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Neutron-Antineutron Oscillations: Discrete Symmetries and Quark Operators

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Search for the neutron-antineutron oscillations was suggested by Vadim Kuzmin in 1970, and such experiments are under active discussion now, see D. G. Phillips, II *et al.*, Phys. Rept. **612**, 1 (2016)

This is a transition where the baryon charge $\mathcal{B}$ is changed by two units. The observation of the transition besides demonstration of the baryon charge non-conservation could be also important for explanation of baryogengesis. Of course, following Sakharov conditions, it should be also accompanied by CP non-conservation.

Thus, discrete symmetries associated with neutron-antineutron mixing are of real interest.

C, P and T symmetries in $|\Delta\mathcal{B}| = 2$ transitions

In our 2015 text Zurab Berezhiani, AV, arXiv:1506.05096
we noted that the parity $\mathbf{P}$, defined in such a way that $\mathbf{P}^2 = 1$, is broken in n-nbar transition as well as $\mathbf{CP}$.
Indeed, eigenvalues of parity $\mathbf{P}$ are ± 1 and opposite for neutron and antineutron. So, n-nbar mixing breaks $\mathbf{P}$.

We noted, however, that it does not automatically imply an existence of CP breaking in absence of interaction.
In September of the same 2015 we presented at the INT workshop in Seattle a modified definition of parity $\mathbf{P}_z$, such that $\mathbf{P}_z^2 = -1$, and parities $\mathbf{P}_z$ are i for both, neutron and antineutron. With this modification all discrete symmetries are preserved in n-nbar transition.

The issue of discrete symmetries was later discussed in few papers in 2016:

K. Fujikawa and A. Tureanu

D. McKeen and A. E. Nelson

S. Gardner and X. Yan,

In particular, Nelson and MaKeen insisted, incorrectly, that one can keep $\mathbf{P^2 = 1}$. Fujikawa and Tereanu also used it, while Gardner and Yan did it right with $\mathbf{P}_z^{\mathbf{2}} = -\mathbf{1}$, following Wolfenstein's and Kayser's applications to neutrino case.

I will present some details of an interesting history of the subject which goes back to Majorana and Racah's papers of 1937.

Dirac Lagrangian for neutron

$$\mathcal{L}_D = i\bar{n}\gamma^\mu\partial_\mu n - m\,\bar{n}n$$

describes free neutron and antineutron and preserves the baryon charge, $\mathcal{B} = 1$ for n, $\mathcal{B} = -1$ for $\bar{n}$.

Continuous $\mathrm{U}(1)_\mathcal{B}$ symmetry:

$$n \to \mathrm{e}^{i\alpha} n, \quad \bar{n} \to \mathrm{e}^{-i\alpha}\bar{n}$$

Another term $-im'\bar{n}\gamma_5 n$ consistent with $\mathcal{B}$ conservation can be rotated away by the chiral rotation, $n \to \mathrm{e}^{i\beta\gamma_5} n$.

Four degenerate states: two spin doublets differ by $\mathcal{B}$.

How does baryon number non-conservation shows up?

At the level of free particles it could be only bilinear $|\Delta\mathcal{B}| = 2$ mass terms:

$$C = i\gamma^2\gamma^0$$

$$n^T C n\,, \quad n^T C\gamma_5 n\,, \quad \bar{n}C\bar{n}^T, \quad \bar{n}C\gamma_5\bar{n}^T$$

At these bilinear in fields the most generic Lorentz invariant modifications reduce by field redefinitions to the only one term, breaking baryon charge by two units,

$$\Delta\mathcal{L}_{\not{B}} = -\frac{1}{2}\,\epsilon\left[n^T C n + \bar{n} C \bar{n}^T\right] \qquad\qquad C = i\gamma^2\gamma^0$$

where ϵ is a real positive parameter. Redefinitions are due to U(2) symmetry of the kinetic term $i\bar{n}\gamma^\mu\partial_\mu n$

What is the status of $\mathbf{C}$, $\mathbf{P}$ and $\mathbf{T}$ discrete symmetries?
Let us start with the charge conjugation $\mathbf{C}$:

$$\mathbf{C}: \quad n \longleftrightarrow n^c = C\bar{n}^T$$

Kind of Z_2 symmetry, $\mathbf{C}^2 = 1$. Most simple in the Majorana representation

$$n^c = n^*\,.$$

Lagrangians can be rewritten as

$$\mathcal{L}_D = \frac{i}{2}\left[\bar{n}\gamma^\mu\partial_\mu n + \overline{n^c}\gamma^\mu\partial_\mu n^c\right] - \frac{m}{2}\left[\bar{n}n + \overline{n^c}n^c\right],$$

$$\Delta\mathcal{L}_{\not B} = -\frac{1}{2}\,\epsilon\left[\overline{n^c}\,n + \bar{n}\,n^c\right],$$

what makes C-invariance explicit.

