Ratchet Baryogenesis during Reheating

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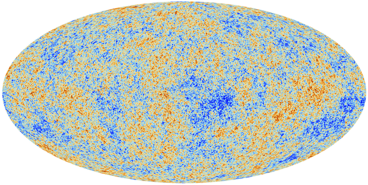
K. Bamba, N. B., A. Sugamoto, T. Takeuchi and K. Yamashita, arxiv:1610.03268, and upcoming work.

Outline

1. Introduction
2. The Mechanism and Dynamics
3. Calculating the Generated η
4. Conclusion and Future Work

Matter-Antimatter Asymmetry



The asymmetry is described quantitatively by,

$$\eta = \frac{n_b - n_{\bar{b}}}{s} \simeq 8.5 \times 10^{-11}$$

The Sakharov Conditions

1. Baryon number violation
2. $\mathcal{C}$ and $\mathcal{CP}$ violation
3. Period of non-equilibrium

Ratchet Mechanism

Inspired by molecular motors in biological systems, and their ability to generate directed motion.

- Consider an inflaton and complex scalar carrying X charge during reheating.
- A derivative coupling between a complex scalar and inflaton.
- Directed motion in the complex scalar phase gives a non-zero X number density.

The Model

- Interplay between the inflaton and complex scalar during reheating.
- Introduce a derivative coupling term describing their interaction.

$$S = \int dx^4 \sqrt{-g} \, [g^{\mu\nu}\partial_\mu\phi\partial_\nu\phi^* - V_0(\phi,\phi^*) \\ + \frac{1}{2} g^{\mu\nu}\partial_\mu\Phi\partial_\nu\Phi - U(\Phi) + \frac{i}{\Lambda} g^{\mu\nu} \left(\phi^* \overleftrightarrow{\partial_\mu} \phi\right) \partial_\nu \Phi \Big],$$

where $V_0(\phi,\phi^*) = \lambda\phi^*\phi(\phi-\phi^*)(\phi^*-\phi) + \ldots$

Satisfying the Sakharov Conditions

1. The complex scalar potential.
2. Derivative coupling interaction.
3. Reheating epoch.

$U(1)$ Global Symmetry and X Charge

- If $\lambda = 0$, the action will be invariant under the global $U(1)$ transformation $(\phi, \phi^*) \to (e^{i\alpha}\phi, e^{-i\alpha}\phi^*)$, with α a constant.
- We identify the $U(1)$ symmetry with X charge, which may be B or L.
- Take ϕ to have a unit charge under this symmetry, and Φ uncharged.
- The associated current,

$$j^\mu = -i\phi^* \partial^\mu \phi \ .$$

Starobinsky Inflation and Reheating

The Starobinsky inflation is consistent with current data, has the potential,

$$U(\Phi) = \frac{3\mu^2 M_p^2}{4}(1 - e^{-\sqrt{2/3}\Phi/M_p})^2$$

- The Starobinsky model approaches $\frac{1}{2}\mu^2\Phi^2$ during reheating.
- An approximate matter dominated epoch ($a \propto t^{2/3}$).

The potential will be expressed as,

$$U(\Phi) \approx \frac{1}{2}\mu^2\Phi^2 \quad \rightarrow \quad \frac{dU(\Phi)}{d\Phi} \approx \mu^2\Phi \ .$$

where $\mu \simeq 1.3 \times 10^{-5} M_p$, $H_i \simeq 6.2 \times 10^{12}\ \mathrm{GeV}$ and $\Phi_i \simeq 0.62 M_p$.

Reparametrising

Taking ϕ in polar form

$$\phi = \frac{1}{\sqrt{2}}\phi_r e^{i\theta},$$

we find

$$V(\phi_r, \theta) = \lambda \phi_r^4 \sin^2\theta + \cdots$$

and

$$n_X = j^0 = \phi_r^2 \dot{\theta}\ .$$

Therefore, a non-zero n_X requires non-zero ϕ_r and $\dot{\theta}$.

