Stability of Electroweak Vacuum in the Standard Model and Beyond

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Chigusa, TM, Shoji, PRL 119 ('17) 211801 [1707.09301]

Chigusa, TM, Shoji, work in progress

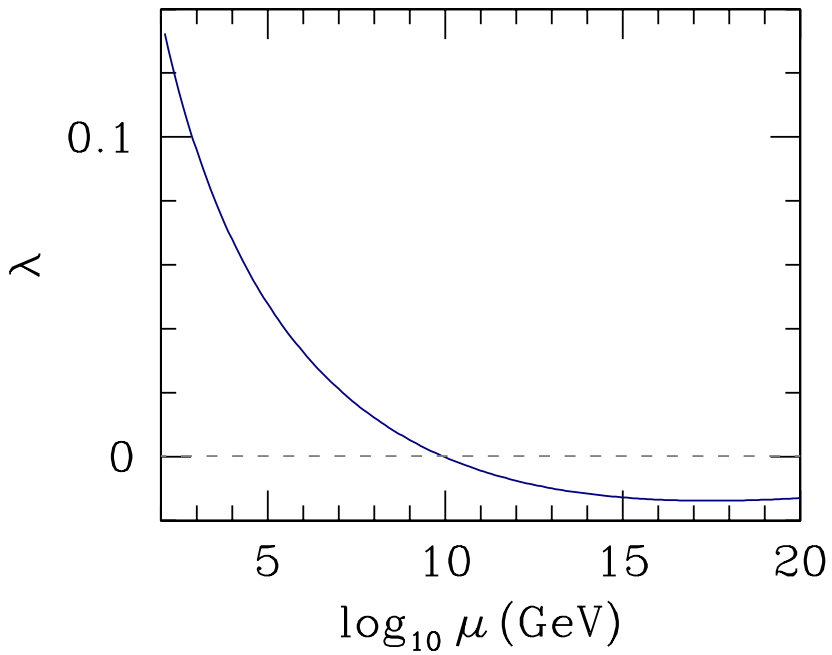
PACIFIC Workshop, Hokkaido, Japan, '18.02.17

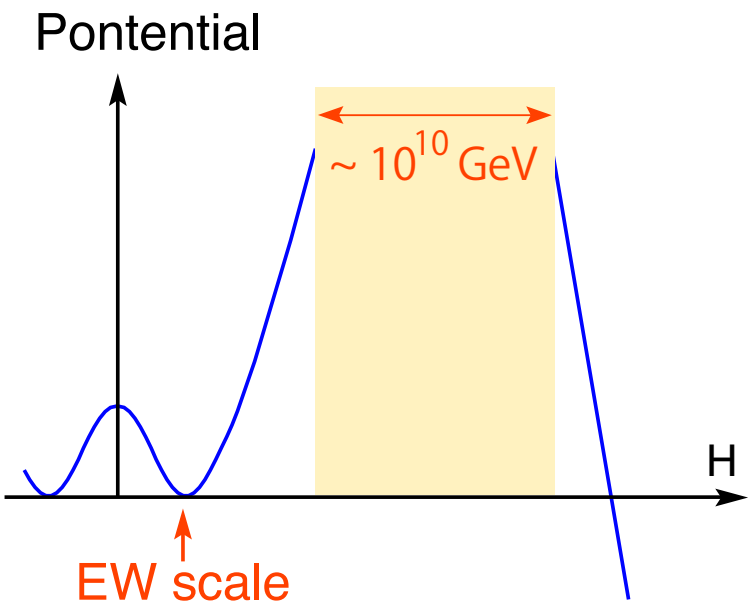
1. Introduction

What we learn from the Higgs mass

$$m_h \simeq 125 \text{ GeV} \quad \Rightarrow \quad V = \lambda(|H|^2 - v^2)^2 \text{ with } \lambda(m_h) \simeq 0.13$$

λ becomes negative at a very high scale





- EW vacuum is not stable in the standard model (SM)
- λ is minimized at $\mu \sim 10^{17}$ GeV

Is the decay rate small enough so that $t_{\text{now}} \simeq 13.6$ Gyr?

$\Rightarrow$ (Probably) yes

[Isidori, Ridolfi & Strumia; Degrassi et al.; Alekhin, Djouadi & Moch; Espinosa et al.; Plascencia & Tamarit; Lalak, Lewicki & Olszewski; Espinosa, Garny, Konstandin & Riotto; $\cdots$]

How precisely can we estimate the decay rate?