Lagrangians are diagonalized in terms of Majorana fields $n_{1,2}$

$$n_{1,2} = \frac{n \pm n^c}{\sqrt{2}}, \qquad \mathrm{C}n_{1,2} = \pm\, n_{1,2}.$$

$$\mathcal{L}_D = \frac{1}{2}\sum_{k=1,2}\left[\bar{n}_k\gamma^\mu\partial_\mu n_k - m\,\bar{n}_k n_k\right],$$

$$\Delta\mathcal{L}_{\not B} = -\frac{1}{2}\,\epsilon\left[\bar{n}_1\,n_1 - \bar{n}_2\,n_2\right].$$

Splitting into two Majorana spin doublets with masses

$$M_1 = m + \epsilon \qquad\qquad M_2 = m - \epsilon.$$

The parity transformation $\mathbf{P}$ involves, besides reflection of space coordinates, the substitution

$$\mathbf{P}: \qquad n \to \gamma^0 n\,, \qquad n^c \to -\gamma^0 n^c\,.$$

We use $\gamma^0 C \gamma^0 = -C$. The opposite signs reflect the opposite parities of fermion and antifermion C.N. Yang '50 V.B. Berestetsky '51

The definition satisfies $\mathbf{P}^2 = 1$ so eigenvalues of $\mathbf{P}$ are ± 1, opposite parities for fermion and antifermion. Different parities of neutron and antineutron implies that their mixing breaks $\mathbf{P}$ parity. Indeed, $\mathbf{P}$ -transformation changes $\Delta\mathcal{L}_{\not{B}}$ to $(-\Delta\mathcal{L}_{\not{B}})$. With $\mathbf{C}$ -invariance it implies that $\Delta\mathcal{L}_{\not{B}}$ is also $\mathbf{CP}$ odd.

This CP-oddness, however, does not translates immediately into observable CP-breaking effects. To get them one needs an interference of amplitudes provided only by interaction.

This subtlety is discussed in number of textbooks, see e.g. V. B. Berestetsky, E. M. Lifshitz and L. P. Pitaevsky,

Let's remind it.

When $\mathcal{B}$ is conserved there is there is no transition between sectors with different $\mathcal{B}$. One can combine $\mathbf{P}$ with a $\mathrm{U(1)}_{\mathcal{B}}$ phase rotation and define $\mathbf{P}_\alpha$

$$\mathbf{P}_\alpha = \mathbf{P}\,\mathrm{e}^{i\mathcal{B}\alpha}: \quad n \to \mathrm{e}^{i\alpha}\gamma^0 n\,, \quad n^c \to -\mathrm{e}^{-i\alpha}\gamma^0 n^c$$

Of course, then $\mathbf{P}_\alpha^2 = \mathrm{e}^{2i\mathcal{B}\alpha} \neq 1$ but the phase is unobservable while $\mathcal{B}$ is is conserved.

When $\mathcal{B}$ is its not conserved the only remnant of $\mathrm{U}(1)_{\mathcal{B}}$ rotations is Z_2 symmetry, $n \to -n$. It means that we can consider a different parity definition P_z, such that $\mathrm{P}_z^2 = -1$.

Thus, choosing $\alpha = \pi/2$ we come to

$$\mathrm{P}_z = \mathrm{P}e^{iB\pi/2} : \qquad n \to i\gamma^0 n\,, \qquad n^c \to i\gamma^0 n^c\,.$$

Moreover, in case of Majorana fermions it is the only possible choice. Indeed, in Majorana representation where

$$\gamma^0 = \begin{pmatrix} 0 & \sigma_2 \\ \sigma_2 & 0 \end{pmatrix}$$

only $i\gamma^0$ preserves reality of the Majorana spinor. Also P_z preserves the Majorana structure of $n_{1,2}$ fields, $n_{1,2} = \dfrac{n \pm n^c}{\sqrt{2}}$,

This was derived by Ettore Majorana and Giulio Racah in 1937.

Now $\mathbf{P}_z$ parities of n and $\bar{n}$ are the same $\cdot i$, so their mixing does not break the $\mathbf{P}_z$ parity. It means that all discrete symmetries, $\mathbf{C}$, $\mathbf{P}_z$ and $\mathbf{T}$ are preserved by $\Delta\mathcal{L}_{\not B}$.

