Properties of a variational equation of state in core-collapse supernovae

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Outline

1 : Introduction

2 : Supernova EOS with realistic nuclear forces

3 : Application to astrophysical objects

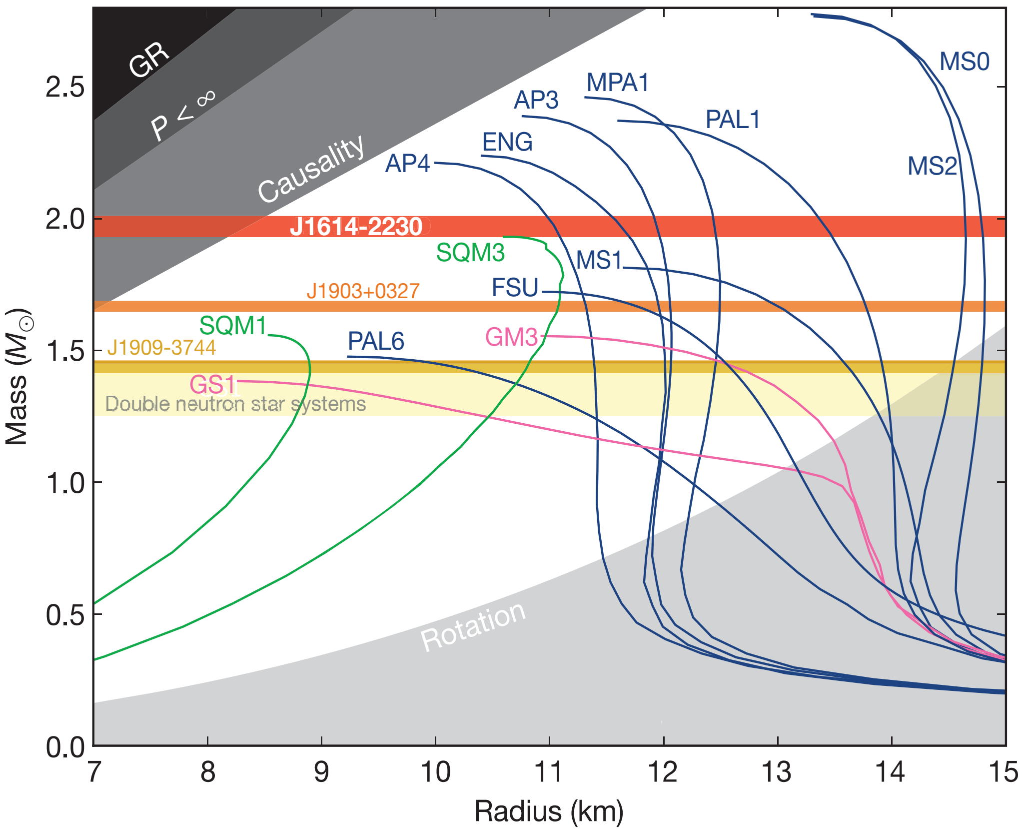
4 : Summary

PACIFIC2018 @ Kiroro, Hokkaido, February 14, 2017

1. Introduction

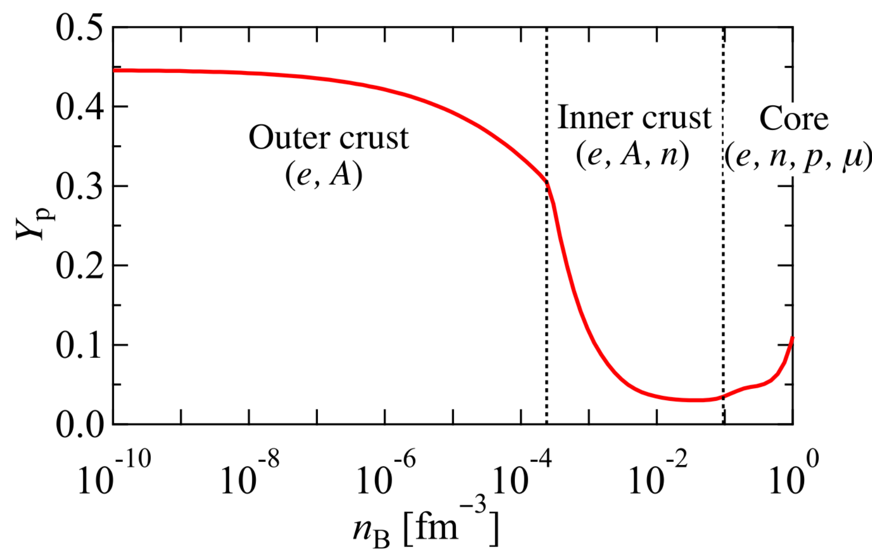
Equation of state (EOS) for infinite nuclear matter
plays important roles for astrophysical studies.

Neutron Stars: Stiffness (EOS at 0 MeV) ⇔ Self-gravity



Mass-radius relation of cold neutron stars

P. B. Demorest et al., NATURE 467 (2010)

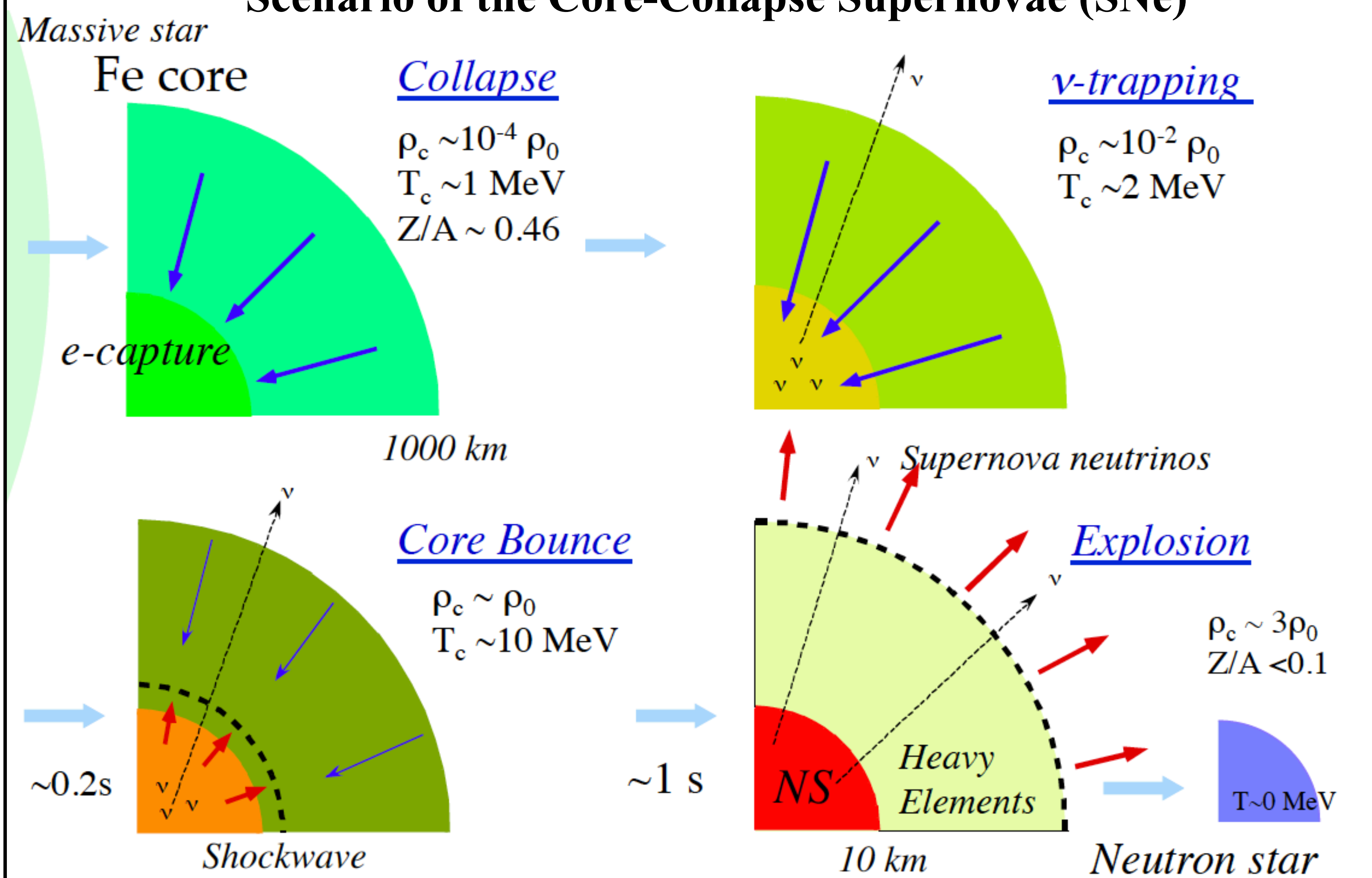


Phase diagram of cold nuclear matter

Nuclear EOS and Core-Collapse Supernovae

Nuclear EOS at finite temperature *is one of the crucial ingredients for the numerical simulations of* ***Core-Collapse Supernovae****.*