- Gauge-invariance of the result was unclear
- Effects of zero modes were not properly taken into account
- There has been progresses in the calculation of the decay rate of false vacuum

  [Endo, TM, Nojiri & Shoji; Chigusa, TM & Shoji; see also Andreassen, Frost & Schwartz]

Today, I discuss

- A calculation of the decay rate of EW vacuum
- Effects of extra matters

Outline

1. Introduction
2. Bounce in the SM
3. Effects of Higgs Mode
4. Total Decay Rate
5. Case with Extra Matters
6. Summary

2. Bounce in the SM

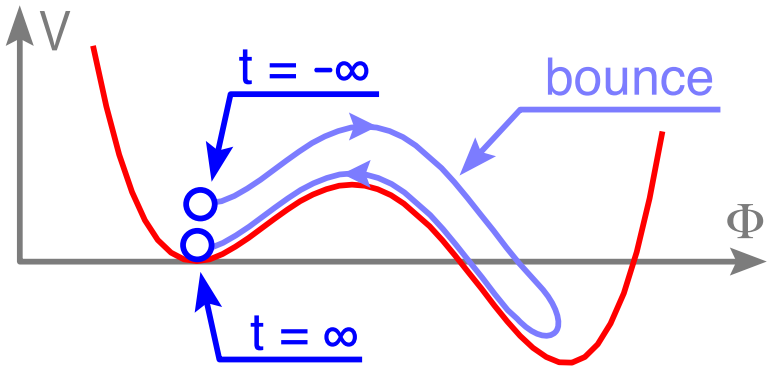
The decay rate is related to 4D Euclidean partition function

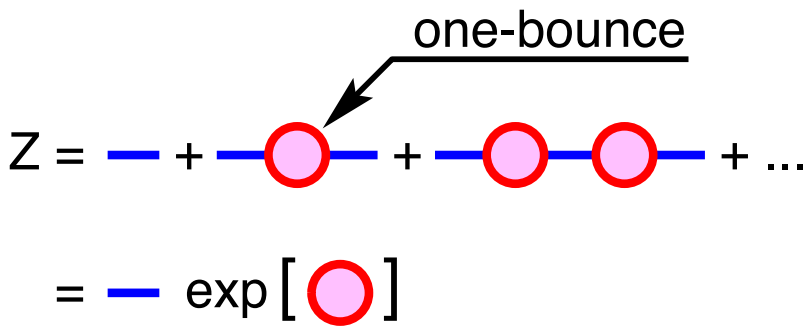
[Coleman; Callan & Coleman]

$$Z = \langle \mathsf{FV} | e^{-HT} | \mathsf{FV} \rangle \propto \exp(i\gamma VT)$$

The path integral is dominated by the "bounce"

Bounce: a saddle-point solution of classical EoM





$$\gamma \simeq \frac{1}{VT} \mathrm{Im} \left[\frac{\int_{\text{1-bounce}} \mathcal{D}\Psi \, e^{-S_{\mathsf{E}}}}{\int_{\text{0-bounce}} \mathcal{D}\Psi \, e^{-S_{\mathsf{E}}}} \right] \equiv \mathcal{A} e^{-\mathcal{B}} \quad \text{with } \mathcal{B} = S_{\mathsf{E}}(\text{Bounce})$$

Main concern of this talk: calculation of the prefactor $\mathcal{A}$

$\mathcal{A}$ contains one-loop (and higher order) effects

We expand the action around the bounce $\bar{\phi}$

$$S_{\mathsf{E}}[\bar{\phi}+\Psi] = S_{\mathsf{E}}[\bar{\phi}] + \frac{1}{2}\int d^4x \Psi \mathcal{M} \Psi + O(\Psi^3)$$

$$S_{\mathsf{E}}[v+\Psi] = S_{\mathsf{E}}[v] + \frac{1}{2}\int d^4x \Psi \widehat{\mathcal{M}} \Psi + O(\Psi^3)$$

Prefactor $\mathcal{A}$ (for bosonic contribution)

$$\mathcal{A} \simeq \frac{1}{VT}\left|\frac{\mathsf{Det}\mathcal{M}}{\mathsf{Det}\widehat{\mathcal{M}}}\right|^{-1/2} \propto \prod_i \sqrt{\frac{1}{\omega_i}}$$ with ω_i = eigenvalue of $\mathcal{M}$

Sometimes $\mathcal{M}$ has zero eigenvalue

$\Rightarrow$ A careful treatment is necessary

Higgs potential in the SM: $V = -m^2 H^\dagger H + \lambda (H^\dagger H)^2$

- We consider very large Higgs amplitude for which $\lambda < 0$
- We will neglect quadratic term because $\lambda < 0$ occurs at a scale much higher than the EW scale

We use the following potential:

$$V = -|\lambda| (H^\dagger H)^2$$

The "bounce solution" (Fubini-Lipatov instanton)

$$H_{\text{bounce}} = \frac{1}{\sqrt{2}} e^{i\sigma^a \theta^a} \begin{pmatrix} 0 \\ \bar{\phi} \end{pmatrix} \quad \text{with } \partial_r^2 \bar{\phi} + \frac{3}{r} \partial_r \bar{\phi} + 3|\lambda| \bar{\phi}^2 = 0$$

