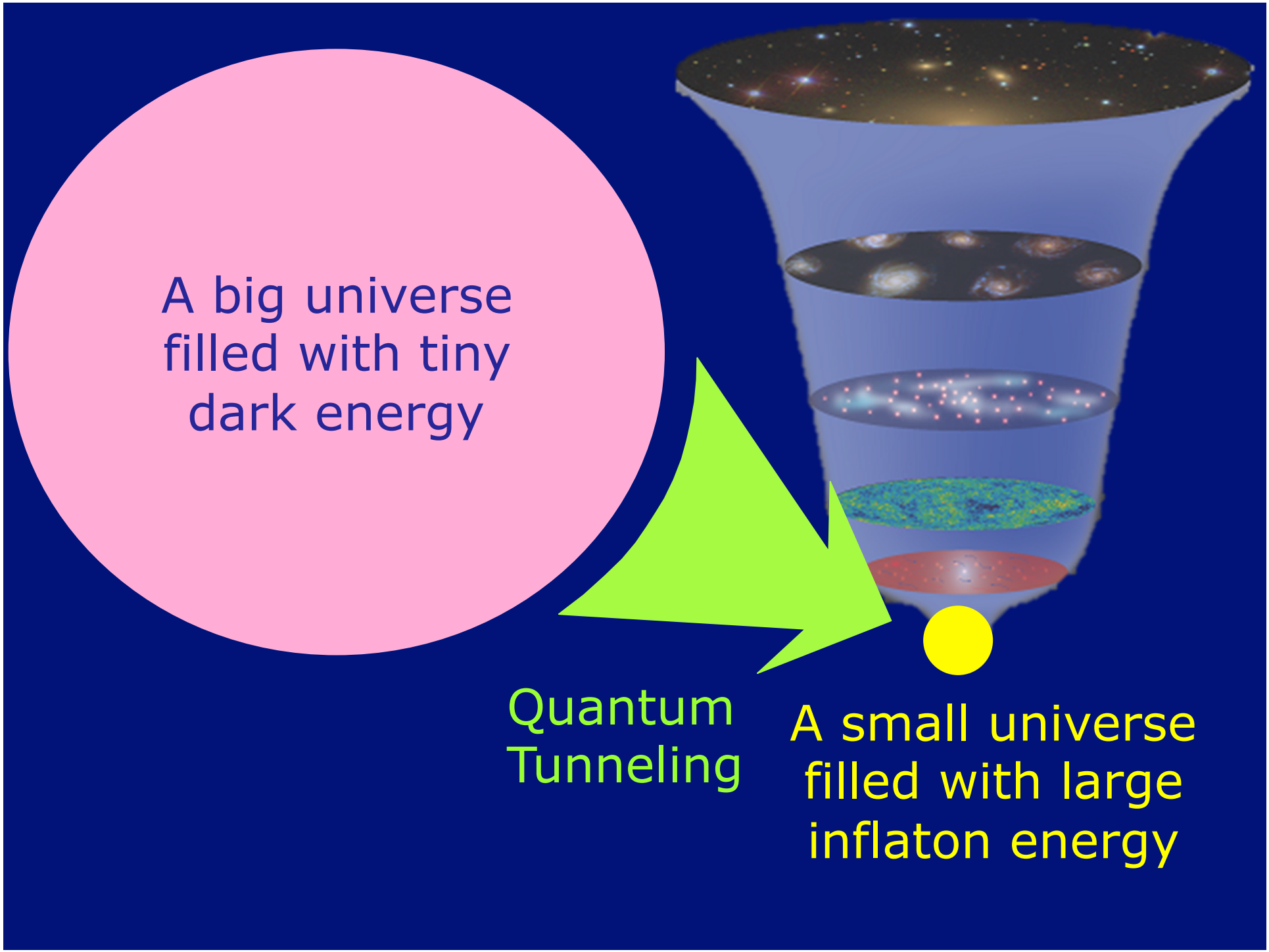
$$= A\exp\left[\frac{3}{8G^2}\left(\frac{1}{V(\phi_{\mathrm{top}})} - \frac{1}{V(\phi_{\mathrm{fv}})}\right)\right]$$

$$= A\exp\left[S(\phi_{\mathrm{top}}) - S(\phi_{\mathrm{fv}})\right]$$

In this picture transition to the potential top is suppressed NOT because it has larger energy density BUT because it has smaller entropy.



Oshita & JY 1603.06671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Reincarnation of the universe

Garriga and Vilenkin 1998

The universe can recycle itself to another universe with possibly different properties.

We may not have to consider the real beginning of the universe.

This scenario prefers a pure cosmological constant/vacuum energy with $w = -1$.