Scenario of the Core-Collapse Supernovae (SNe)



K. Sumiyoshi @ NUFRA 2011

Nuclear EOS and Core-Collapse Supernovae

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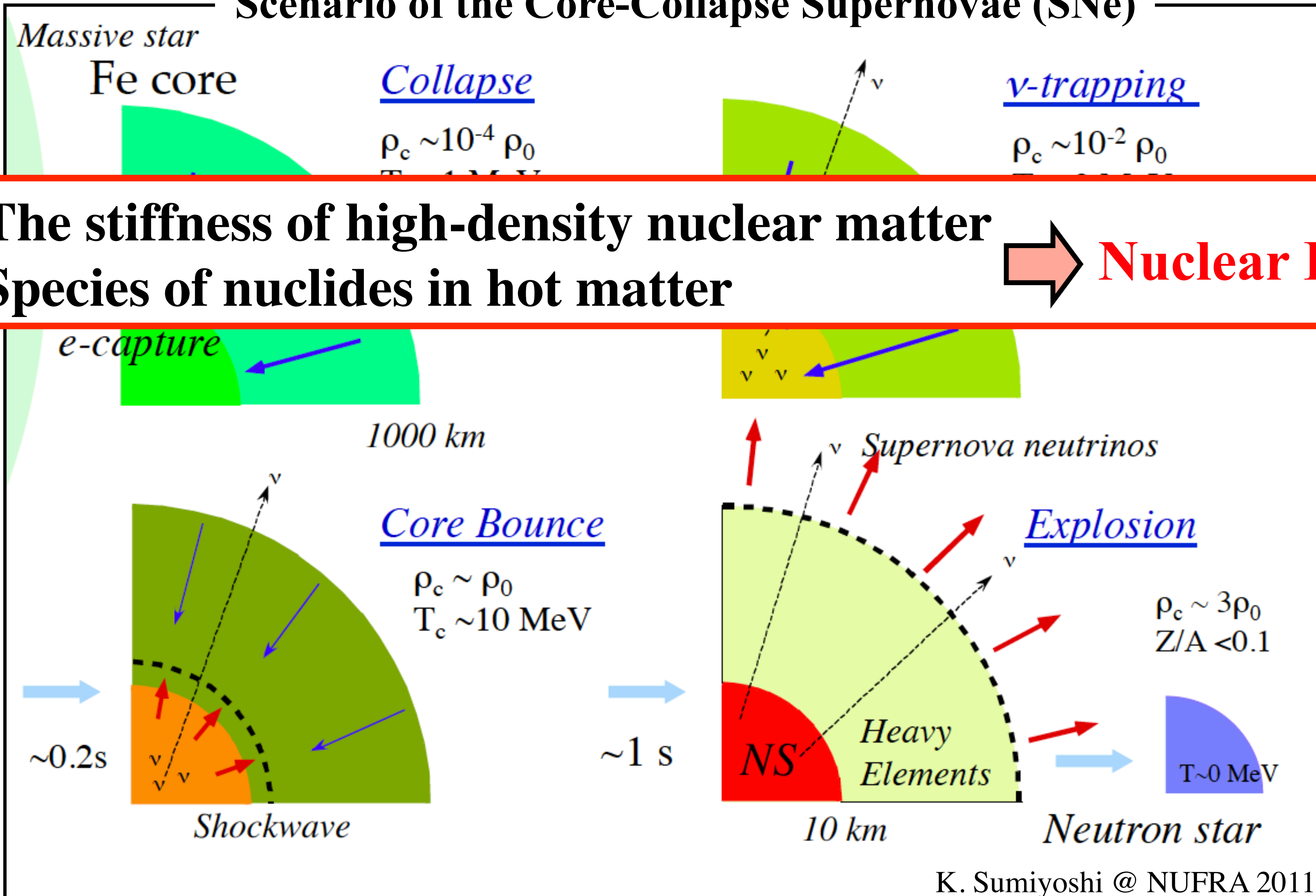


- **The stiffness of high-density nuclear matter**
- **Species of nuclides in hot matter**

⇒ **Nuclear EOS**

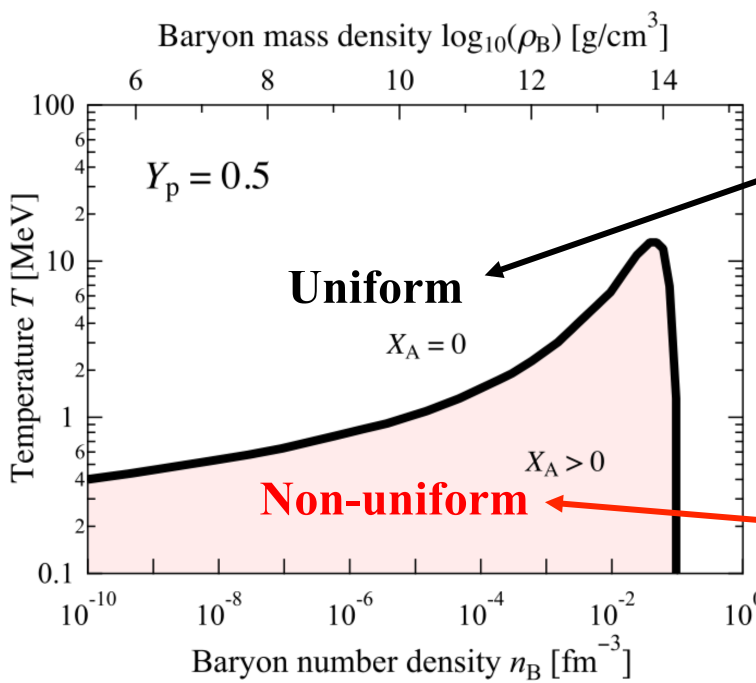
K. Sumiyoshi @ NUFRA 2011

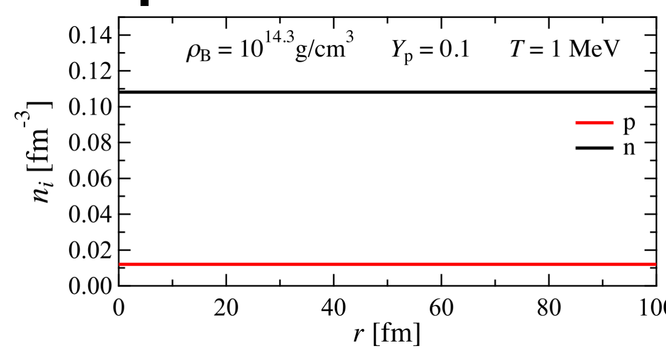
Nuclear EOS for supernova simulations

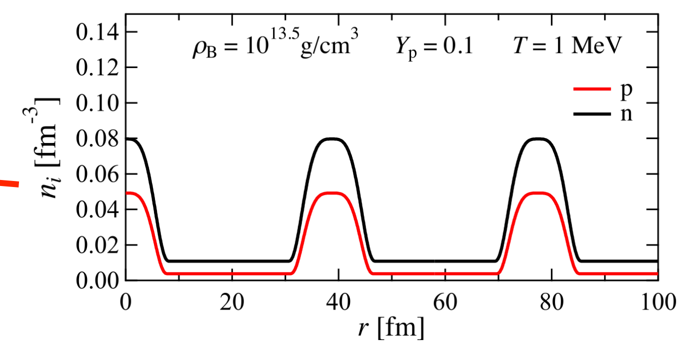
- SN-EOS should provide thermodynamic quantities in the wide ranges.

- Temperature T : $0 \le T \le 30$ MeV
- Density ρ : $10^{5.1} \le \rho_B \le 10^{15.0}$g/cm^3
- Proton fraction Y_p : $0 \le Y_p \le 0.50$

- SN matter contains uniform and non-uniform phases.