$\Rightarrow$ Explicit form of the bounce:

$$\bar{\phi}(r) = \sqrt{\frac{8}{|\lambda|}} R^{-1} \frac{1}{1 + R^{-2} r^2} \quad \text{with } R = \text{(free parameter)}$$

Bounce action for the SM

$$\mathcal{B} = \frac{8\pi^2}{3|\lambda|}$$

Possible deformations of the bounce

- Dilatation: parameterized by R
- $SU(2)$ transformation: parameterized by θ^a

  Effects of zero modes in association with these transformations were not properly taken into account before

- Translation

  [Callan & Coleman]

Expansion around the bounce:

$$H = \frac{1}{\sqrt{2}} e^{i\sigma^a\theta^a} \begin{pmatrix} \varphi^1 + i\varphi^2 \\ \bar{\phi} + h - i\varphi^3 \end{pmatrix}, \qquad W^a_\mu = w^a_\mu, \qquad B_\mu = b_\mu$$

3. Effects of the Higgs Mode

We need to calculate the functional determinant of $\mathcal{M}^{(h)}$

$$\mathcal{L} \ni \frac{1}{2} h \left(-\partial^2 - 3|\lambda|\bar{\phi}^2\right) h = \frac{1}{2} h \, \mathcal{M}^{(h)} \, h$$

Expansion of h w.r.t. 4D spherical harmonics $\mathcal{Y}_{J,m_A,m_B}$

$$h(x) = \sum_{J,m_A,m_B,n} \alpha_{n,J,m_A,m_B} \, \rho_{n,J}(r) \, \mathcal{Y}_{J,m_A,m_B}(\hat{\mathbf{r}})$$

$J = 0,\ 1/2,\ 1,\ 3/2,\ \cdots$

$\rho_{n,J}$: radial mode function

α_{n,J,m_A,m_B}: expansion coefficient (integration variable)

Fluctuation operator for angular-momentum eigenstates:

$$\mathcal{M}_J^{(h)} \equiv -\left(\Delta_J + 3|\lambda|\bar{\phi}^2\right) \equiv -\left[\partial_r^2 + \frac{3}{r}\partial_r - \frac{4J(J+1)}{r^2} + 3|\lambda|\bar{\phi}^2\right]$$

Higgs-mode contribution to the prefactor $\mathcal{A}$

$$\mathcal{A}^{(h)} = \left[\frac{\mathrm{Det}\mathcal{M}^{(h)}}{\mathrm{Det}\widehat{\mathcal{M}}^{(h)}}\right]^{-1/2} = \prod_J \left[\frac{\mathrm{Det}\mathcal{M}_J^{(h)}}{\mathrm{Det}\widehat{\mathcal{M}}_J^{(h)}}\right]^{-(2J+1)^2/2}$$

The ratio of the functional determinants can be evaluated with so-called Gelfand-Yaglom theorem

Zero modes exist for $\mathcal{M}^{(h)}$

- Dilatational zero mode (for $J = 0$)

$$\rho_{\mathrm{D}}(r) \propto \frac{\partial\bar{\phi}}{\partial R} \quad \Leftrightarrow \quad \mathcal{M}_0^{(h)}\rho_{\mathrm{D}}(r) = 0 \text{ and } \rho_{\mathrm{D}}(r \to \infty) = 0$$

- Translational zero modes (for $J = 1/2$)

[Callan & Coleman]

Path integral over dilatational zero mode = integral over R

$$H \ni \bar{\phi} + h = \bar{\phi} + \alpha_{\mathsf{D}} \mathcal{N}_{\mathsf{D}} \frac{\partial \bar{\phi}}{\partial R} + \cdots \simeq \bar{\phi}\Big|_{R \to R + \alpha_{\mathsf{D}} \mathcal{N}_{\mathsf{D}}} + \cdots$$

$$\Rightarrow \int \mathcal{D}h^{(\text{dilatation})} \equiv \int d\alpha_{\mathsf{D}} \to \int \frac{dR}{\mathcal{N}_{\mathsf{D}}}$$

$$\Rightarrow \left[\frac{\mathsf{Det}\mathcal{M}_0^{(h)}}{\mathsf{Det}\widehat{\mathcal{M}}_0^{(h)}}\right]^{-1/2} \to \int \frac{dR}{\mathcal{N}_{\mathsf{D}}} \left[\frac{\mathsf{Det}'\mathcal{M}_0^{(h)}}{\mathsf{Det}\widehat{\mathcal{M}}_0^{(h)}}\right]^{-1/2}$$

Det': zero eigenvalue is omitted from the Det

Higgs-mode contribution:

[Chigusa, TM & Shoji; Andreassen, Frost & Schwartz]

$$\mathcal{A}^{(h)} \to \int d\ln R \left(\frac{16\pi}{|\lambda|}\right)^{1/2} \prod_{J \geq 1/2} \left[\frac{\mathsf{Det}\mathcal{M}_J^{(h)}}{\mathsf{Det}\widehat{\mathcal{M}}_J^{(h)}}\right]^{-(2J+1)^2/2}$$

Comment on gauge and NG contribution

- Gauge fixing function in old calculations

$$\mathcal{F} = \partial_\mu B_\mu - 2\xi g_1 (\mathrm{Re} H_0)(\mathrm{Im} H_0), \quad \cdots$$

$\Rightarrow$ Gauge and NG fields couple in the EoM

$\Rightarrow$ General form of the bounce is more complicated

- We adopt the following gauge fixing function

[Kusenko, Lee & Weinberg]

$$\mathcal{F} = \partial_\mu B_\mu, \;\; \mathcal{F}^a = \partial_\mu W^a_\mu \quad \Rightarrow H_{\text{bounce}} = \frac{1}{\sqrt{2}} e^{i\sigma^a \theta^a} \begin{pmatrix} 0 \\ \bar{\phi} \end{pmatrix}$$

- Path integral over gauge zero modes = integral over θ^a

$$\left[\frac{\mathrm{Det}\mathcal{M}^{(W,Z,\mathrm{NG})}}{\mathrm{Det}\widehat{\mathcal{M}}^{(W,Z,\mathrm{NG})}} \right]^{-1/2} \to \mathcal{V}_{SU(2)} \left(\frac{16\pi}{|\lambda|} \right)^{3/2} \prod_{J \geq 1/2} \left[\frac{\mathrm{Det}\mathcal{M}_J^{(W,Z,\mathrm{NG})}}{\mathrm{Det}\widehat{\mathcal{M}}_J^{(W,Z,\mathrm{NG})}} \right]^{-1/2}$$

4. Total Decay Rate

Decay rate:

$$\gamma = \int d\ln R \left[I^{(h)} I^{(W,Z,\mathsf{NG})} I^{(t)} e^{-\mathcal{S}_{\mathsf{C.T.}}} e^{-\mathcal{B}} \right]_{\mu \sim 1/R}$$

We derived complete and gauge-invariant expressions of $I^{(X)}$

$I^{(h)}$: Higgs contribution

$I^{(W,Z,\mathsf{NG})}$: gauge and NG contribution

$I^{(t)}$: top contribution

We take the renormalization scale as $\mu \sim 1/R$

$\Rightarrow$ The effects of μ-dependent terms from higher loops, i.e., $\sim \ln^p(\mu R)$, are expected to be minimized

$\Rightarrow$ This is important for the convergence of the integral

We use:

- $m_h = 125.09 \pm 0.24$ GeV
- $m_t = 173.1 \pm 0.6$ GeV
- $\alpha_s(m_Z) = 0.1181 \pm 0.0011$
- 2- or 3-loop RGEs (with relevant threshold corrections)

Decay rate of the EW vacuum (taking $\mu = 1/R$)