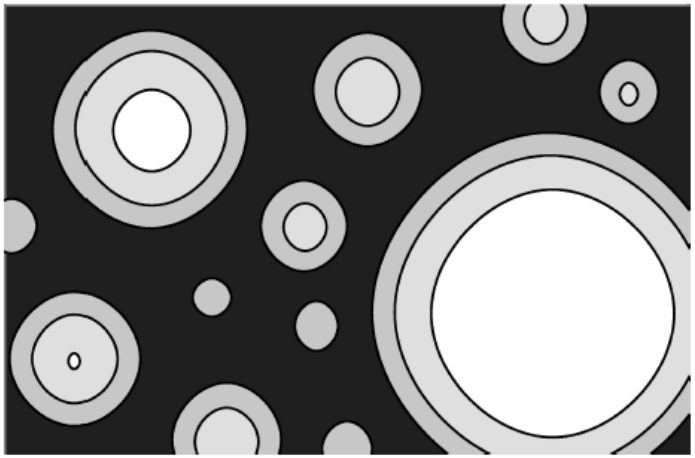
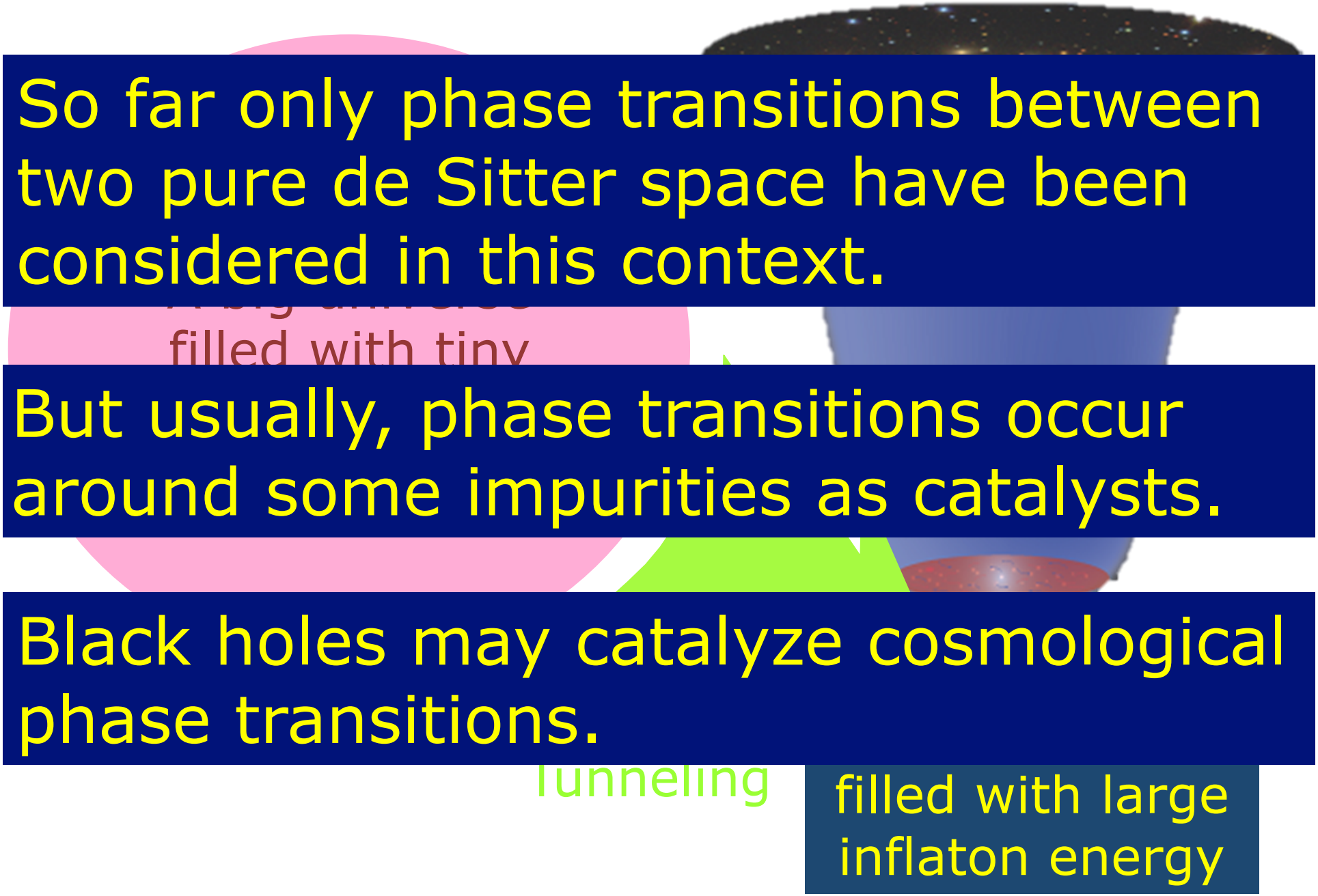


FIG. 1. True vacuum bubbles (white) nucleating in false vacuum (black). The shaded rings represent slow roll regions (external ring) and matter or radiation dominated regions (internal ring).

So far only phase transitions between two pure de Sitter space have been considered in this context.

But usually, phase transitions occur around some impurities as catalysts.

Black holes may catalyze cosmological phase transitions.



Studies on phase transitions in the early Universe around a black hole was pioneered by Hiscock in 1987. He assumed that the black hole mass does not change during bubble nucleation around a black hole.

More recently, Gregory, Moss, & Withers (2014) revisited the problem. They started with a Schwarzschild de Sitter space and considered nucleation of a thin-wall bubble of true vacuum with a remnant black hole in the center. cf Coleman De Luccia (1980)

They calculated Euclidean actions before and after bubble nucleation, postulating that the nucleation rate is given by

$$\Gamma = Ae^{-B}, \quad B = I_{\odot} - I_{SdS}$$

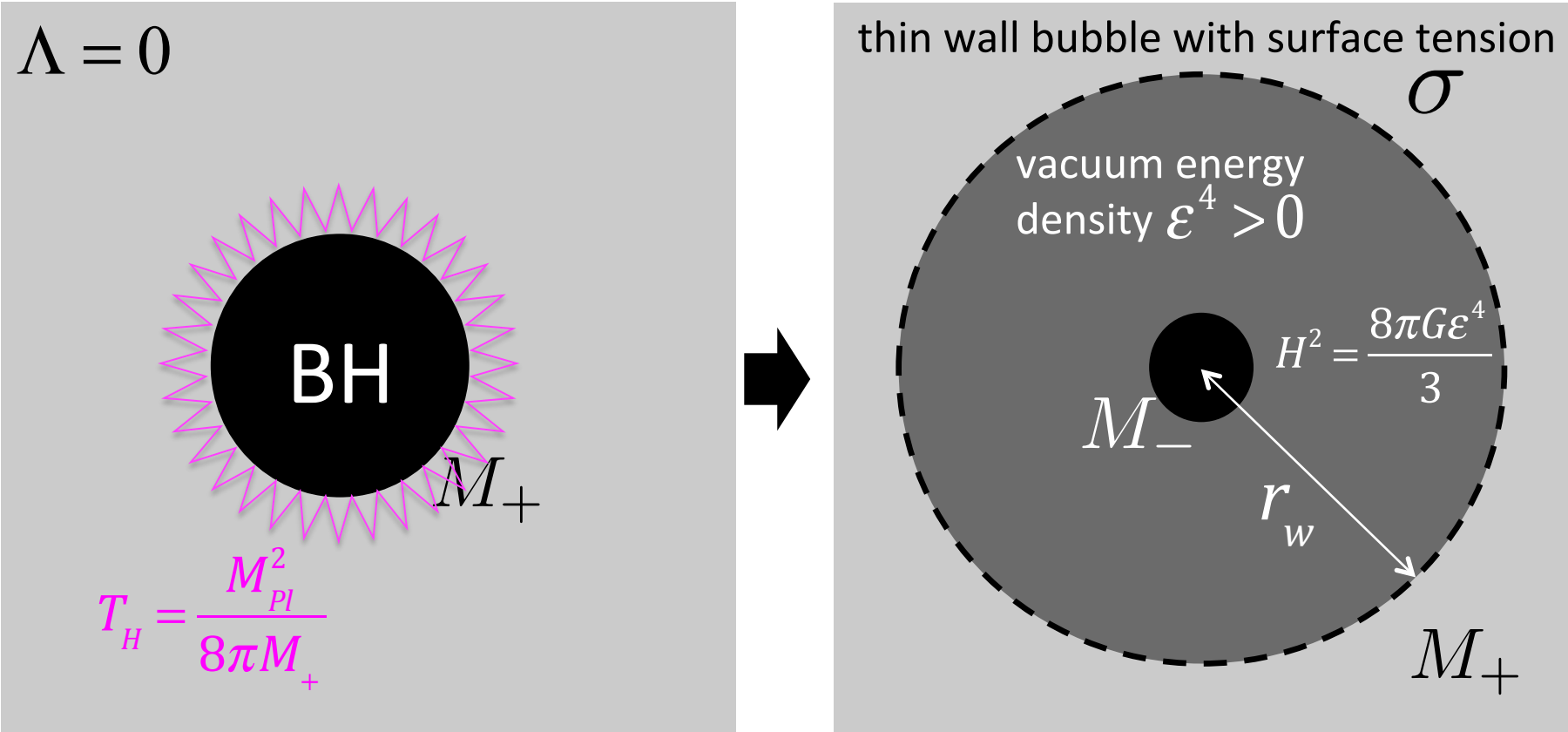
Euclidean action of a true vacuum bubble surrounded by false vacuum with a BH in the center

They have taken the effects of conical deficits properly, obtaining a term proportional to the surface area of the horizon.