The action is now,

$$S = \int d^4x \sqrt{-g}\Big[\frac{\phi_r^2}{2} g_{\mu\nu}\partial^\mu\theta\partial^\nu\theta - \lambda\phi_r^4 \sin^2\theta + \tfrac{1}{2} g_{\mu\nu}\partial^\mu\Phi\partial^\nu\Phi - U(\Phi) - \tfrac{\phi_r^2}{\Lambda} g_{\mu\nu}\partial^\mu\theta\partial^\nu\Phi\Big]$$

Analysis of the Equations of Motion

Equations of Motion for θ and Φ

Equations of motion including the inflaton decay rate Γ,

$$\begin{aligned}(\ddot{\Phi}+3H\dot{\Phi})+\left(\Gamma\,\dot{\Phi}+\frac{dU(\Phi)}{d\Phi}\right)-\frac{\phi_r^2}{\Lambda}(\,\ddot{\theta}+3H\dot{\theta}\,) &= 0\,,\\ (\ddot{\theta}+3H\dot{\theta})+\lambda\phi_r^2\sin(2\theta)-\frac{1}{\Lambda}(\,\ddot{\Phi}+3H\dot{\Phi}\,) &= 0\,.\end{aligned}$$

Rescaling coefficients by $\left(1-\dfrac{\phi_r^2}{\Lambda^2}\right)$ and rearranging, in reheating epoch,

$$\begin{aligned}\left(\ddot{\Phi}+\frac{2}{t}\,\dot{\Phi}\right)+\left(\tilde{\Gamma}\,\dot{\Phi}+\tilde{\mu}^2\Phi\right)+\frac{\tilde{\lambda}\phi_r^4}{\Lambda}\sin(2\theta) &= 0\,,\\ \left(\ddot{\theta}+\frac{2}{t}\,\dot{\theta}\right)+\frac{1}{\Lambda}\left(\tilde{\Gamma}\,\dot{\Phi}+\tilde{\mu}^2\Phi\right)+\tilde{\lambda}\phi_r^2\sin(2\theta) &= 0\,.\end{aligned}$$

Behaviour of the Inflaton

Want $\Phi(t)$ to be unaffected by the dynamics of θ.

Require that $\mu^2\Phi(t) \gg \frac{\lambda\phi_r^4}{\Lambda}\sin(2\theta)$.

$$\ddot{\Phi} + \left(3H + \tilde{\Gamma}\right)\dot{\Phi} + \tilde{\mu}^2\Phi \; = \; 0 \, .$$

When $\tilde{\Gamma} \ll \tilde{\mu}$, the approximate solution to this equation is

$$\frac{\Phi(t)}{\Phi_i} \; \simeq \; \left(\frac{t_i}{t}\right) e^{-\Gamma(t-t_i)/2} \cos\left[\mu(t-t_i)\right] \, ,$$

We now require that $\phi_r^2/\Lambda^2 \ll 1 \quad \Rightarrow \quad \tilde{\lambda} \approx \lambda$, $\tilde{\mu} \approx \mu$, and $\tilde{\Gamma} \approx \Gamma$.

Behaviour of the θ

The EoM for θ is,

$$\ddot{\theta} + (\Gamma_\theta + 3H)\,\dot{\theta} + \lambda\phi_r^2 \sin(2\theta) + \frac{1}{\Lambda}\left(\Gamma\dot{\Phi}(t) + \mu^2\Phi(t)\right) \;=\; 0\,.$$

where we have added a decay term for θ, defined by Γ_ϕ.

Utilising the solution to the inflaton EoM, and neglecting Φ decay term,

$$\ddot{\theta} + (\Gamma_\theta + 3H)\,\dot{\theta} + p\sin(2\theta) + q(t)\cos\big[\mu(t - t_i)\big] \;=\; 0\,,$$

where

$$p \;=\; \lambda\phi_r^2\,, \qquad q(t) \;=\; \frac{\mu^2\Phi_i}{\Lambda}\left(\frac{t_i}{t}\right)e^{-\Gamma(t-t_i)/2}\,.$$

Possible Cases

$p \ll q(t)$,

$$\left(\ddot{\theta} + 3H\dot{\theta}\right) = \frac{1}{t^2}\frac{d}{dt}\left(t^2\dot{\theta}\right) = -q(t)\cos\left[\mu(t - t_i)\right],$$

which can be integrated to yield,

$$\dot{\theta}(t) = \left(\frac{\mu^2\Phi_i}{\Lambda}\right)\frac{t_i}{t^2}\left[e^{-\Gamma(t-t_i)/2}\left\{\cos\left[\mu(t - t_i)\right] + \mu t \sin\left[\mu(t - t_i)\right]\right\}\right] + \cdots$$

$\dot{\theta}$ simply oscillates around zero, because the X violation has vanished.

$p \gg q(t)$,

$$\ddot{\theta} + 3H\dot{\theta} + p\sin(2\theta) = 0$$

Friction term damps $\dot{\theta}$ until θ settles into a minima $\Rightarrow$ no persistent non-zero $\dot{\theta}$, as $\mathcal{C}$ and $\mathcal{CP}$ breaking term ignored.