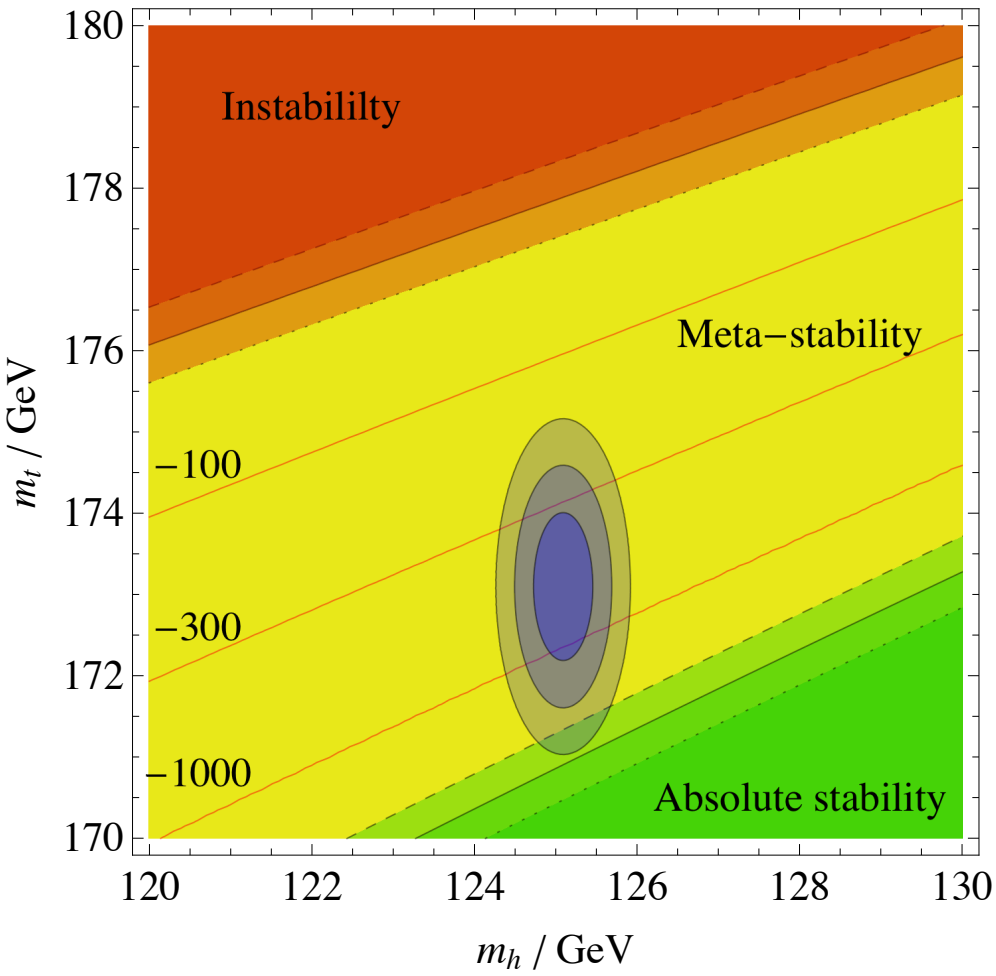
- $\log_{10}[\,\gamma\ (\mathrm{Gyr}^{-1}\mathrm{Gpc}^{-3})\,] \simeq -564^{+38+173+137}_{-43-312-208}$

For the present universe:

- Cosmic age: $t_0 \simeq 13.6$ Gyr
- Horizon scale: $H_0^{-1} \simeq 4.5$ Gpc

$\log_{10}[\gamma\ (\mathrm{Gyr}^{-1}\mathrm{Gpc}^{-3})]$ on m_h vs. m_t plane (with $\mu = 1/R$)

[Chigusa, TM & Shoji, in preparation]



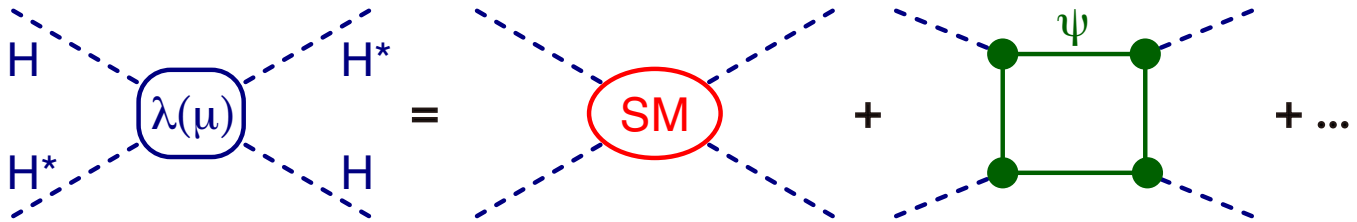
- Instability: $\gamma > H_{\text{now}}^4$
- Metastability: $\gamma < H_{\text{now}}^4$
- Absolute stability: $\lambda > 0$

5. Case with Extra Matters

Let us consider vector-like fermions coupled to H

$$\mathcal{L} = \mathcal{L}_{\text{SM}} + y_\psi H \psi_L \psi_R + y_{\bar{\psi}} H^* \bar{\psi}_L \bar{\psi}_R + M_\psi \bar{\psi}_L \psi_L + M_\psi \bar{\psi}_R \psi_R + \cdots$$

RGE for λ



$$\frac{d\lambda}{d\ln\mu} = \left[\frac{d\lambda}{d\ln\mu}\right]_{\text{SM}} - \frac{1}{4\pi^2}\sum_\psi N_C^{(\psi)} y_\psi^4 + \cdots$$