Phase diagram of nuclear matter [based on HT *et al.*, NPA 961 (2017) 78]

Current status of SN-EOS

Model	Nuclear Interaction	Degrees of Freedom	$M_{\max}$ ($M_\odot$)	$R_{1.4M_\odot}$ (km)	Ξ	publ. avail.	References
H&W	SKa	$n, p, \alpha, \{(A_i, Z_i)\}$	2.21[a]	13.9 [a]		n	El Eid and Hillebrandt (1980); Hillebrandt *et al.* (1984)
LS180	LS180					y	Lattimer and Swesty (1991)
LS220	LS220					y	Lattimer and Swesty (1991)
LS375	LS375	$n, p, \alpha, (A, Z)$	2.72	14.5	0.32	y	Lattimer and Swesty (1991)
STOS	TM1	$n, p, \alpha, (A, Z)$	2.23	14.5	0.26	y	Shen *et al.* (1998); Shen *et al.* (1998, 2011)
FYSS	TM1	$n, p, d, t, h, \alpha, \{(A_i, Z_i)\}$	2.22	14.4	0.26	n	Furusawa *et al.* (2013b)
HS(TM1)	TM1*	$n, p, d, t, h, \alpha, \{(A_i, Z_i)\}$	2.21	14.5	0.26	y	Hempel and Schaffner-Bielich (2010); Hempel *et al.* (2012)
HS(TMA)	TMA*	$n, p, d, t, h, \alpha, \{(A_i, Z_i)\}$	2.02	13.9	0.25	y	Hempel and Schaffner-Bielich (2010)
HS(FSU)	FSUgold*	$n, p, d, t, h, \alpha, \{(A_i, Z_i)\}$	1.74	12.6	0.23	y	Hempel and Schaffner-Bielich (2010); Hempel *et al.* (2012)
HS(NL3)	NL3*					y	Hempel and Schaffner-Bielich (2010); Fischer *et al.* (2014a)
HS(DD2)	DD2					y	Hempel and Schaffner-Bielich (2010); Fischer *et al.* (2014a)
HS(IUFSU)	IUFSU*	$n, p, d, t, h, \alpha, \{(A_i, Z_i)\}$	1.95	12.7	0.25	y	Hempel and Schaffner-Bielich (2010); Fischer *et al.* (2014a)
SFHo	SFHo	$n, p, d, t, h, \alpha, \{(A_i, Z_i)\}$	2.06	11.9	0.30	y	Steiner *et al.* (2013a)
SFHx	SFHx	$n, p, d, t, h, \alpha, \{(A_i, Z_i)\}$	2.13	12.0	0.29	y	Steiner *et al.* (2013a)
SHT(NL3)	NL3	$n, p, \alpha, \{(A_i, Z_i)\}$	2.78	14.9	0.31	y	Shen *et al.* (2011b)
SHO(FSU)	FSUgold	$n, p, \alpha, \{(A_i, Z_i)\}$	1.75	12.8	0.23	y	Shen *et al.* (2011a)
SHO(FSU2.1)	FSUgold2.1	$n, p, \alpha, \{(A_i, Z_i)\}$	2.12	13.6	0.26	y	Shen *et al.* (2011a)

Skyrme-type effective interaction

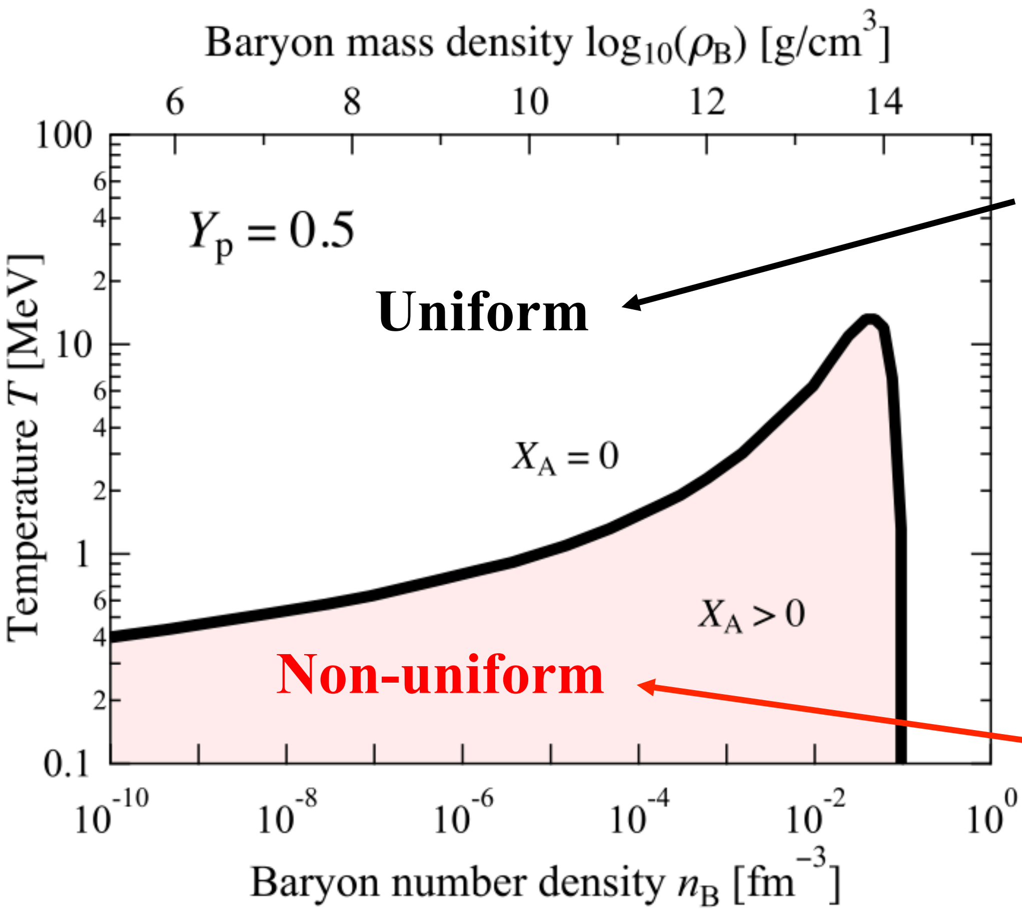
Relativistic Mean Field Theory

(M. Oertel et al., Rev. Mod. Phys. 89 (2017) 015007)

There is no SN-EOSs based on the microscopic many-body theory.

We aim to construct a new SN-EOS with the variational method starting from bare nuclear forces.

Our procedure to construct a new EOS table



1: **Cluster variational method with AV18 + UIX potentials**

2: **Thomas-Fermi calculation for non-uniform matter**

2. Supernova EOS with realistic nuclear forces

Nuclear Hamiltonian

$$H = -\sum_{i=1}^{N} \frac{\hbar^2}{2m}\nabla^2 + \sum_{i<j}^{N} V_{ij} + \sum_{i<j<k}^{N} V_{ijk}$$