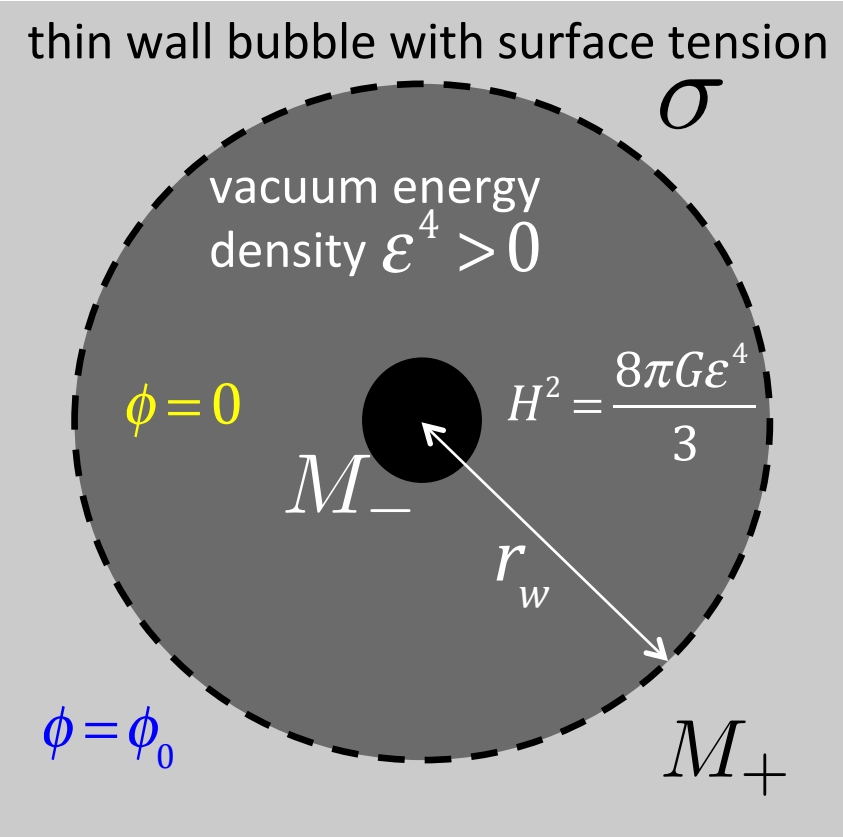
They have also observed that the BH mass may change.

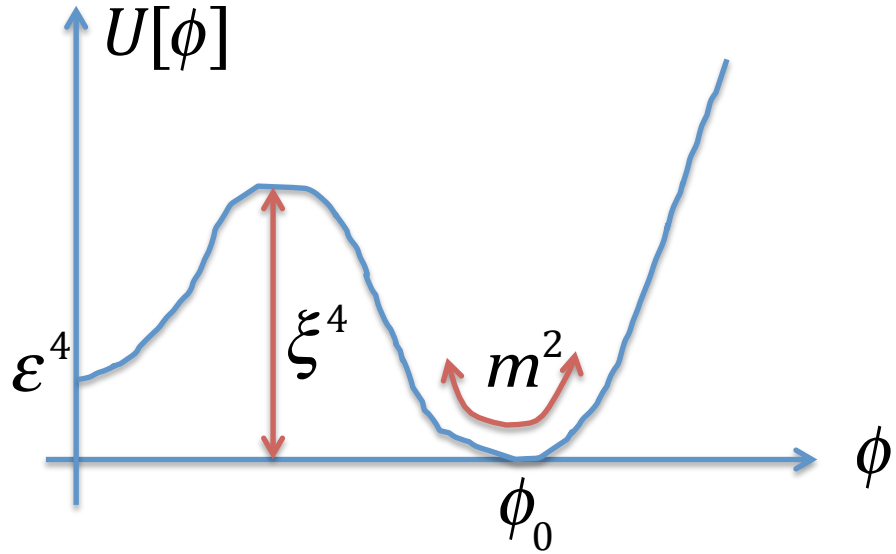
Formation of a false vacuum bubble around a black hole emitting Hawking radiation.

As the Hawking temperature increases to the mass scale m of a Higgs-like scalar field ϕ, its symmetry is restored and a false vacuum bubble is created around the black hole and supported by outward pressure. Moss (1985)



The outer geometry remains the same Schwarzschild space even after the phase transition, thanks to the Birkhoff's theorem.





Particles coupled to ϕ are massless inside the bubble and reflected by the wall unless they have a large enough energy.

With appropriate choices of couplings we find thermalized radiation inside the bubble with temperature T substantially smaller than the Hawking temperature, and the radius of the bubble is determined by thermodynamic consideration.

Moss (1985)

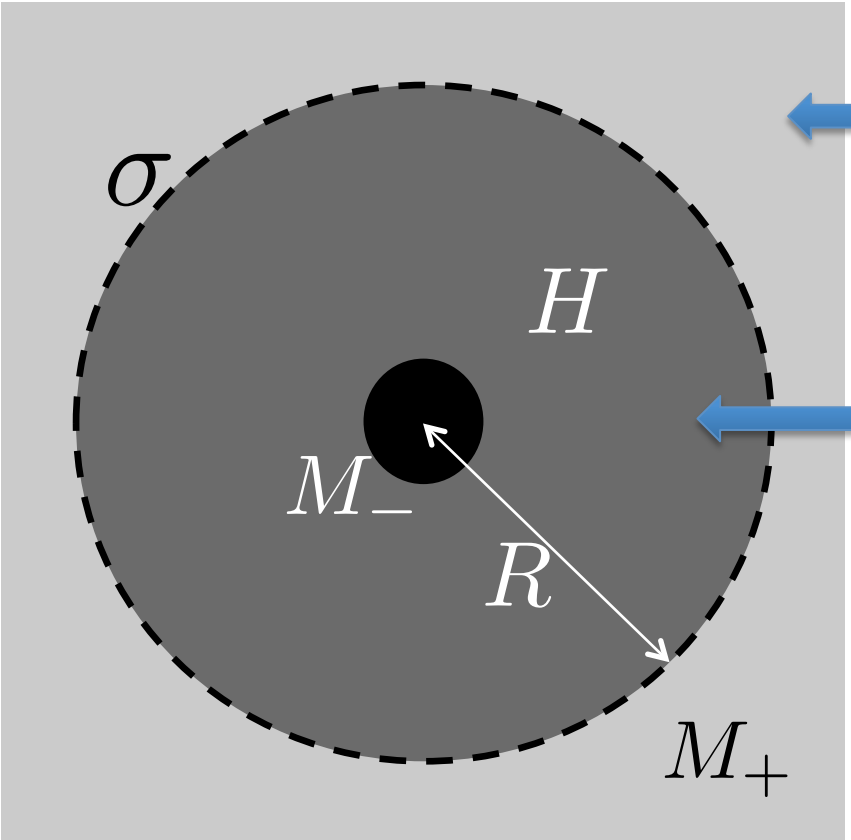
$$r_w = \sqrt{\frac{3q}{2}}\frac{m}{\varepsilon^2}, \quad T = 6\sqrt{2}\frac{\varepsilon^2}{m}$$

q : coupling parameter O(1)

$\varepsilon^4 \gg T^4$ is possible.