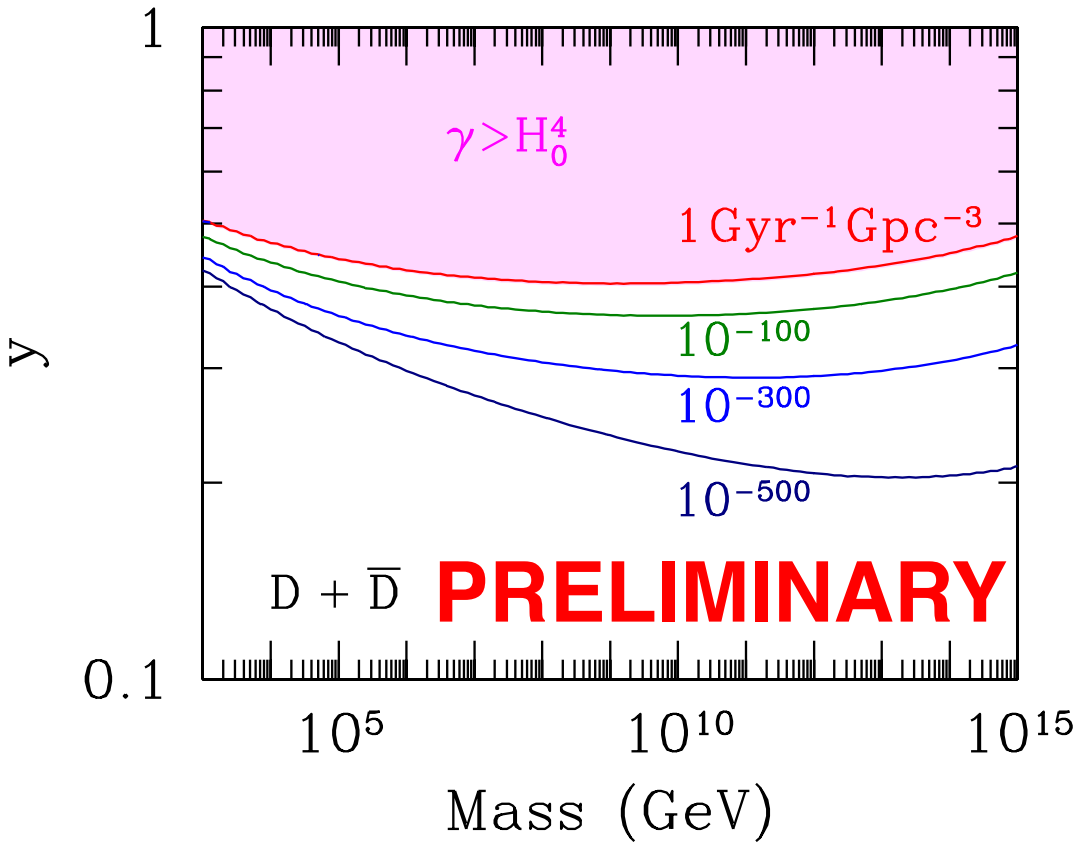
With extra fermions, λ may become smaller (at high scale)

$\Rightarrow$ Enhancement of the decay rate

C.f., $\gamma = \mathcal{A} e^{-\mathcal{B}}$ with $\mathcal{B} = \dfrac{8\pi^2}{3|\lambda|}$

Case 1: Down-quark-like colored fermions

$\Rightarrow \psi_L(\mathbf{3}, \mathbf{2}, 1/6)$ and $\psi_R(\bar{\mathbf{3}}, \mathbf{1}, -1/3)$