A few comments. First, preservation of $\mathbf{T}$ follows from $\mathbf{CPT}$ theorem provided by Lorentz invariance and locality. Second, it is amusing that the same parity for n and n^c equal to $\cdot i$ is consistent with the notion of the opposite parities for fermion and antifermion: the product of their parities is (-1). Third, $\mathbf{P}_z$ commutes with $\mathbf{C}$, i.e. $\mathbf{CP}_z = \mathbf{P}_z\mathbf{C}$, in contrast with $\mathbf{P}$ which anticommutes, $\mathbf{CP} = -\mathbf{PC}$

Similar effects for neutrino were noted by Wolfenstein '81. as well as by Kayser '82.

Weyl spinor description

Two right-handed Weyl spinors, forming a flavor doublet,

$$\psi^{i\,\alpha}\,, \qquad i = 1,2, \quad \alpha = 1,2\,,$$

together with with their two complex conjugates, left-handed spinors,

$$\bar\psi_i^{\dot\alpha} = (\psi^{i\,\alpha})^*\,, \qquad i = 1,2, \quad \dot\alpha = 1,2$$

In the chiral basis four-component neutron spinor is

$$n = \begin{pmatrix} \psi^1 \\ -i\sigma^2(\psi^2)^* \end{pmatrix} = \begin{pmatrix} \psi^{1\alpha} \\ \bar\psi_{2\dot\alpha} \end{pmatrix}, \quad n^c = \begin{pmatrix} \psi^{2\alpha} \\ \bar\psi_{1\dot\alpha} \end{pmatrix}$$

The most general Lorentz invariant Lagrangian, quadratic in fields is

$$\mathcal{L} = \bar\psi_i^{\dot\alpha}\, i\,\partial_{\alpha\dot\alpha}\,\psi^{i\,\alpha} - \frac{1}{2}\left[m_{ik}\psi^{i\alpha}\psi_\alpha^k + \overline{m}^{ki}\,\bar\psi_{k\,\dot\alpha}\,\bar\psi_i^{\dot\alpha}\right]$$

$$\partial_{\alpha\dot\alpha} = \sigma^\mu_{\alpha\dot\alpha}\partial_\mu\,,\ \sigma^\mu = \{1, \vec\sigma\} \qquad m_{ik} = m_{ki} \qquad \overline{m}^{ik} = (m_{ik})^*$$

The symmetry of the kinetic term is apparently U(2). It is broken by mass terms. Mass matrix m_{ik} can be viewed as transforming under U(2). Its overall phase rotation related to U(1). Under SU(2) it is the adjoint, i.e., isovector, $\mu^a,\ a=1,2,3,$

$$m^i_k = \varepsilon^{ij} m_{jk} = \mu^a (\tau^a)^i_k\,, \quad a=1,2,3$$

μ^a is complex, two real isovectors $\{\mathrm{Re}\,\mu^a,\ \mathrm{Im}\,\mu^a\}$

We can can orient mass matrix

$$m_{ik} = \begin{pmatrix} -\mu^1 - i\mu^2 & \mu^3 \\ \mu^3 & \mu^1 - i\mu^2 \end{pmatrix}$$

in the convenient way: $\mu^1 = 0,\ \mathrm{Re}\,\mu^2 = 0,\ \mathrm{Im}\,\mu^3 = 0.$

$$\widehat{m}_0 = \begin{pmatrix} \mathrm{Im}\,\mu^2 & \mathrm{Re}\,\mu^3 \\ \mathrm{Re}\,\mu^3 & \mathrm{Im}\,\mu^2 \end{pmatrix} = \begin{pmatrix} \epsilon & m \\ m & \epsilon \end{pmatrix}$$

It finishes the proof of generality.

Thus, we show that a generic mass matrix can be rotated to the standard form $\widehat{m}_0$ by a certain U(2) transformation V ,

$$\widehat{m}_0 = V^T \widehat{m} V$$

How discrete transformations look in the Weyl description? Let us start with the charge conjugation $\mathbf{C}$:

$$\mathbf{C}: \quad \psi^{1\,\alpha} \longleftrightarrow \psi^{2\,\alpha}, \quad \bar{\psi}_{1\dot{\alpha}} \longleftrightarrow \bar{\psi}_{2\dot{\alpha}}$$

In terms of U(2) transformations it can written as

$$\mathbf{C}: \quad \psi \to U_C\,\psi, \quad U_C = \mathrm{e}^{-i\pi/2}\mathrm{e}^{i\pi\tau^1/2} = \tau^1$$