AV18 two-body nuclear potential

UIX three-body nuclear potential

Jastrow wave function

$$\Psi = \mathrm{Sym}\left[\prod_{i<j} f_{ij}\right]\Phi_{\mathrm{F}}$$

f_{ij}: Correlation function

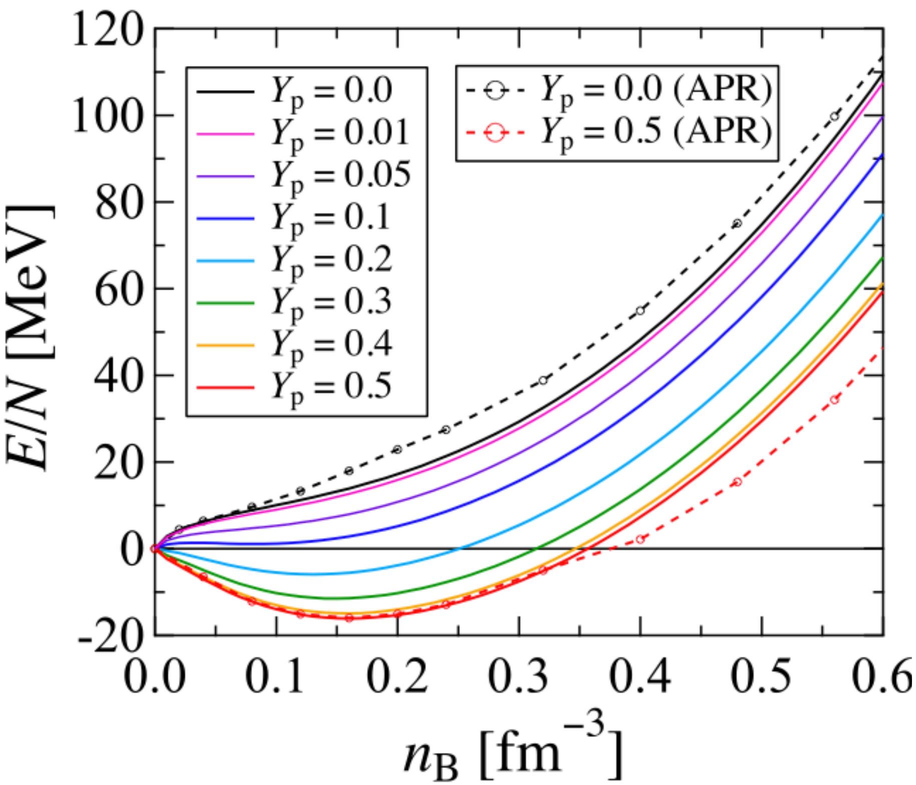
Φ_{F}: Fermi-gas wave function

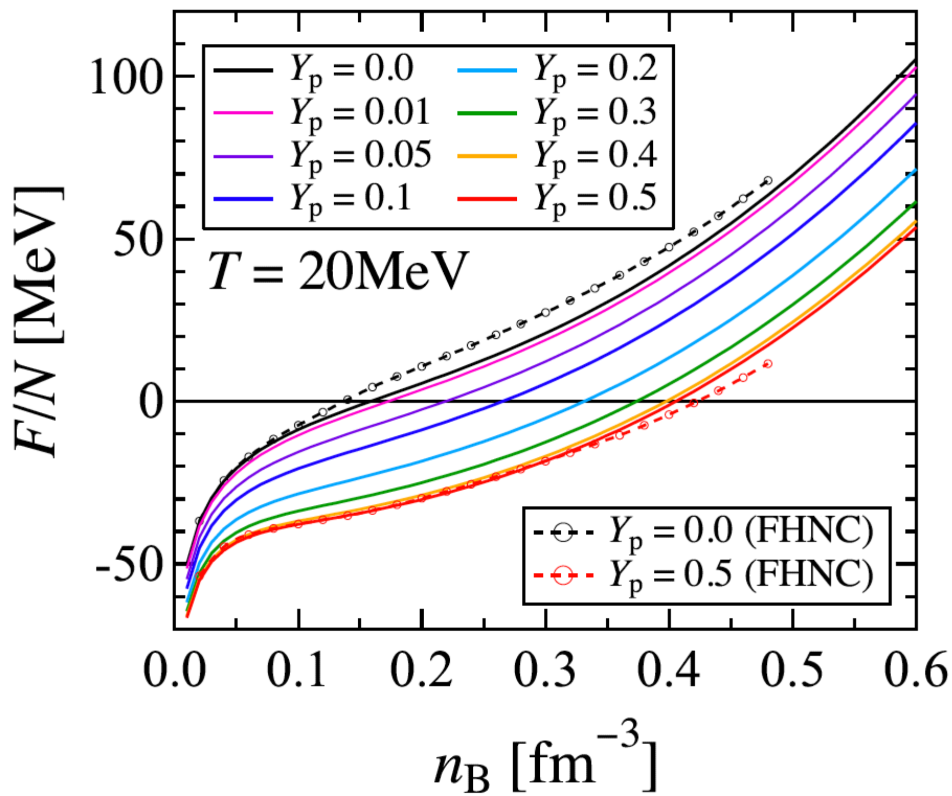
- The expectation value of the Hamiltonian is calculated in ***the two-body cluster approximation***.

- The prescription by Schmidt and Pandharipande is employed to obtain the free energy ***at finite temperature***.

(Phys. Lett. 87B(1979) 11, PRC 75(2007) 035802)

Nuclear EOS for uniform matter





n_0[fm^{-3}]	E_0[MeV]	K [MeV]	E_{sym}[MeV]
0.16	−16.1	245	30.0

Our EOS : HT and M. Takano, NPA 902 (2013) 53

APR : A. **A**kmal, V. R. **P**andharipande, D. G. **R**avenhall, PRC 58 (1998) 1804

FHNC : A. Mukherjee, PRC 79(2009) 045811

Nuclear EOS for non-uniform matter

*We use the **Thomas-Fermi method** by Shen et al.*

(PTP 100 (1998) 1013, APJS 197(2011) 20)

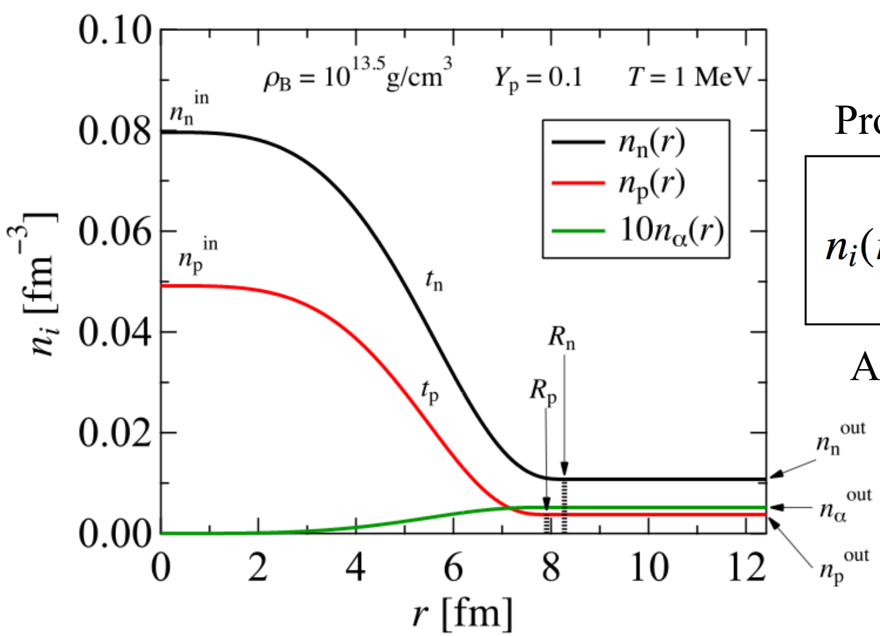
Free energy of a Wigner-Seitz cell

Bulk energy **Gradient energy**

$$F = \int d\boldsymbol{r} f(n_\mathrm{p}(r), n_\mathrm{n}(r), n_\alpha(r)) + F_0 \int d\boldsymbol{r} |\nabla(n_\mathrm{p}(r) + n_\mathrm{n}(r))|^2$$
$$+ \frac{e^2}{2} \int d\boldsymbol{r} \int d\boldsymbol{r}' \frac{[n_\mathrm{p}(r) + 2n_\alpha(r) - n_\mathrm{e}][n_\mathrm{p}(r') + 2n_\alpha(r') - n_\mathrm{e}]}{|\boldsymbol{r} - \boldsymbol{r}'|} + c_\mathrm{bcc} \frac{(Ze)^2}{a}$$

Coulomb energy

Free energy density of uniform matter: $f = f_\mathrm{N} + f_\alpha$



Particle number density distributions

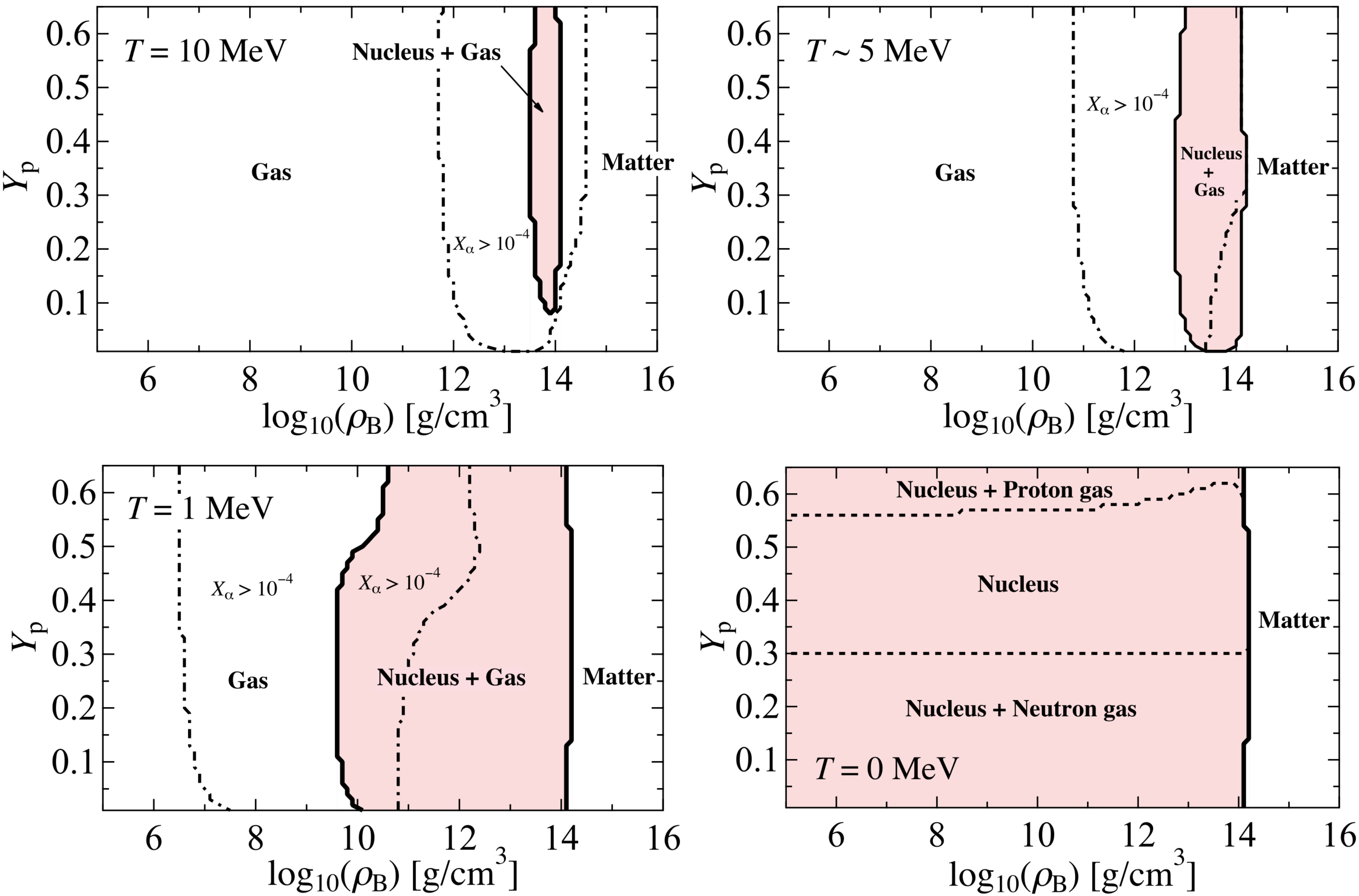
Protons and neutrons (i = p, n)

$$n_i(r) = \begin{cases} (n_i^\mathrm{in} - n_i^\mathrm{out})[1 - (r/R_i)^{t_i}]^3 + n_i^\mathrm{out} & (0 \le r \le R_i) \\ n_i^\mathrm{out} & (R_i \le r \le R_\mathrm{cell}) \end{cases}$$