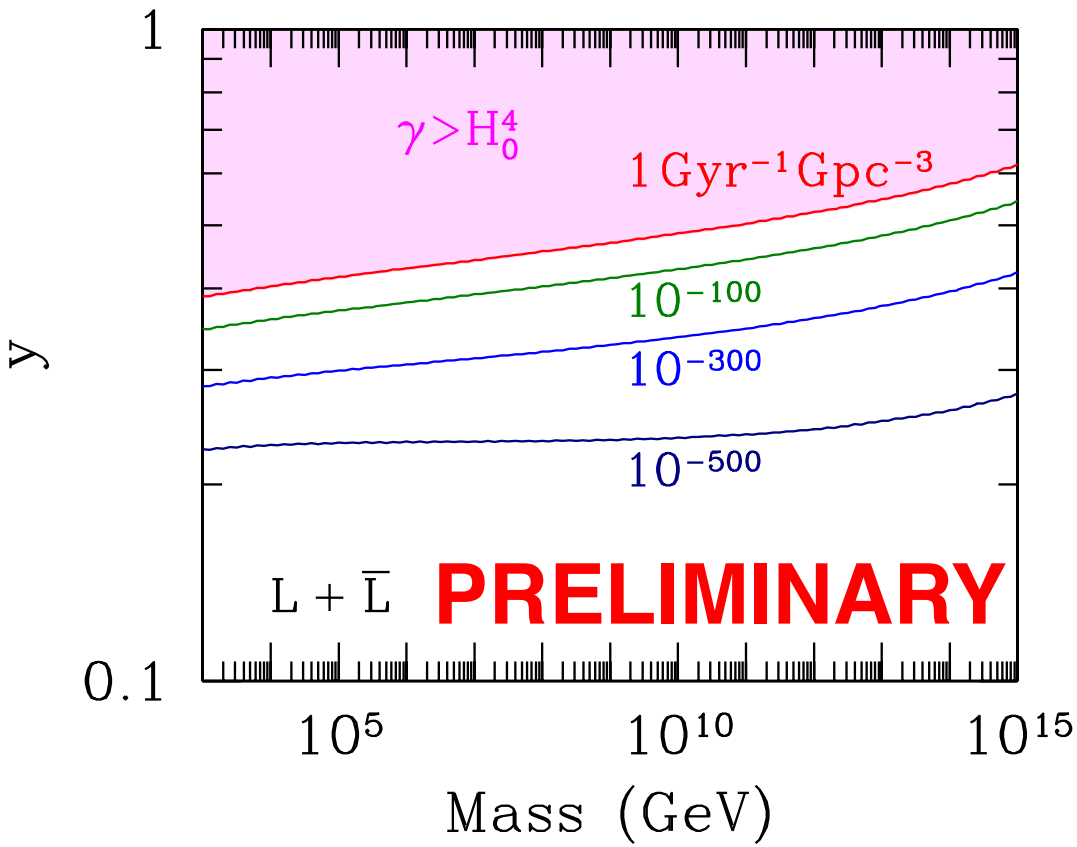


- $\bar{\phi}_C^{(\mathrm{max})} = M_{\mathrm{Pl}}$

$\Rightarrow$ Yukawa coupling larger than $\sim 0.4 - 0.5$ is dangerous

Case 2: Charged-lepton-like fermions

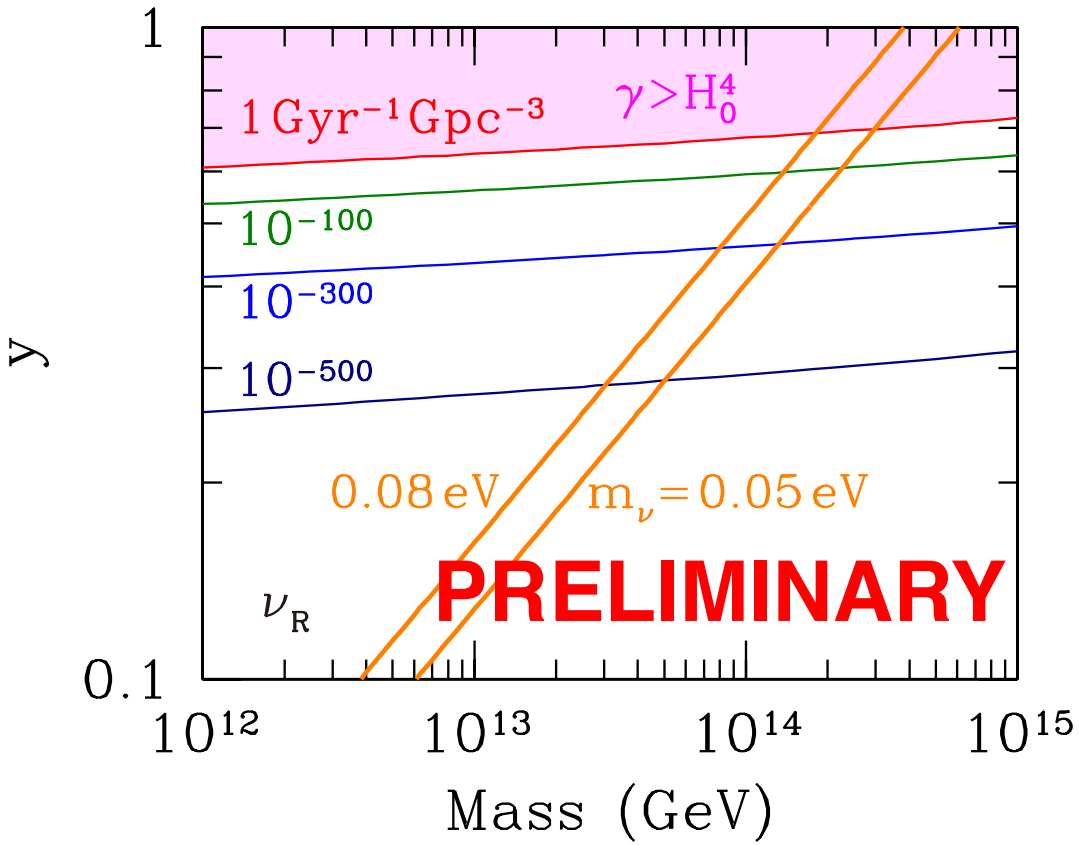
$\Rightarrow \psi_L(\mathbf{1}, \mathbf{2}, 1/2)$ and $\psi_R(\mathbf{1}, \mathbf{1}, -1)$



- $\bar{\phi}_C^{(\mathrm{max})} = M_{\mathrm{Pl}}$

Case 3: Right-handed neutrino

$$\mathcal{L} = \mathcal{L}_{\mathsf{SM}} + y_\nu H \ell_L \nu_R^c + \frac{1}{2} M_\nu \nu_R^c \nu_R^c + \cdots$$