In generic basis U_C becomes $U_C = V\tau^1 V^\dagger$. Thus, $\mathbf{C}$ is a component of Z_2- the discrete survivor of broken U(2),

$$U_C^T \widehat{m} U_C = \widehat{m}$$

Parity transformation, besides inversion of spatial coordinates, acts as

$$\mathbf{P}_z: \quad \psi^{1\alpha} \to i\,\bar{\psi}_{2\dot{\alpha}}\,, \quad \psi^{2\alpha} \to i\,\bar{\psi}_{1\dot{\alpha}},$$
$$\bar{\psi}_{1\dot{\alpha}} \to i\psi^{2\alpha}\,, \quad \bar{\psi}_{2\dot{\alpha}} \to i\psi^{1\alpha}\,.$$

Again, it can be written as

$$\mathbf{P}_z: \quad \psi \to i\,\bar{\psi}\,U_P\,, \quad \bar{\psi} \to i\,U_P^\dagger\,\psi\,,$$

with $U_P = \tau^1$ for the special basis and $U_P = V\,\tau^1\,V^T$ for arbitrary one. Clearly, $\mathbf{P}_z^2 = -1$.

For $\mathbf{CP}_z$, $\mathbf{T}$, $\mathbf{CP}_z\mathbf{T}$ we get

$$\mathbf{CP}_z: \; \psi \to i\,\bar{\psi}U_C^\dagger U_P = i\,\bar{\psi}VV^T, \qquad \bar{\psi} \to U_P^\dagger U_C\psi = iV^*V^\dagger\psi\,.$$

$$\mathbf{T}: \quad \psi \to \bar{\psi}VV^T, \quad \bar{\psi} \to -V^*V^\dagger\psi^{i\alpha}\,,$$

$$\mathbf{CP}_z\mathbf{T}: \quad \psi^{1\alpha} \to i\psi^{i\alpha}, \quad \bar{\psi}_{i\alpha} \to -i\,\bar{\psi}_{i\alpha}\,.$$

with $V = 1$ for the special basis of the mass matrix.

Six-quarks operators: discrete symmetries

New physics beyond the Standard Model, leading to $|\Delta B| = 2$ transitions, induces the effective six-quark interaction,

$$\mathcal{L}\,(\Delta\mathcal{B} = -2) = \frac{1}{M^5}\sum c_i \mathcal{O}^i\,,$$

$$\mathcal{O}^i = T^i_{A_1 A_2 A_3 A_4 A_5 A_6} q^{A_1} q^{A_2} q^{A_3} q^{A_4} q^{A_5} q^{A_6}\,,$$

where coefficients T^i account for color, flavor and spinor structures.

In particular, for n-nbar mixing

$$\langle\bar{n}|\,\mathcal{L}\,(\Delta\mathcal{B} = -2)\,|n\rangle = -\frac{1}{2}\,\epsilon\, v^T_{\bar{n}} C\, u_n$$

it lead to an estimate .

$$\epsilon = \frac{1}{\tau_{n\bar{n}}} \sim \frac{\Lambda^6_{\rm QCD}}{M^5}\,.$$

For u and d quarks of the first generation the full list of operators was determined S. Rao and R. Shrock, W. E. Caswell, J. Milutinovic and G. Senjanovic

$$\mathcal{O}^1_{\chi_1\chi_2\chi_3} = u^{iT}_{\chi_1} C u^j_{\chi_1} d^{kT}_{\chi_2} C d^l_{\chi_2} d^{mT}_{\chi_3} C d^n_{\chi_3} \left[\epsilon_{ikm}\epsilon_{jln} + \epsilon_{ikn}\epsilon_{jlm} + \epsilon_{jkm}\epsilon_{nil} + \epsilon_{jkn}\epsilon_{ilm}\right],$$

$$\mathcal{O}^2_{\chi_1\chi_2\chi_3} = u^{iT}_{\chi_1} C d^j_{\chi_1} u^{kT}_{\chi_2} C d^l_{\chi_2} d^{mT}_{\chi_3} C d^n_{\chi_3} \left[\epsilon_{ikm}\epsilon_{jln} + \epsilon_{ikn}\epsilon_{jlm} + \epsilon_{jkm}\epsilon_{nil} + \epsilon_{jkn}\epsilon_{ilm}\right],$$

$$\mathcal{O}^3_{\chi_1\chi_2\chi_3} = u^{iT}_{\chi_1} C d^j_{\chi_1} u^{kT}_{\chi_2} C d^l_{\chi_2} d^{mT}_{\chi_3} C d^n_{\chi_3} \left[\epsilon_{ijm}\epsilon_{kln} + \epsilon_{ijn}\epsilon_{klm}\right].$$

