Properties of a variational equation of state in core-collapse supernovae

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Outline

1: Introduction
2: Supernova EOS with realistic nuclear forces
3: Application to astrophysical objects
4: Summary
1. Introduction

Equation of state (EOS) for infinite nuclear matter plays important roles for astrophysical studies.

Neutron Stars: Stiffness (EOS at 0 MeV) ⇔ Self-gravity

Mass-radius relation of cold neutron stars

Phase diagram of cold nuclear matter
Nuclear EOS and Core-Collapse Supernovae

Nuclear EOS at finite temperature is one of the crucial ingredients for the numerical simulations of Core-Collapse Supernovae.

Scenario of the Core-Collapse Supernovae (SNe)

- **Fe core**
  - **Collapse**
    - \( \rho_c \sim 10^{-4} \rho_0 \)
    - \( T_c \sim 1 \) MeV
    - \( Z/A \sim 0.46 \)
  - **\( v \)-trapping**
    - \( \rho_c \sim 10^{-2} \rho_0 \)
    - \( T_c \sim 2 \) MeV

- **e-capture**

- **1000 km**
- **Core Bounce**
  - \( \rho_c \sim \rho_0 \)
  - \( T_c \sim 10 \) MeV
- **Explosion**
  - \( \rho_c \sim 3\rho_0 \)
  - \( Z/A < 0.1 \)

- **Neutron star**

**K. Sumiyoshi @ NUFRA 2011**
Nuclear EOS and Core-Collapse Supernovae

*Nuclear EOS at finite temperature* is one of the crucial ingredients for the numerical simulations of *Core-Collapse Supernovae*.

- The stiffness of high-density nuclear matter
- Species of nuclides in hot matter

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### Scenario of the Core-Collapse Supernovae (SNe)

*Massive star*  
Fe core  
**Collapse**  
\[ \rho_c \sim 10^{-4} \rho_0 \]  
**\( \nu \)-trapping**  
\[ \rho_c \sim 10^{-2} \rho_0 \]

- e-capture

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**Core Bounce**  
\[ \rho_c \sim \rho_0 \]  
\[ T_c \sim 10 \text{ MeV} \]

- Shockwave

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**Explosion**  
\[ \rho_c \sim 3\rho_0 \]  
\[ Z/A < 0.1 \]

- Neutron star

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K. Sumiyoshi @ NUFRA 2011
Nuclear EOS for supernova simulations

- SN-EOS should provide thermodynamic quantities in the wide ranges.
  
  - Temperature $T$ : $0 \leq T \leq 30$ MeV
  - Density $\rho$ : $10^{5.1} \leq \rho_B \leq 10^{15.0}\text{g/cm}^3$
  - Proton fraction $Y_p$ : $0 \leq Y_p \leq 0.50$

- SN matter contains uniform and non-uniform phases.

Phase diagram of nuclear matter [based on HT et al., NPA 961 (2017) 78]
Current status of SN-EOS

<table>
<thead>
<tr>
<th>Model</th>
<th>Nuclear Interaction</th>
<th>Degrees of Freedom</th>
<th>$M_{\text{max}}$ (M$_{\odot}$)</th>
<th>$R_{1.4M_{\odot}}$ (km)</th>
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<td>H&amp;W</td>
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<td>FYSS</td>
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<td>HS(TM1)</td>
<td>TM1*</td>
<td>$n, p, d, t, h, \alpha, {A_i, Z_i}$</td>
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<td>$n, p, d, t, h, \alpha, {A_i, Z_i}$</td>
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<td>y Shen et al. (2011a)</td>
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(M. Oertel et al., Rev. Mod. Phys. 89 (2017) 015007)

There is no SN-EOSs based on the microscopic many-body theory.

We aim to construct a new SN-EOS with the variational method starting from bare nuclear forces.
Our procedure to construct a new EOS table

1: Cluster variational method with AV18 + UIX potentials

2: Thomas-Fermi calculation for non-uniform matter
2. Supernova EOS with realistic nuclear forces

Nuclear Hamiltonian

\[ H = -\sum_{i=1}^{N} \frac{\hbar^2}{2m} \nabla^2 + \sum_{i<j} V_{ij} + \sum_{i<j<k} V_{ijk} \]

AV18 two-body nuclear potential

UIX three-body nuclear potential

Jastrow wave function

\[ \Psi = \text{Sym} \left[ \prod_{i<j} f_{ij} \right] \Phi_F \]

\( f_{ij} \): Correlation function

\( \Phi_F \): Fermi-gas wave function

- The expectation value of the Hamiltonian is calculated in \textit{the two-body cluster approximation}.

- The prescription by Schmidt and Pandharipande is employed to obtain the free energy \textit{at finite temperature}.

Nuclear EOS for uniform matter

Our EOS: HT and M. Takano, NPA 902 (2013) 53
FHNC: A. Mukherjee, PRC 79(2009) 045811

<table>
<thead>
<tr>
<th>$n_0$ [fm$^{-3}$]</th>
<th>$E_0$ [MeV]</th>
<th>$K$ [MeV]</th>
<th>$E_{\text{sym}}$ [MeV]</th>
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<tr>
<td>0.16</td>
<td>-16.1</td>
<td>245</td>
<td>30.0</td>
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</table>
Nuclear EOS for non-uniform matter

We use the Thomas-Fermi method by Shen et al.