Geometries $$ds^2 = -f_\pm(r)dt^2 + \frac{dr^2}{f_\pm(r)} + r^2 d\Omega^2$$

3-sphere



Outer Schwarzschild

$$f_+ = 1 - \frac{2GM_+}{r}$$

Inner Schwarzschild de Sitter

$$f_- = 1 - \frac{2GM_-}{r} - H^2 r^2$$

Wall trajectory

$$(t_\pm(\tau), r_\pm(\tau), \theta, \phi)$$

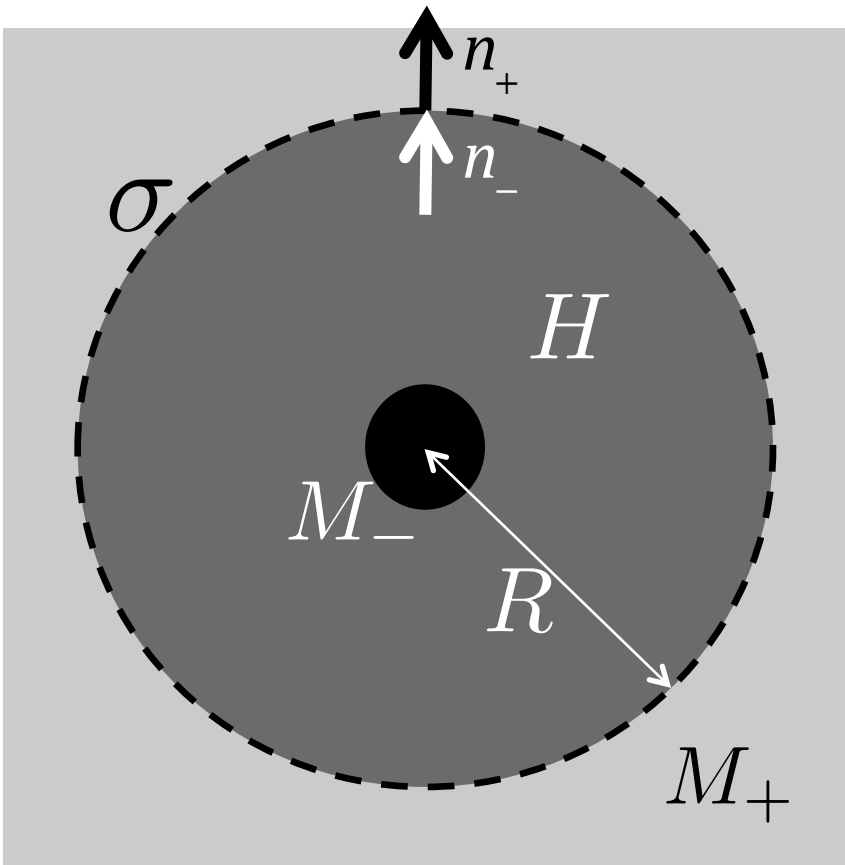
τ : proper time of the wall

$$r_+ = r_- \equiv R \qquad d\tau^2 = f_\pm(R)dt^2 - \frac{dR^2}{f_\pm(R)} - R^2 d\Omega^2 \qquad f_\pm(R)\dot{t}_\pm^2(\tau) - \frac{\dot{R}_\pm^2(\tau)}{f_\pm(R)} = 1$$

We analyze the dynamics of this thin wall shell using Israel's formalism and consider its quantum tunneling to form a wormhole configuration.

Israel's junction condition

normal vector $n_\pm \propto d(r - R(\tau)) \propto (-\dot{R}, \dot{t}_\pm, 0, 0)$ $\qquad \dot{t} = \frac{\partial t}{\partial \tau}$



Extrinsic curvature

$$K_{\pm\theta\theta} = r f_\pm(r) n_r \equiv r\beta_\pm$$

$$\beta_\pm = f_\pm(R) n_{r\pm} = f_\pm(R)\dot{t}_\pm$$

Junction condition

$$K_{+\theta\theta} - K_{-\theta\theta} = -4\pi G\sigma R$$

$$\beta_- - \beta_+ = 4\pi G\sigma R \equiv \Sigma R$$

$$\beta_- - \beta_+ = 4\pi G\sigma R \equiv \Sigma R$$

square

$$f_\pm(R)\dot{t}_\pm^2(\tau) - \frac{\dot{R}_\pm^2(\tau)}{f_\pm(R)} = 1 \qquad \beta_\pm^2 = f_\pm^2(R)\dot{t}_\pm^2 = f_\pm^2 + \dot{R}^2$$

an Analogue of “energy conservation”

$$M_+ = M_- + \frac{4\pi}{3}R^3\varepsilon^4 + 4\pi\sigma R^2\frac{\beta_+ + \beta_-}{2}$$

Bulk energy of the bubble $\gg$ Surface contribution

thin wall condition

$$M_+ \cong M_- + \frac{4\pi}{3}r_w^3\varepsilon^4$$

from the initial condition of the bubble.

$$\beta_- - \beta_+ = 4\pi G\sigma R \equiv \Sigma R$$

$$\beta_+ = f_+(R)\dot{t}_+ = \pm\sqrt{f_+^2 + \dot{R}^2}$$

$$\beta_- = f_-(R)\dot{t}_- = \pm\sqrt{f_-^2 + \dot{R}^2}$$

square
square

$$\left(\frac{dz}{d\tau'}\right)^2 - \frac{\gamma^2}{1-s}\frac{1}{z} - \left(\frac{1-z^3}{z^2}\right)^2 = -\frac{\gamma^2}{(2GM_+\chi)^{\frac{3}{2}}(1-s)^{\frac{3}{2}}}$$

"potential" V(z) total "energy" E

Thin shell's motion = 1 dimensional system in quantum field theory

$$z \equiv \frac{\chi^{2/3}}{(2GM_+)^{1/3}(1-s)^{1/3}}R \qquad \tau' = \frac{\chi^2}{8\pi G\sigma}\tau \qquad s \equiv \frac{M_-}{M_+}$$