Alpha-particles

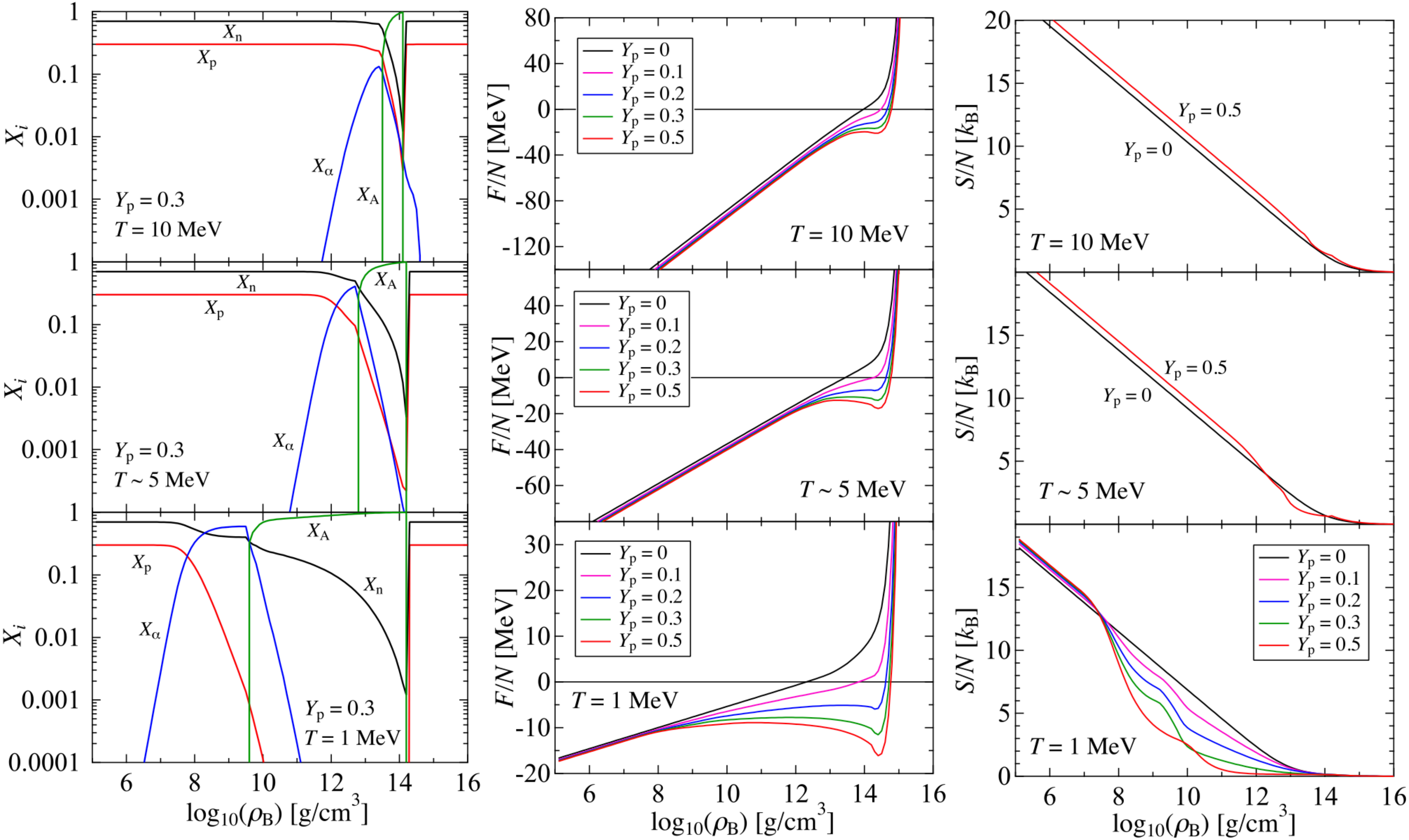
$$n_\alpha(r) = \begin{cases} -n_\alpha^\mathrm{out}[1 - (r/R_\mathrm{p})^{t_\mathrm{p}}]^3 + n_\alpha^\mathrm{out} & (0 \le r \le R_\mathrm{p}) \\ n_\alpha^\mathrm{out} & (R_\mathrm{p} \le r \le R_\mathrm{cell}) \end{cases}$$

Phase Diagram of Nuclear Matter



HT *et al.*, NPA 961 (2017) 78

EOS for Hot Nuclear Matter



Particle fraction *Free energy* *Entropy*

HT *et al.*, NPA 961 (2017) 78

Home Page of Variational EOS Table

http://www.np.phys.waseda.ac.jp/EOS/

Equation of state for nuclear matter with the variational method

Equation of state (EOS) based on the variational many-body theory with realistic nuclear forces is provided. For uniform matter, the EOS is constructed with the cluster variational method starting from the Argonne v18 two-body nuclear potential and the Urbana IX three-body nuclear potential. Non-uniform nuclear matter is treated in the Thomas-Fermi approximation. Alpha particle mixing is also taken into account. See Togashi et al., Nucl. Phys. A 961 (2017) 78 for details. This EOS table is open for general use in any studies for nuclear physics and astrophysics, provided that our paper is referred to in your publication.

User's Guide (read me first)

guide.pdf

EOS tables

eos.zip

Contact

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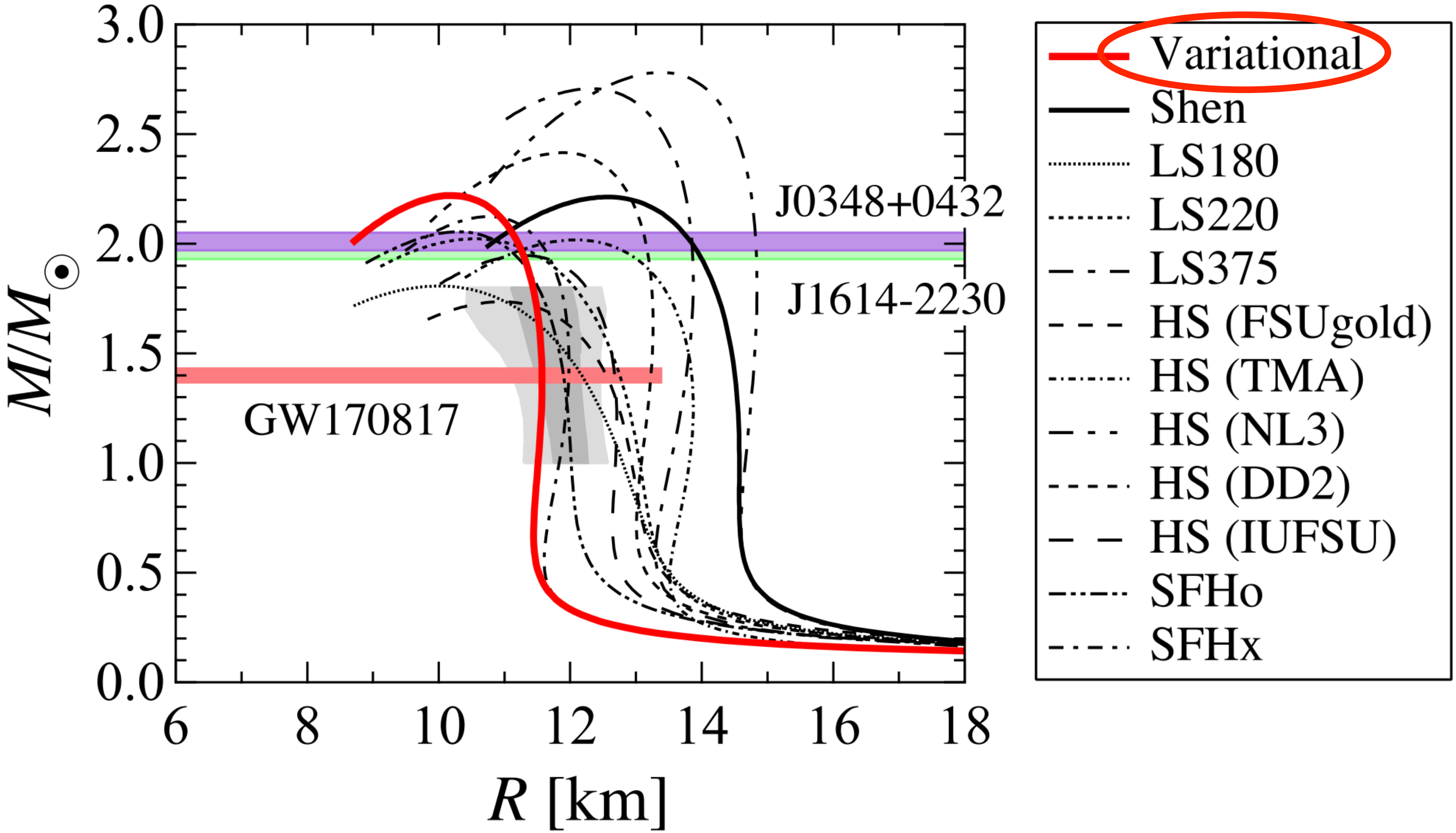
(HT *et al.*, NPA961 (2017) 78)

Table A.1: Ranges of temperature T, proton fraction Y_p, and baryon mass density ρ_B in the table of the variational EOS. At the top of the last column, "+1" represents the case at $T = 0$ MeV.