Here χ_i stand for L or R quark chirality. Accounting for relations

$$\mathcal{O}^1_{\chi LR} = \mathcal{O}^1_{\chi RL}, \quad \mathcal{O}^{2,3}_{LR\chi} = \mathcal{O}^{2,3}_{RL\chi},$$

$$\mathcal{O}^2_{\chi\chi'} - \mathcal{O}^1_{\chi\chi'} = 3\mathcal{O}^3_{\chi\chi'},$$

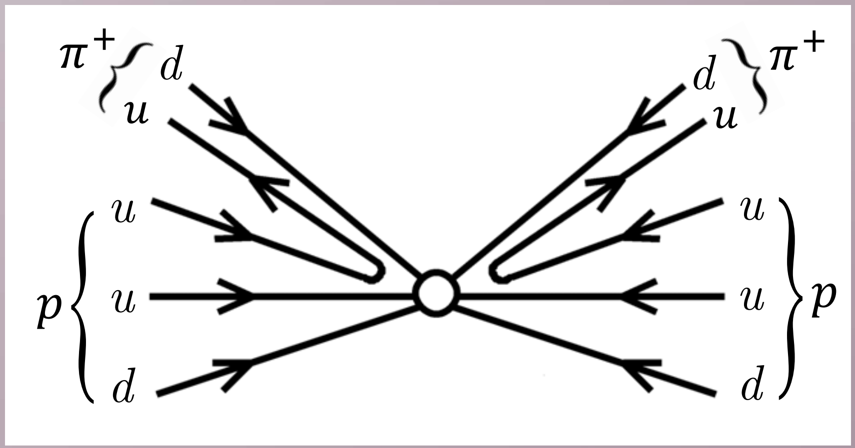
we deal with 14 operators for $\Delta\mathcal{B} = -2$ transitions.

Only combinations of operators which are $\mathbf{P}_z$ even contributes to n-nbar mixing. The $\mathbf{P}_z$ reflection interchanges L and R chiralities χ_i in the operators $O^i_{\chi_1\chi_2\chi_3}$.
Thus, only 7 combinations

$$O^i_{\chi_1\chi_2\chi_3} + L \leftrightarrow R$$

of 14 operators contribute to n-nbar mixing.

What about remaining $\mathbf{P}_z$ odd combinations $(O^i_{\chi_1\chi_2\chi_3} - L \leftrightarrow R)$? Although they do not contribute to n-nbar mixing their effect shows up in instability of nuclei. This source of instability is due to two nucleon annihilation into pions.



The charge conjugation $\mathbf{C}$ transforms operators $O^i_{\chi_1\chi_2\chi_3}$ into Hermitian conjugated $[O^i_{\chi_1\chi_2\chi_3}]^\dagger$. So, we have 14 C-even operators, $O^i_{\chi_1\chi_2\chi_3} + \mathbf{H.c.}$, and 14 C-odd ones, $O^i_{\chi_1\chi_2\chi_3} - \mathbf{H.c.}$

In total, we break all 28 operators in four sevens with different $\mathbf{P}_z$, $\mathbf{C}$ and $\mathbf{CP}_z$ features,

$$[O^i_{\chi_1\chi_2\chi_3} + L \leftrightarrow R] + \mathbf{H.c.}, \quad \mathbf{P}_z = +, \ \mathbf{C} = +, \ \mathbf{CP}_z = +$$

$$[O^i_{\chi_1\chi_2\chi_3} + L \leftrightarrow R] - \mathbf{H.c.}, \quad \mathbf{P}_z = +, \ \mathbf{C} = -, \ \mathbf{CP}_z = -$$

$$[O^i_{\chi_1\chi_2\chi_3} - L \leftrightarrow R] + \mathbf{H.c.}, \quad \mathbf{P}_z = -, \ \mathbf{C} = +, \ \mathbf{CP}_z = -$$

$$[O^i_{\chi_1\chi_2\chi_3} - L \leftrightarrow R] - \mathbf{H.c.}, \quad \mathbf{P}_z = -, \ \mathbf{C} = -, \ \mathbf{CP}_z = +$$

Only the first seven which are both $\mathbf{P}_z$ and $\mathbf{C}$ even contribute to n-nbar mixing.

Conclusions

We demonstrate that Lorentz and CPT invariance lead to the unique $|\Delta B| = 2$ operator in the neutron-antineutron mixing. This operator preserves all discrete symmetries, C, P and T.

The subtlety is that P should be defined as $\mathbf{P}_z$ with $\mathbf{P}_z^2 = -1$. Then, parities of both, neutron and and antineutron, are the same i, and their mixing is consistent with conservation of parity.

Our classification of $|\Delta B| = 2$ operators coming from new physics could be useful in association with Sakharov conditions for theory of baryogenisis.