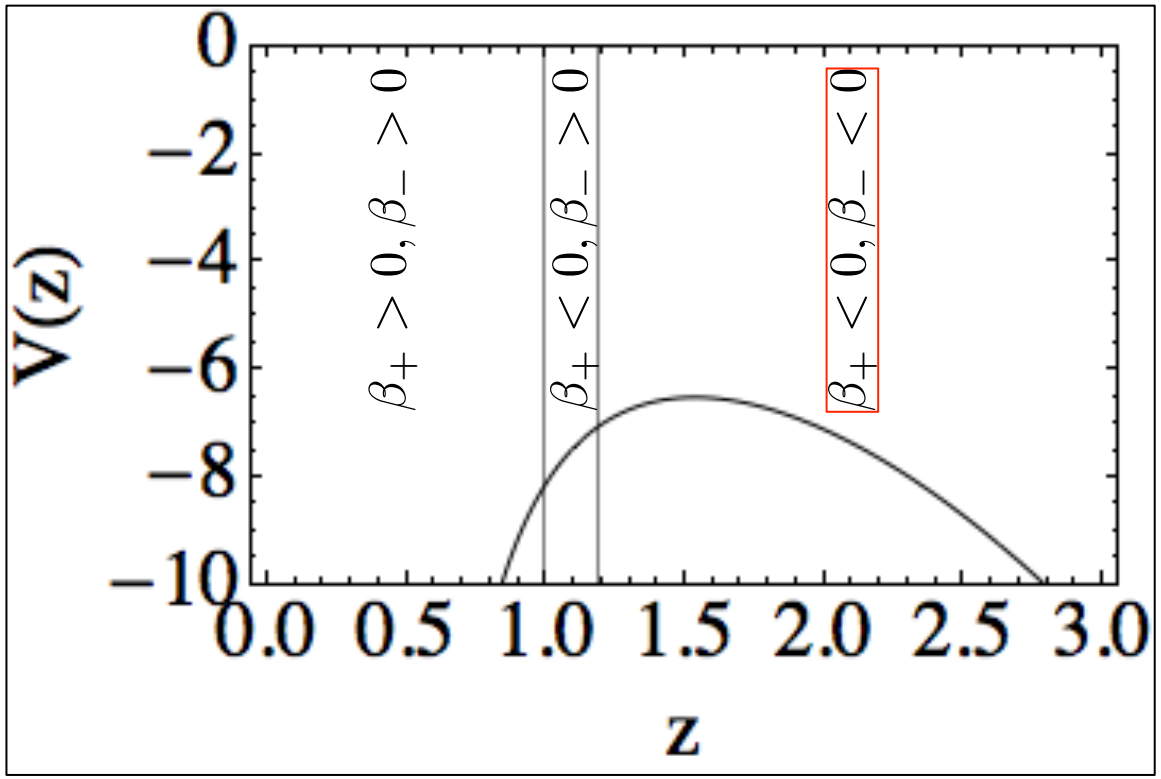
$$\chi \equiv (H^2 + \Sigma^2)^{\frac{1}{2}} \approx \frac{\varepsilon^2}{M_{Pl}}$$

$$\gamma \equiv \frac{2\Sigma}{\chi} \approx \frac{8\pi G\sigma}{H} \approx \frac{\xi^4}{m\varepsilon^2 M_{Pl}} \sim \frac{\varepsilon}{M_{Pl}} \ll 1$$

From $\beta_- - \beta_+ = 4\pi G\sigma R \equiv \Sigma R$ we also find

$$\beta_\pm(r) = \frac{f_-(r) - f_+(r) \mp 16\pi^2\sigma^2 r^2}{8\pi\sigma r}$$

The sign (and value) of $\beta_\pm$ is determined solely by r or z.