- $\bar{\phi}_C^{(\mathsf{max})} = M_{\mathsf{Pl}}$

6. Summary

I have discussed the decay rate of the EW vacuum

Path integrals over the dilatational and gauge zero modes are properly performed

Numerical result

$$\log_{10}[\,\gamma\ (\mathrm{Gyr}^{-1}\mathrm{Gpc}^{-3})\,] \simeq -564^{+38+173+137}_{-43-312-208}$$

The decay rate is extremely small: $\gamma \ll H_0^4$

$\Rightarrow$ We will fall into another vacuum if we wait $\sim 10^{562}$ Gyr (assuming that the dark energy is cosmological constant)

Extra fermions may change the above conclusion

$\Rightarrow$ $y \gtrsim 0.4 - 0.6$ is dangerous

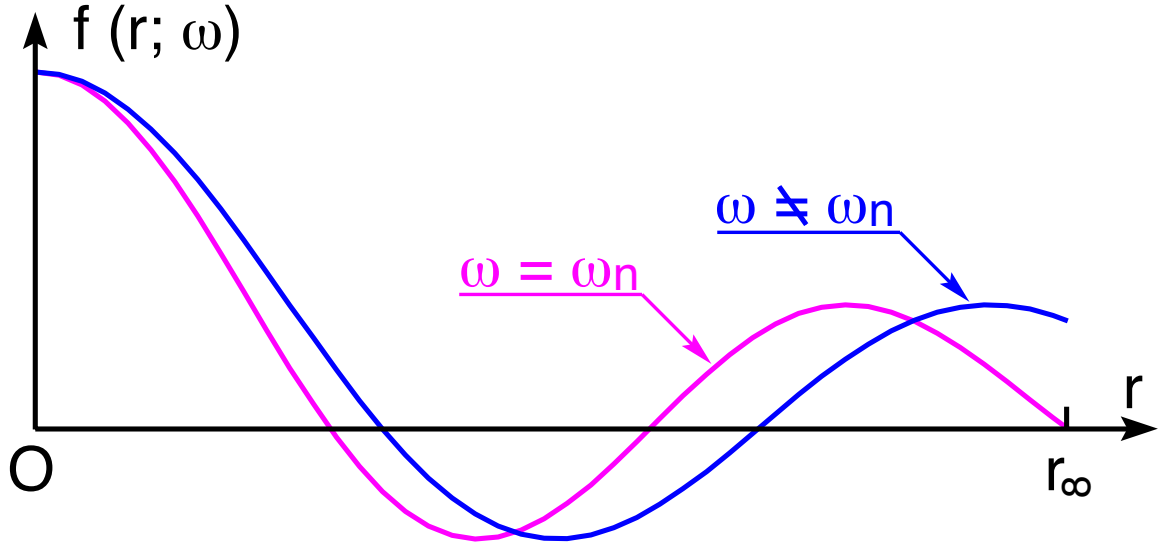
Back Up

Functional determinant for operators defined in $0 \leq r \leq r_\infty$

$$\mathrm{Det}\mathcal{M} \simeq \prod_n \omega_n \text{ with } \begin{cases} \mathcal{M}\rho_n = \omega_n \rho_n \text{ with } \mathcal{M} = -\Delta_J + \delta W(r) \\ \rho_n(0) < \infty \\ \rho_n(r_\infty) = 0 \end{cases}$$

We introduce a function f which obeys: $\mathcal{M} f(r;\omega) = \omega f(r;\omega)$

- $f(r = r_\infty;\omega)|_{\omega=\omega_n} = 0$
- $\mathrm{Det}(\mathcal{M} - \omega)|_{\omega=\omega_n} = 0$



Gelfand-Yaglom theorem

[Gelfand & Yaglom; Coleman; Dashen, Hasslacher & Neveu; Kirsten & McKane; $\cdots$]

$$\frac{\mathrm{Det}(\mathcal{M}-\omega)}{\mathrm{Det}(\widehat{\mathcal{M}}-\omega)} = \frac{f(r=r_\infty;\omega)}{\widehat{f}(r=r_\infty;\omega)} \text{ with } \begin{cases} \mathcal{M}f(r;\omega) = \omega f(r;\omega) \\ \widehat{\mathcal{M}}\widehat{f}(r;\omega) = \omega \widehat{f}(r;\omega) \\ f(r=0) = \widehat{f}(r=0) < \infty \end{cases}$$

$\Rightarrow$ Notice: LHS and RHS have the same analytic behavior

- LHS and RHS have same zeros and infinities
- LHS and RHS becomes equal to 1 when $\omega \to \infty$

We take $r_\infty \to \infty$ at the end of calculation

$\Rightarrow$ The results converge (in the case of our interest)