Sweet Spot Condition and Driven Motion

We need $p \simeq q(t) \Rightarrow X$, $\mathcal{C}$ and $\mathcal{CP}$ violating terms all contribute.

$$\text{SSC:} \;\; \lambda\phi_r^2 \;\simeq\; \frac{\mu^2\Phi_i}{\Lambda}\frac{H_d}{H_i}$$

We thus obtain the following simplified equation:

$$\ddot{\theta} + (\Gamma_\theta + 3H_d)\dot{\theta} + p\sin(2\theta) \;=\; -q(t_d)\cos\left[\mu(t - t_i)\right] \;,$$

This is analogous to a forced pendulum where

- LHS represents acceleration, damping, and gravitation,
- RHS is a sinusoidal driving torque with amplitude q and frequency μ.

Nature of the Driven Motion

Phase Locked States

- Solutions increasing monotonously in time with small amplitude modulations.
- Known as "phase-locked states" in the study of chaotic behaviour of the forced pendulum.
- Studies of electric current passing through Josephson junctions.

Reparameterise to:

$$\Theta \equiv 2\theta\,, \quad \tau \equiv \sqrt{2p}\left[(t-t_i)-\frac{\pi}{\mu}\right]\,, \quad \omega \equiv \frac{\mu}{\sqrt{2p}}\,, \quad Q \equiv \sqrt{\frac{p}{2}}\,t_d$$

The EoM becomes,

$$\ddot{\Theta}+\frac{1}{Q}\dot{\Theta}+\sin\Theta \;=\; \gamma\cos(\omega\tau)\;,$$

where, $\gamma \equiv \dfrac{q(t_d)}{p}$.

Solution and X Number Density

The generic phase-locked state solution to the above equation has the form

$$\Theta(\tau) \;=\; \Theta_0 + n\omega\tau - \sum_{m=1}^{\infty} \alpha_m \sin(m\omega\tau - \phi_m)\;,$$

where (n, m) are integers.

For these solutions, we can calculate the X number density n_X as

$$\begin{aligned} n_X &= \phi_r^2\,\langle\dot{\theta}\rangle = \sqrt{\frac{p}{2}}\phi_r^2\langle\dot{\Theta}\rangle = \sqrt{\frac{p}{2}}\phi_r^2 n\omega \\ &= \left(\mu\phi_r^2\right)\frac{n}{2}. \end{aligned}$$

where $n/2$ is the number of rotations of the phase θ per oscillation of Φ.

Generated Asymmetry Parameter

We assume that there is no additional production of entropy after reheating

$$\eta^{\mathrm{reh}} = \frac{n_X}{s} \approx 0.01 n \times \left(\frac{\mu \phi_r^2}{T_{\mathrm{reh}}^3}\right)\left(\frac{a_d}{a_{rh}}\right)^3 .$$

- Using numerical calculations it is possible to determine the generated baryon asymmetry.
- n depends on the validity of the phase-locked state and its stability, so can be determined by numerical simulations.

Now to attempt to approximate n analytically.

Estimation of n

Starting with the EoM for θ in the simplified form, where the friction term has been dropped, and the SSC satisfied,

$$\ddot{\theta} + \lambda\phi_r^2 \left(\sin(2\theta) + \cos(\mu t)\right) = 0 \ ,$$

Reparametrising using $\tau = \mu t$ and $\xi = 2\theta \frac{\mu^2}{2\lambda\phi_r^2}$,

$$\xi'' + \sin\left(\frac{2\lambda\phi_r^2}{\mu^2}\xi\right) + \cos(\tau) = 0 \ ,$$

Let $n_0 = \frac{2\lambda\phi_r^2}{\mu^2}$,

$$\xi'' + \sin\left(n_0\xi\right) + \cos(\tau) = 0 \ .$$

Assume that directed motion is present and of the form $\xi \propto \tau + \ldots$.

Estimation of n

Assume the oscillating terms of ξ are sub-dominant,

$$\xi'' + \sin(n_0\tau) + \cos(\tau) = 0 \ ,$$

The above equation of motion is easily solved and has the general solution,

$$\xi = a + b\tau + \frac{\sin(n_0\tau)}{n_0^2} + \cos(\tau) \ ,$$

Consistent with our initial assumptions,

$$< \xi' >= 1 \quad \Rightarrow \quad < \dot{\theta} >= \frac{\lambda\phi_r^2}{\mu} \quad \Rightarrow \quad n = \frac{2\lambda\phi_r^2}{\mu^2} \ ,$$

which is consistent with numerical calculations.