Parameter	Minimum	Maximum	Mesh	Number
$\log_{10}(T)$ [MeV]	-1.00	2.60	0.04	$91+1$
Y_p	0	0.65	0.01	66
$\log_{10}(\rho_B)$ [g/cm^3]	5.1	16.0	0.10	110

3. Application to astrophysical objects

Mass-Radius relation of neutron stars



J0348+0432: Science 340 (2013) 1233232

J1614-2230: Nature 467 (2010) 1081

GW170817: PRL 119 (2017) 161101arXiv:1711.02644

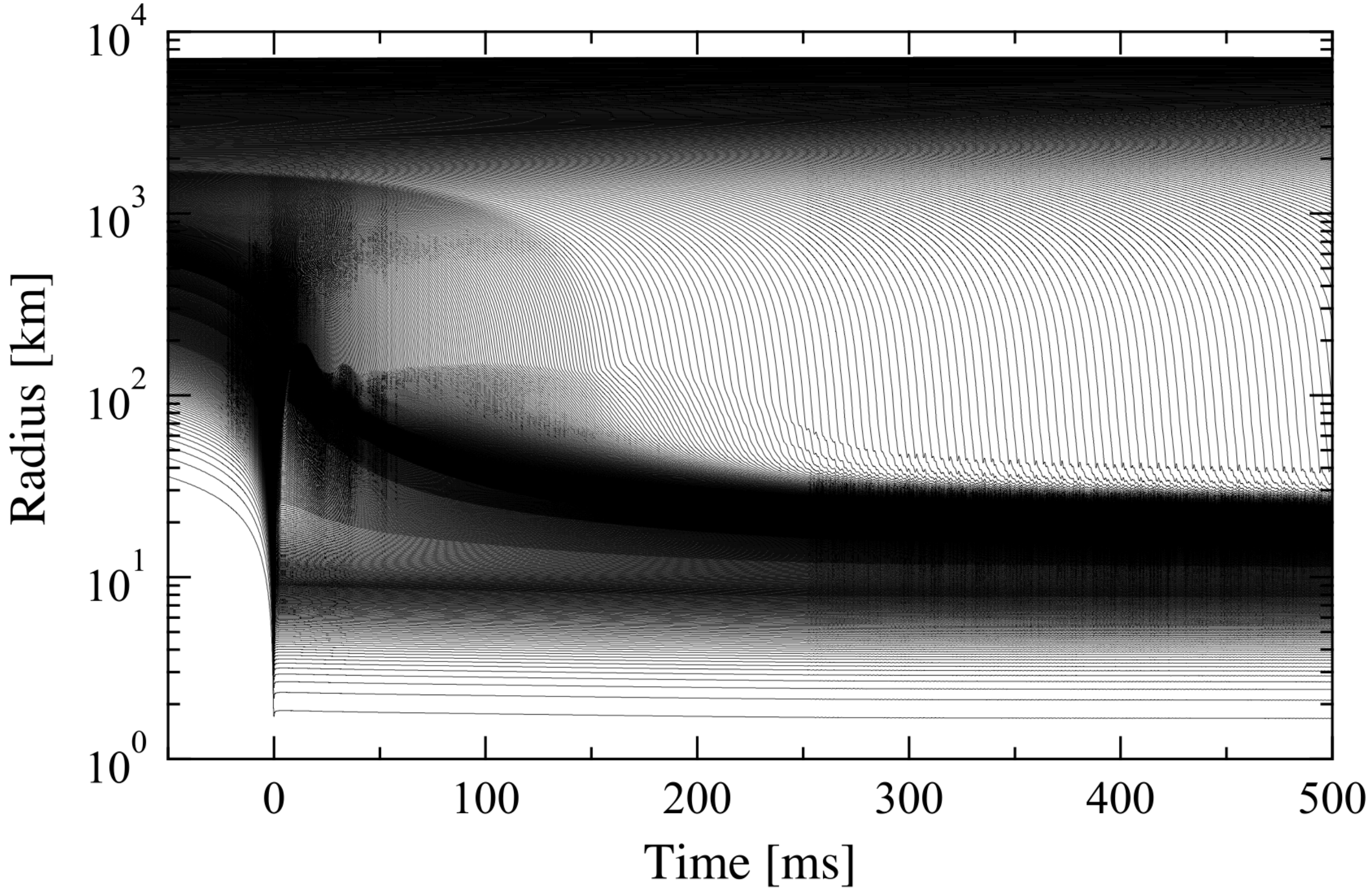
Shaded region: Astrophys. J. 722 (2010) 33

Application to Core-Collapse Supernovae

1D neutrino-radiation hydrodynamics simulations

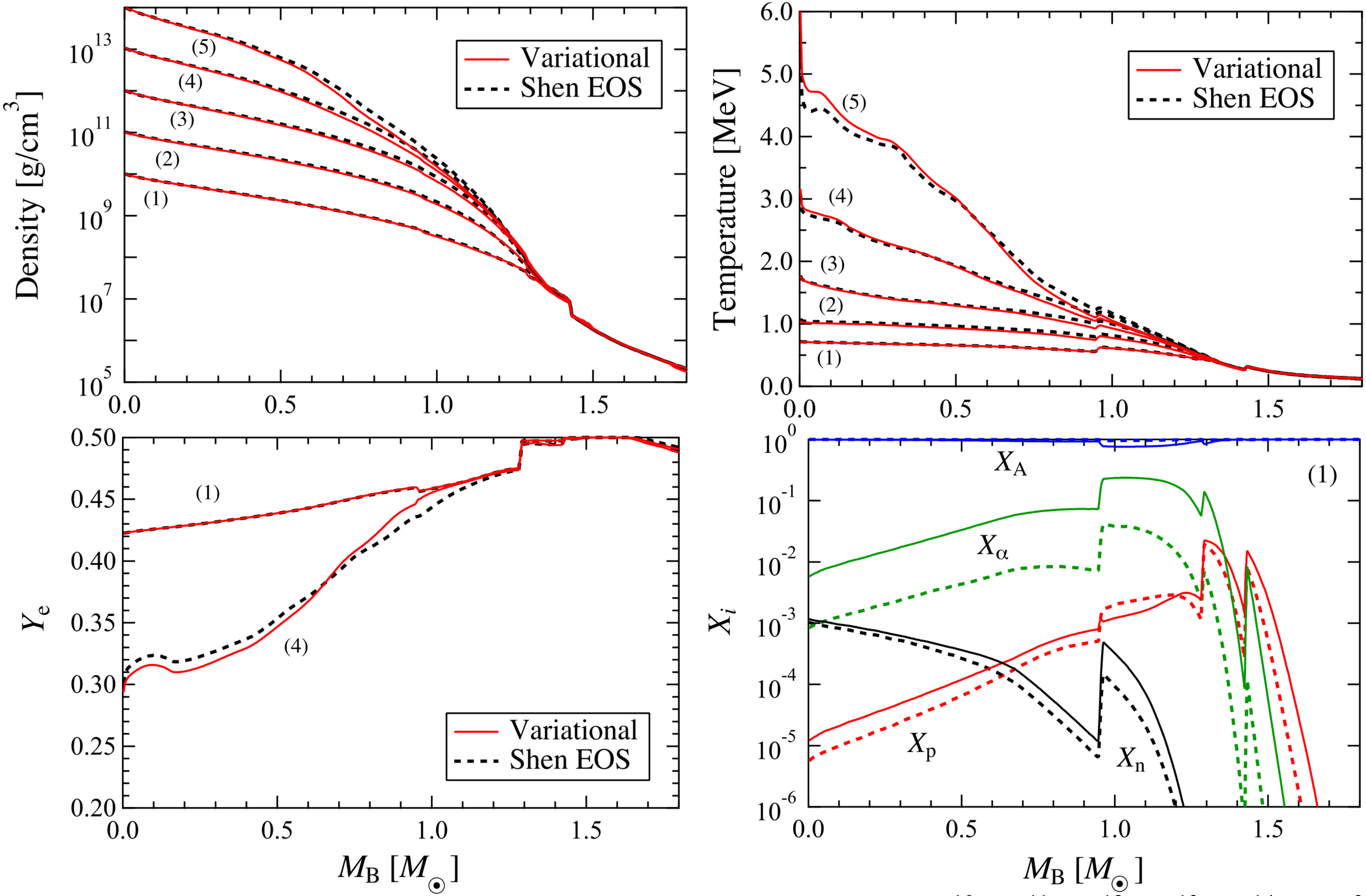
Progenitor: Woosley Weaver 1995, $15M_{\odot}$ Astrophys. J. Suppl. 101 (1995) 181