(PTP 100 (1998) 1013, APJS 197(2011) 20)

Free energy of a Wigner-Seitz cell

\[ F = \int dr f(n_p(r), n_n(r), n_\alpha(r)) + F_0 \int dr |\nabla (n_p(r) + n_n(r))|^2 \]

\[ + \frac{e^2}{2} \int dr \int dr' \frac{[n_p(r) + 2n_\alpha(r) - n_e][n_p(r') + 2n_\alpha(r') - n_e]}{|r - r'|} + c_{bcc} \frac{(Ze)^2}{a} \]

Free energy density of uniform matter: \( f = f_N + f_\alpha \)

Particle number density distributions

Protons and neutrons (\( i = p, n \))

\[ n_i(r) = \begin{cases} (n_i^{in} - n_i^{out})[1 - (r/R_i)^3] + n_i^{out} & (0 \leq r \leq R_i) \\ n_i^{out} & (R_i \leq r \leq R_{cell}) \end{cases} \]

Alpha-particles

\[ n_\alpha(r) = \begin{cases} -n_\alpha^{out}[1 - (r/R_p)^3] + n_\alpha^{out} & (0 \leq r \leq R_p) \\ n_\alpha^{out} & (R_p \leq r \leq R_{cell}) \end{cases} \]
Phase Diagram of Nuclear Matter

$T = 10$ MeV

$T = 5$ MeV

$T = 1$ MeV

$T = 0$ MeV

HT et al., NPA 961 (2017) 78
EOS for Hot Nuclear Matter

Particle fraction

Free energy

Entropy

$Y_p = 0$
$Y_p = 0.1$
$Y_p = 0.2$
$Y_p = 0.3$
$Y_p = 0.5$

$T = 10$ MeV

$T = 5$ MeV

$T = 1$ MeV

$X_p$
$X_n$
$X_A$
$X_{\alpha}$

HT et al., NPA 961 (2017) 78
Equation of state for nuclear matter with the variational method

Equation of state (EOS) based on the variational many-body theory with realistic nuclear forces is provided. For uniform matter, the EOS is constructed with the cluster variational method starting from the Argonne v18 two-body nuclear potential and the Urbana IX three-body nuclear potential. Non-uniform nuclear matter is treated in the Thomas-Fermi approximation. Alpha particle mixing is also taken into account. See Togashi et al., Nucl. Phys. A 961 (2017) 78 for details. This EOS table is open for general use in any studies for nuclear physics and astrophysics, provided that our paper is referred to in your publication.

User’s Guide (read me first)

guide.pdf

EOS tables
eoszip

Contact

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  2-1 Hirosawa, Wako, Saitama 351-0198, Japan

(HT et al., NPA961 (2017) 78)

Table A.1: Ranges of temperature $T$, proton fraction $Y_p$, and baryon mass density $\rho_B$ in the table of the variational EOS. At the top of the last column, ”+1” represents the case at $T = 0$ MeV.

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Minimum</th>
<th>Maximum</th>
<th>Mesh</th>
<th>Number</th>
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</thead>
<tbody>
<tr>
<td>$\log_{10}(T)$ [MeV]</td>
<td>-1.00</td>
<td>2.60</td>
<td>0.04</td>
<td>91 + 1</td>
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<tr>
<td>$Y_p$</td>
<td>0</td>
<td>0.65</td>
<td>0.01</td>
<td>66</td>
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<tr>
<td>$\log_{10}(\rho_B)$ [g/cm$^3$]</td>
<td>5.1</td>
<td>16.0</td>
<td>0.10</td>
<td>110</td>
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</table>
3. Application to astrophysical objects

Mass-Radius relation of neutron stars

J0348+0432: Science 340 (2013) 1233232

Application to Core-Collapse Supernovae

1D neutrino-radiation hydrodynamics simulations


Radial trajectories of mass elements
The numbers (1)–(5) :the times when the central density reaches $10^{10}$, $10^{11}$, $10^{12}$, $10^{13}$, $10^{14}$ g/cm$^3$
Heavy Nuclei in Supernova Matter

The density derivative coefficient of the symmetry energy $L$

(Our EOS: $L = 35$ MeV  Shen EOS: $L = 111$ MeV)

Energies of uniform nuclear matter at 0MeV
Comparison of Results (Postbounce Phase)

$4.7 \times 10^{14} \text{ g/cm}^3$ (Variational)

$3.6 \times 10^{14} \text{ g/cm}^3$ (Shen EOS)
Stiffness of the nuclear EOS

Variational EOS is softer than the Shen EOS.

The density profiles at $t_{pb} = 0$ ms

The adiabatic indices at $t_{pb} = 0$ ms

Adiabatic index: $\Gamma = \left. \frac{d \log P}{d \log \rho} \right|_S$

Variational EOS is softer than the Shen EOS.

→ Our EOS is advantageous for supernova explosion!?
Summary

Nuclear EOS for supernova simulations is constructed with realistic nuclear forces (AV18 + UIX).

Uniform nuclear matter: Cluster variational method
Non-uniform nuclear matter: Thomas-Fermi approximation

Our SN-EOS is available at
http://www.np.phys.waseda.ac.jp/EOS/

- Neutron star structure is consistent with observational data.
- Our EOS is softer than Shen EOS in 1D supernova simulation.

Future Plans

- Multi-dimensional supernova simulations
- Application to other astrophysical simulations
- Hyperon mixing in high-density matter