β_+ changes its sign at

$$z = 1$$

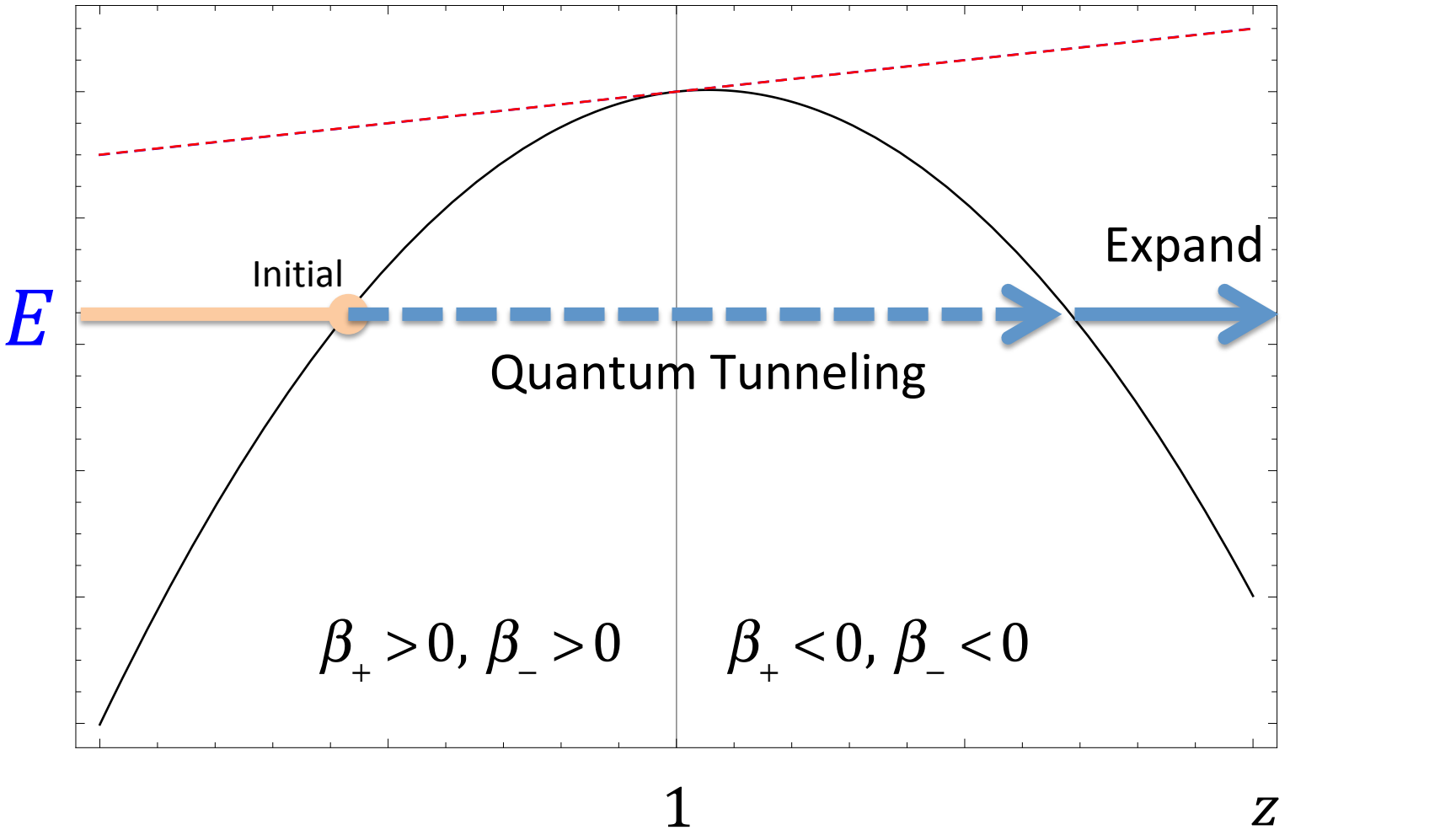
β_- changes its sign at

$$z = (1 - \gamma^2/2)^{-1/3} \equiv z_c$$

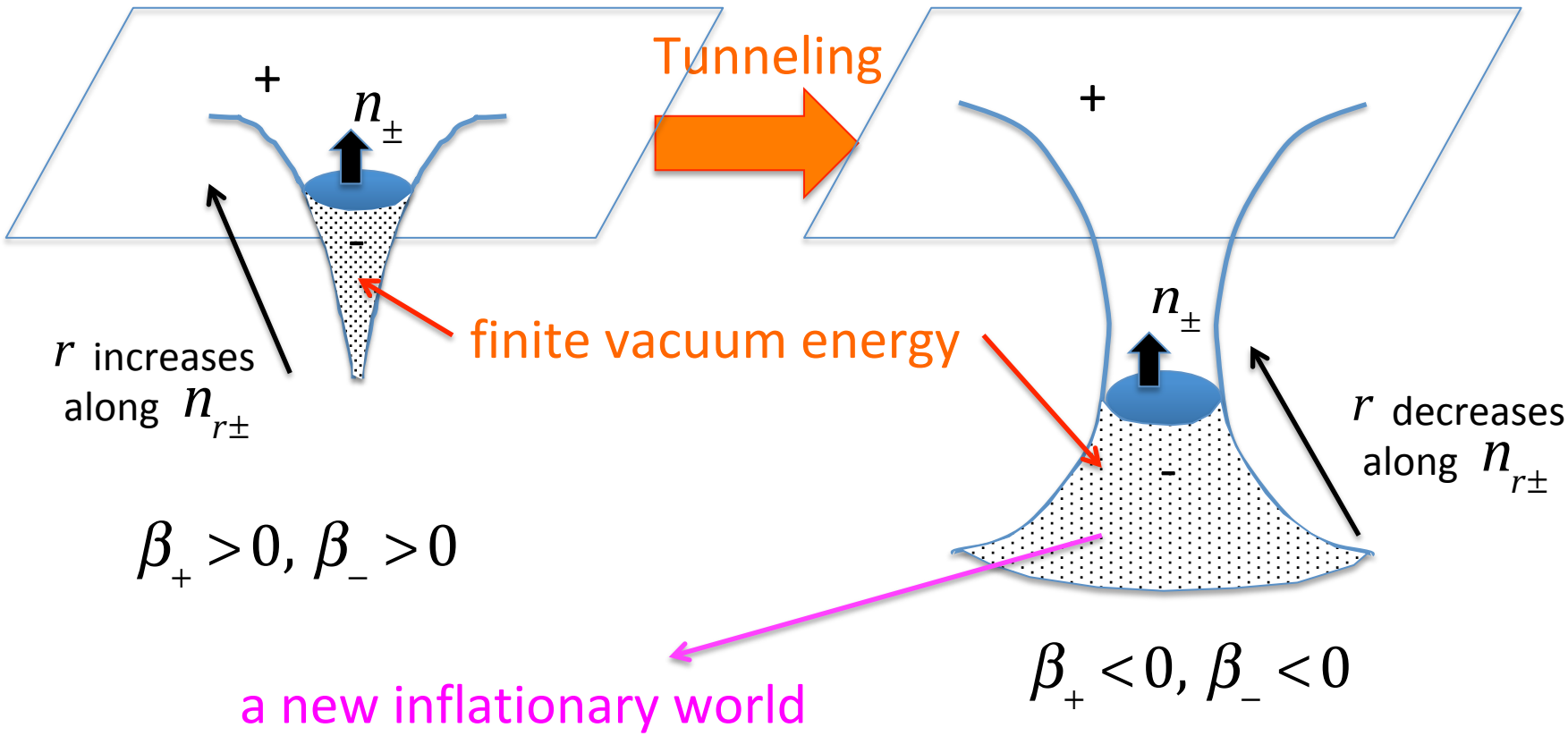
Since $\gamma^2 \ll 1$, both changes sign at $z = 1$ within the thickness of the wall.

$\gamma = 1,\ s = 0.9$ for illustrative purpose

$$V[z] \quad \left(\frac{dz}{d\tau'}\right)^2 - \frac{\gamma^2}{1-s}\frac{1}{z} - \left(\frac{1-z^3}{z^2}\right)^2 = -\frac{\gamma^2}{(2GM_+\chi)^{\frac{3}{2}}(1-s)^{\frac{3}{2}}} \equiv E$$



$$\beta_{\pm} = f_{\pm}(R) n_{r\pm} = f_{\pm}(R)\dot{t}_{\pm}$$ positive to negative