SN simulation numerical code: K. Sumiyoshi, et al., NPA 730 (2004) 227



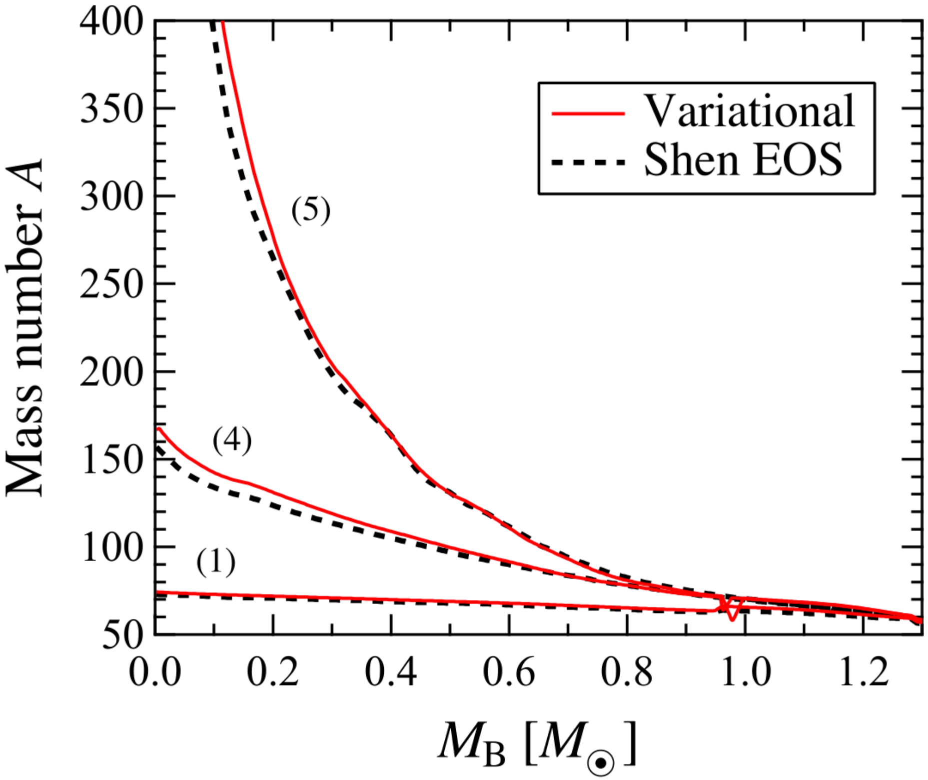
Radial trajectories of mass elements

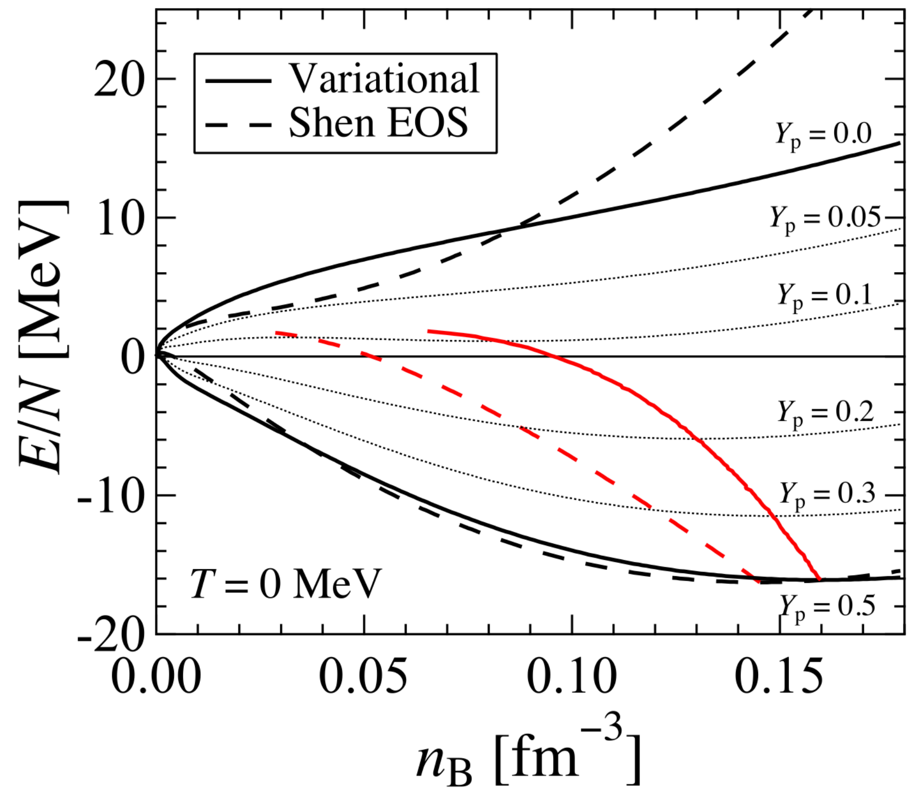
Comparison of Results (Collapse Phase)



The numbers (1)–(5) :the times when the central density reaches 10^{10}, 10^{11}, 10^{12}, 10^{13}, 10^{14} g/cm^3

Heavy Nuclei in Supernova Matter





Energies of uniform nuclear matter at 0MeV

→ **The density derivative coefficient of the symmetry energy *L***

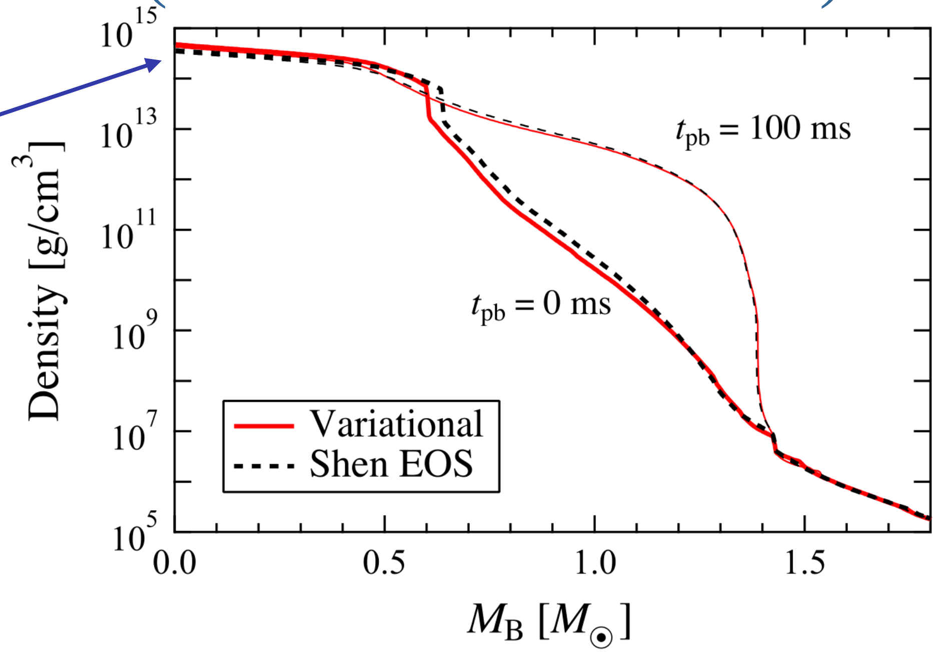
(Our EOS: L = 35 MeV   Shen EOS: L = 111 MeV)