Now $n > 1 \quad \Rightarrow \quad \phi_r > \sqrt{\frac{1}{2\lambda}}\mu$, as a requirement for driven motion.

Dynamics of θ after SSC Violated

- Driven motion ends $\rightarrow$ friction term pushes θ towards the minima,
- SSC violated $\Rightarrow$ ϕ can't produce a non-zero n,
- θ oscillates around minima due to inflaton,
- If we assume $\sin\theta \approx \theta$ after the driven motion has ended,
- Can approximate the amplitude of oscillations as,

$$\sin 2\theta \approx \frac{H}{H_d} \ ,$$

- After the SSC is violated there is no net production or washout of the X asymmetry; cannot realise simultaneous violation of $\mathcal{C}$, $\mathcal{CP}$ and X.

The Generated Asymmetry

Allowed Parameter Ranges

Maximum scalar baryon energy density,

$$3M_p^2H^2 \gg \lambda\phi_r^4 \sin^2\theta \ .$$

Utilising the SSC, this gives,

$$\lambda \gg 8 \times 10^{-6} \ .$$

Require Φ is unaffected by θ,

$$\mu^2\Phi(t) \gg \frac{\lambda\phi_r^4}{\Lambda}\frac{H}{H_d} \quad \Rightarrow \quad \frac{\phi_r^2}{\Lambda^2} \ll 1 \ .$$

Allowed range for ϕ_r,

$$10^{15}\ \mathrm{GeV} > \phi_r > \mu/\sqrt{2\lambda} \quad \Rightarrow \quad 2 \times 10^3\lambda > n > 1 \ .$$

Limits on H_d ,

$$H_0 > H_d > 2 \times 10^{10}\ \mathrm{GeV} \quad \Rightarrow \quad 310 > n > 1 \ .$$

Estimation of the Generated Asymmetry

Generated asymmetry,

$$\eta^{\rm reh} \; = \; \frac{n_X}{s} \; \approx \; 0.02 \left(\frac{\lambda \phi_r^4}{\mu T_{\rm reh}^3} \right) \left(\frac{a_d}{a_{rh}} \right)^3 \; .$$

where the dilution factor is given by,

$$\left(\frac{a_d}{a_{rh}} \right)^3 = \left(\frac{\pi^2 g_*}{90} \right) \left(\frac{T_{rh}^4}{H_d^2 M_p^2} \right) \; .$$

Using the SSC, $H_d = \frac{\lambda \phi_r^2 H_0 \Lambda}{\mu^2 \Phi_0} \simeq 2n \times 10^{10} \ \mathrm{GeV}$,

$$\eta = 7 \times 10^{-11} \left(\frac{T_{rh}}{\lambda \times 10^8 \ \mathrm{GeV}} \right) \; .$$

Coupling ϕ to the SM

- Interpret the charge as lepton number,
- Introduce the lepton-number preserving dimension four interaction,

$$\Delta\mathcal{L}_{\rm int} = y_L \phi^* \bar{\nu}_R^c \nu_R + \mathrm{h.c.}$$

  describing the decay of the complex scalar lepton into a $\nu_R \nu_R$ pair.

- Can generate a large Majorana mass when $\langle\phi\rangle = \phi_r/\sqrt{2}$, depending on the value of y_L.
- The leptonic scalar identified as part of the seesaw mechanism.

Coupling ϕ to the SM

- Interpret as a baryonic scalar,
- Introduce the following interaction,

$$\Delta\mathcal{L}_{\rm int} = \frac{i}{\Lambda^2}(\phi^* \overleftrightarrow{\partial}_\mu \phi)\ \frac{1}{3}(\bar{u}\gamma^\mu u + \bar{d}\gamma^\mu d)\ ,$$

which becomes,

$$\Delta\mathcal{L}_{\rm int} = -\frac{\phi_r^2}{\Lambda^2}\dot{\theta}\ \frac{1}{3}(u^\dagger u + d^\dagger d)\ ,$$

- Analogous to a chemical potential coupling to the baryonic current, shifting in favour of matter or antimatter, where $\mu_B = -\frac{\phi_r^2}{\Lambda^2}\dot{\theta}$.

Conclusion and Future Work

- Interplay between inflaton and scalar baryon/lepton during reheating,
- Driven motion can be modelled as a forced pendulum,
- Presence of phase-locked states,
- Asymmetry linearly dependent on the reheating temperature.

Future work

- Further exploration of the mechanism
- Investigation of efficiency required.
- Other cosmological implications