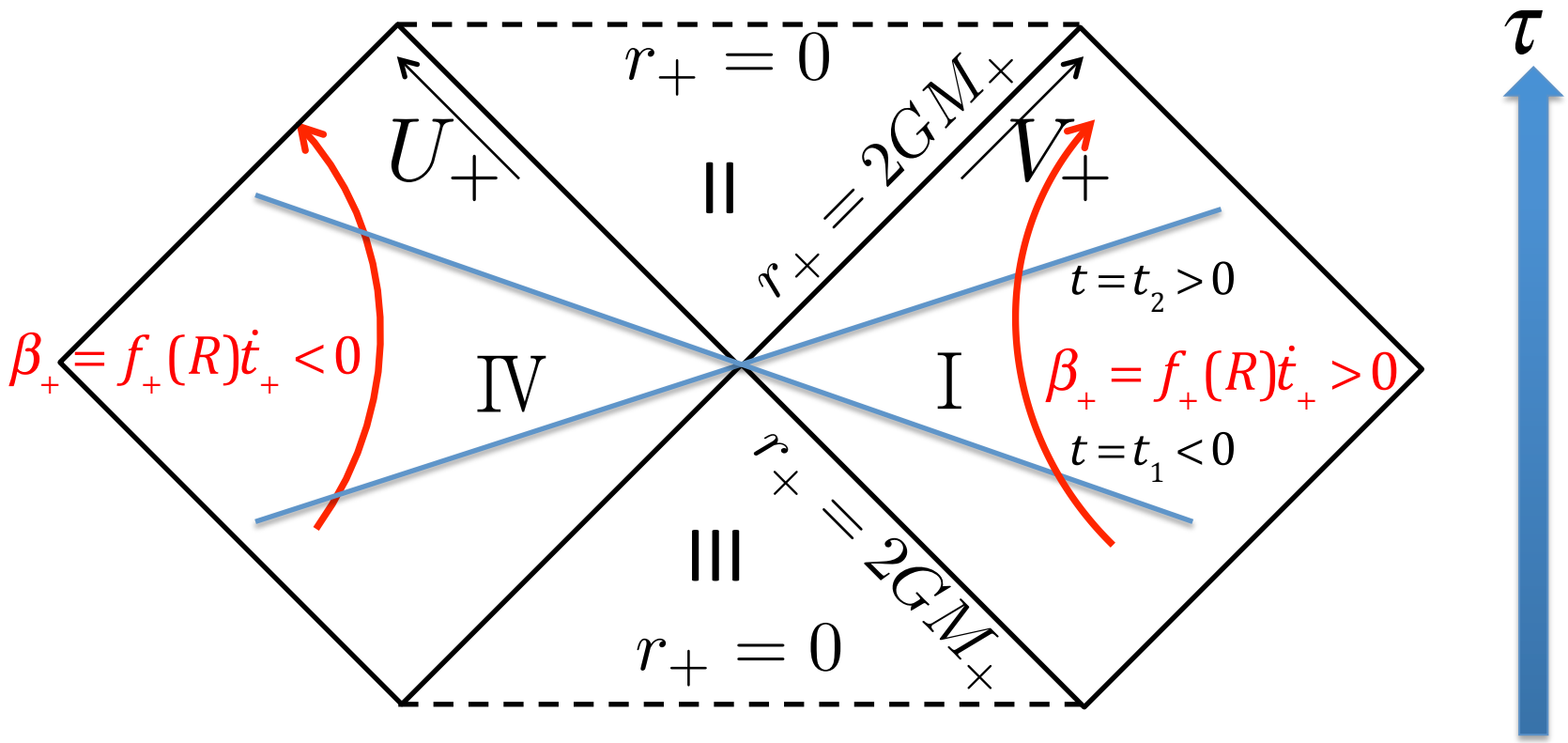
$$\beta_{+} > 0,\ \beta_{-} > 0$$

$$\beta_{+} < 0,\ \beta_{-} < 0$$

A wormhole with a new de Sitter-like domain is created

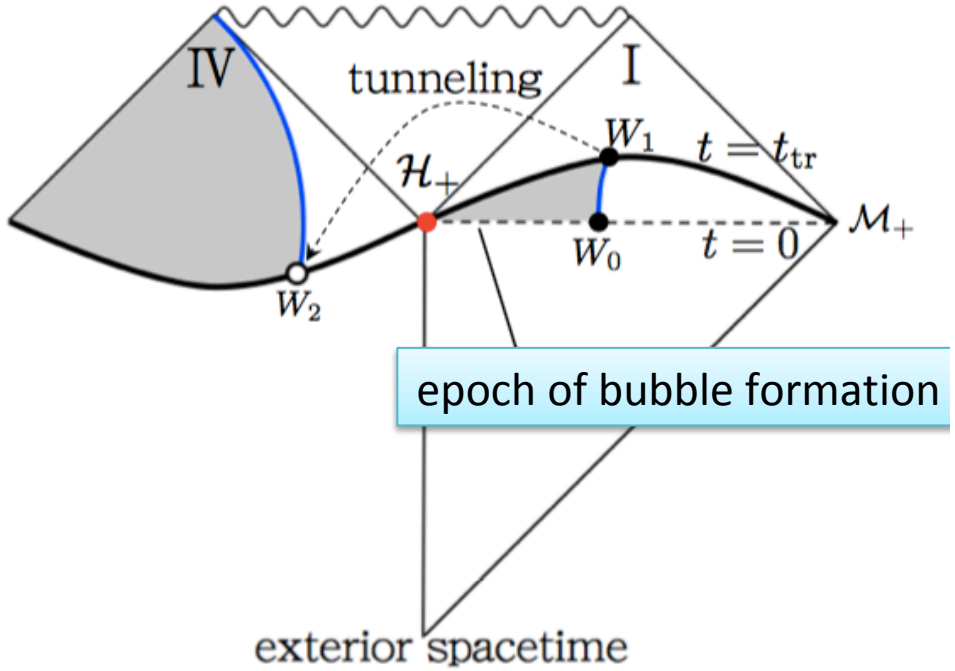
Wormhole formation by cosmological phase transition has also been discussed by Sato, Sasaki, Kodama, Maeda (1981), Blau, Guendelman, Guth (1987), Berezin, Kuzmin, Tkachev (1987), Ansoldi, Guendelman (2008), Ansodi, Tanaka (2015), Chen, Hu, Yeom (2016) etc.

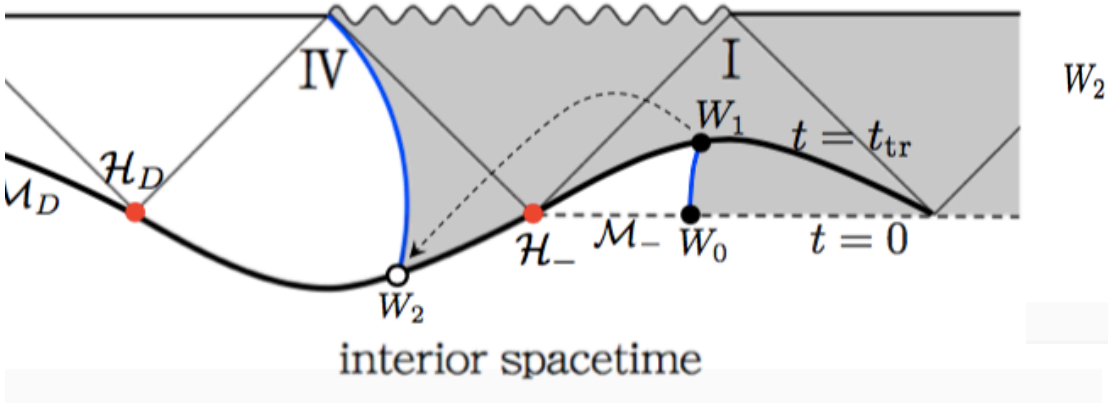
Penrose diagram of the Schwarzschild spacetime