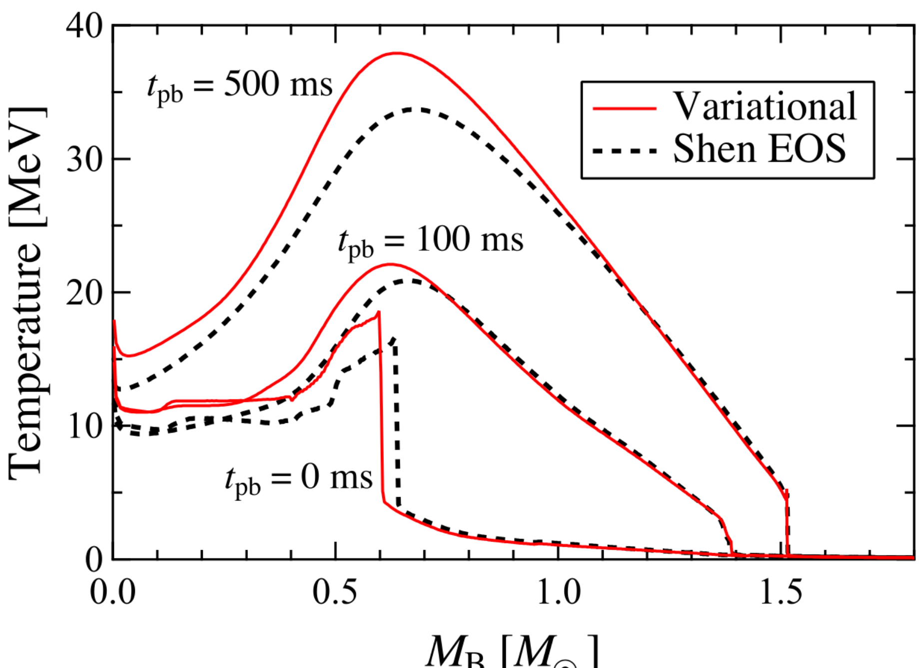
HT *et al.*, NPA 961 (2017) 78

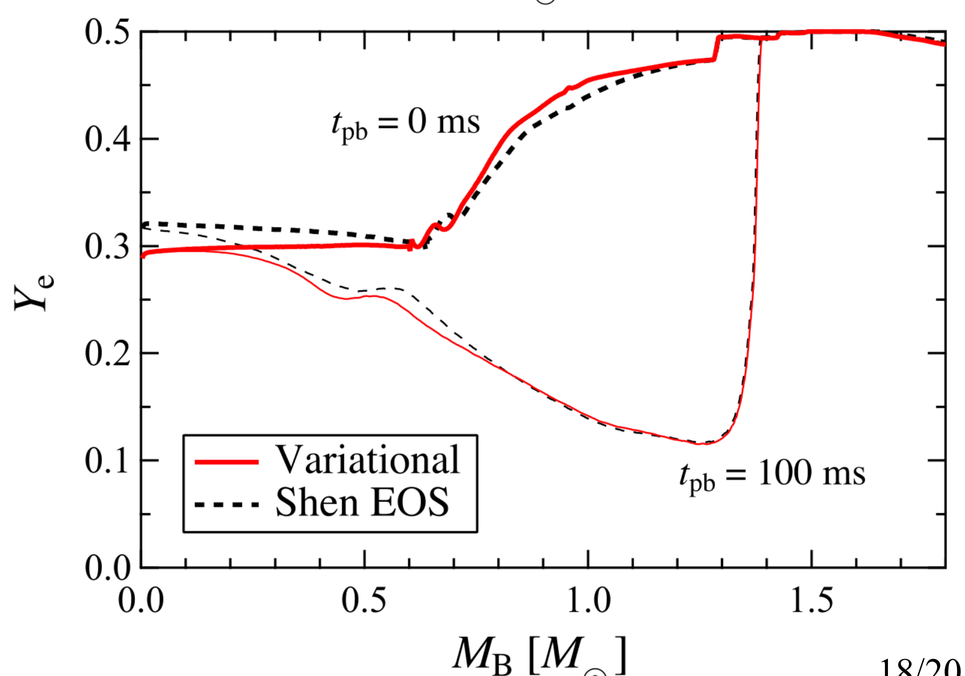
Comparison of Results (Postbounce Phase)

4.7×10^{14} g/cm^3 (Variational)

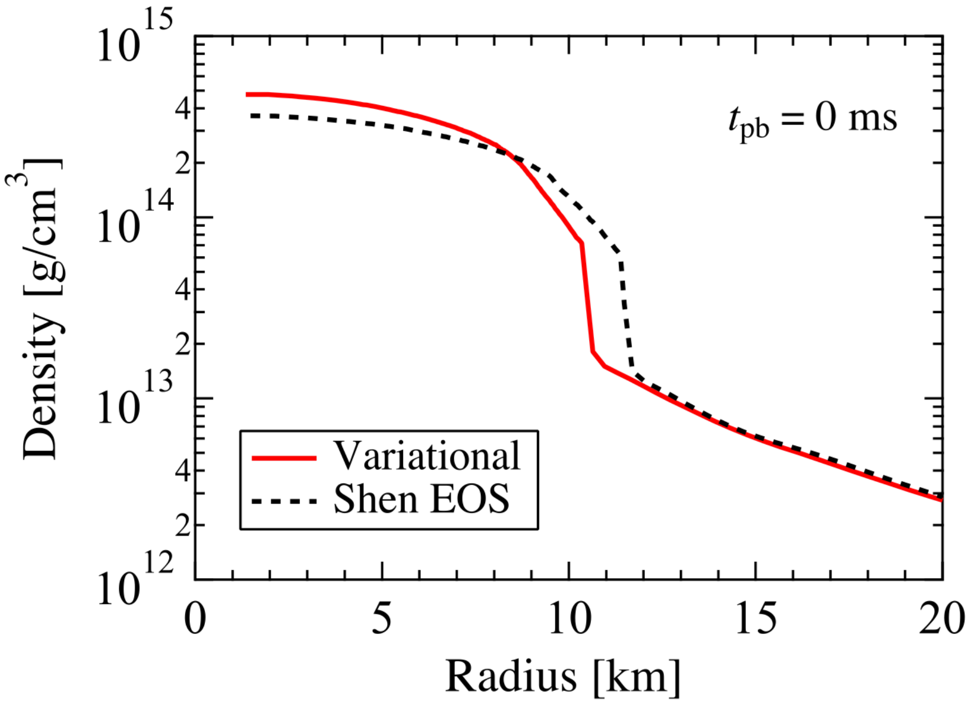
3.6×10^{14} g/cm^3 (Shen EOS)



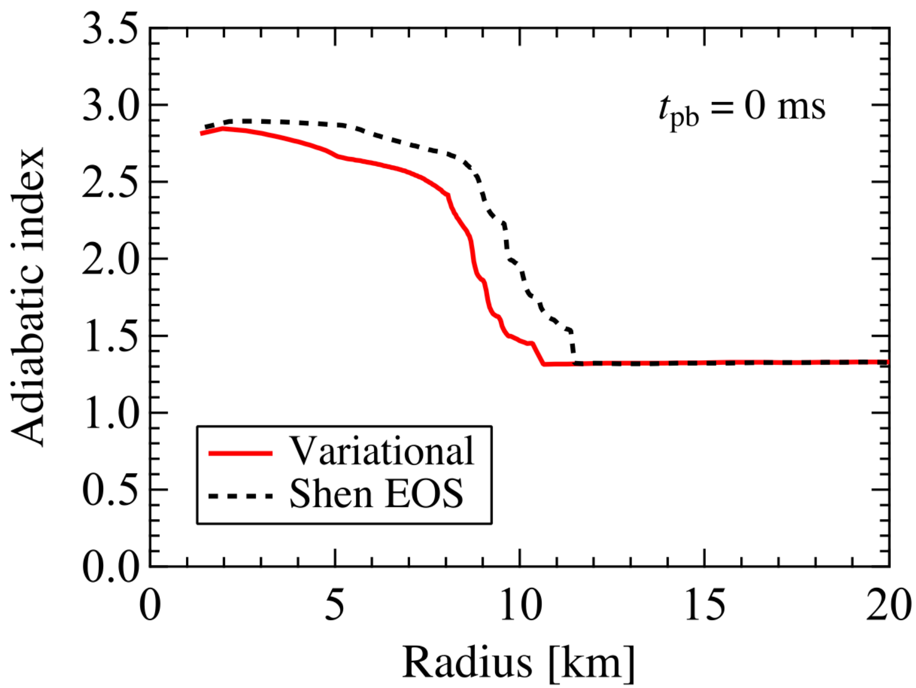




Stiffness of the nuclear EOS



The density profiles at $t_{pb} = 0$ ms



The adiabatic indices at $t_{pb} = 0$ ms

Adiabatic index: $\Gamma = \left.\frac{d \log P}{d \log \rho}\right|_S$

Variational EOS is softer than the Shen EOS.

→ Our EOS is advantageous for supernova explosion!?

Summary

Nuclear EOS for supernova simulations is constructed with realistic nuclear forces (AV18 + UIX).

Uniform nuclear matter : Cluster variational method

Non-uniform nuclear matter : Thomas-Fermi approximation

Our SN-EOS is available at

`http://www.np.phys.waseda.ac.jp/EOS/`

- *Neutron star structure is consistent with observational data.*
- *Our EOS is softer than Shen EOS in 1D supernova simulation.*

Future Plans

- Multi-dimensional supernova simulations
- Application to other astrophysical simulations
- Hyperon mixing in high-density matter