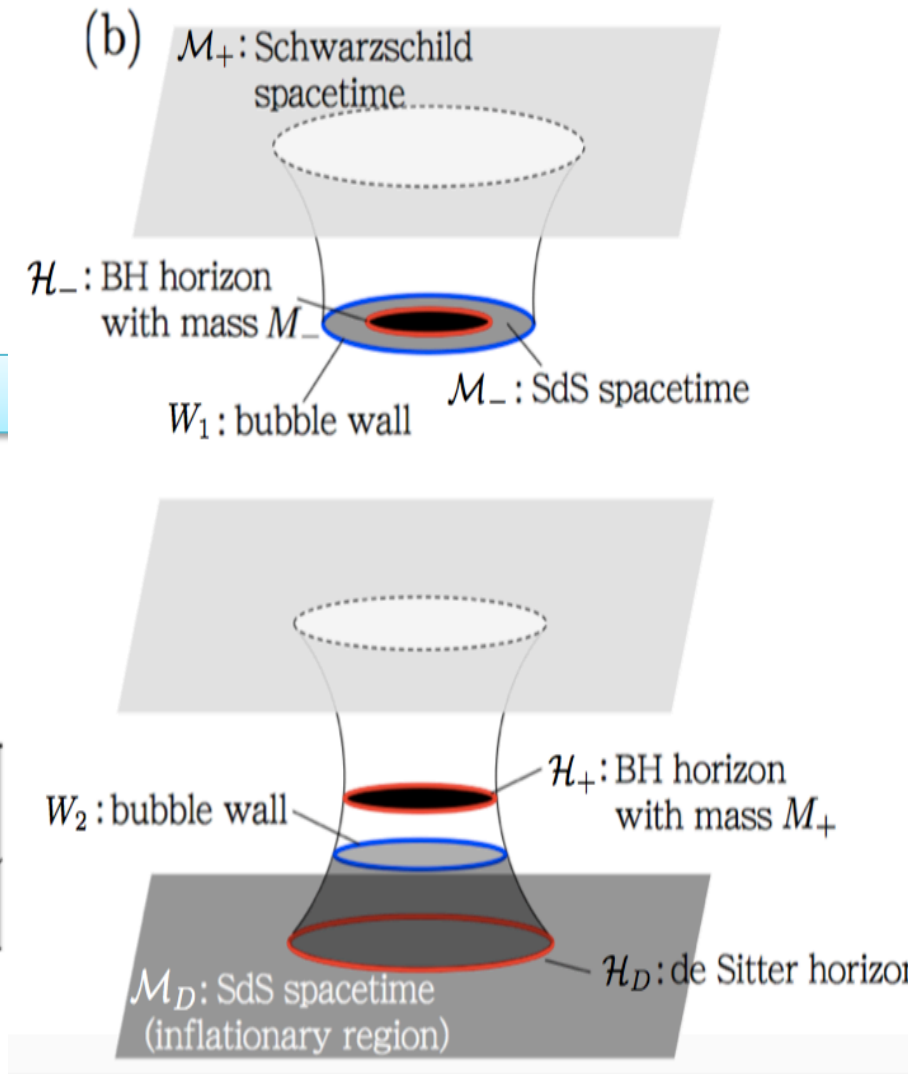


I Our World outside the BH horizon
II Inside the Black Hole
III Inside the White Hole
IV Another World causally disconnected from ours

Our Black Hole ➡ Wormhole causal structure







Calculate the Euclidean action following Gregory et al (2014).

$$I = I_{\mathcal{B}} + I_{-} + I_{+} + I_{\mathcal{W}},$$

$$I_{\mathcal{W}} = -\int_{\mathcal{W}} \mathcal{L}_m(g,\phi) = \int_{\mathcal{W}} \sigma$$ Wall

$$I_{\pm} = -\frac{1}{16\pi G}\int_{\mathcal{M}_{\pm}} \mathcal{R} - \int_{\mathcal{M}_{\pm}} \mathcal{L}_m(g,\phi) + \frac{1}{8\pi G}\int_{\partial\mathcal{M}_{\pm}} K.$$ Bulk

$$I_{\mathcal{B}} = -\frac{1}{16\pi G}\int_{\mathcal{B}} \mathcal{R} + \frac{1}{8\pi G}\int_{\partial\mathcal{B}} K$$ Conical deficits (giving rise to a contribution proportional to the horizon area)

Using the static metric

$$B = -\frac{A_{deSitter}}{4G} - \frac{A_{+BHhorizon}}{4G} + \int d\tau_E \left[(2R - 6GM_{+})\dot{t}_{E+} - (2R - 6GM_{-})\dot{t}_{E-} \right] + \frac{A_{-BHhorizon}}{4G}$$

$$\Gamma \propto e^{-B}\,?$$

B as a whole takes a negative value!!

$$B = \int d\tau_E \left[(2R - 6GM_+)\dot{t}_{E+} - (2R - 6GM_-)\dot{t}_{E-} \right] - \frac{A_{deSitter}}{4G} - \frac{A_{+BHhorizon}}{4G} + \frac{A_{-BHhorizon}}{4G}$$

$$\equiv B_{tunnel} - \Delta S \qquad \boxed{\Delta S = \frac{A_{final}}{4G} - \frac{A_{initial}}{4G}}$$

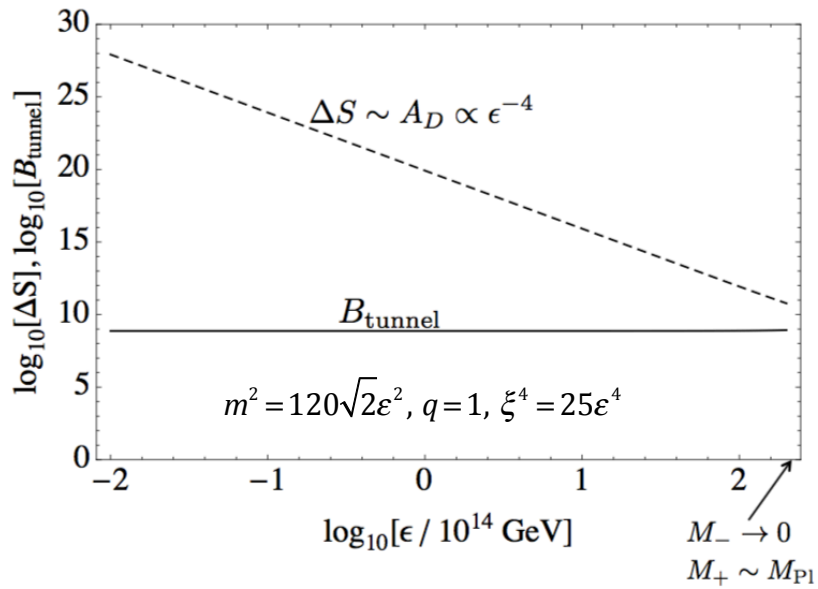
$$\Gamma \propto e^{-B} = e^{-B_{tunnel} + \Delta S} \qquad e^{\Delta S} = \frac{W_{final}}{W_{initial}}$$ # of states

may be interpreted as a transition rate from one microscopic state with a statistical weight $1/W_{initial}$ to a final state with W_{final} macroscopic degrees of freedom.

Mathur (2010)

The transition rate from one microscopic state to another microscopic final state is given by $\Gamma_{micro} \propto e^{-B_{tunnel}} \ll 1$

so semiclassical calculation would be OK.



Conclusion

Our Universe may have been created from a black hole of previous universe !?