

Supersymmetric Higgs-Sneutrino Inflation and Reheating

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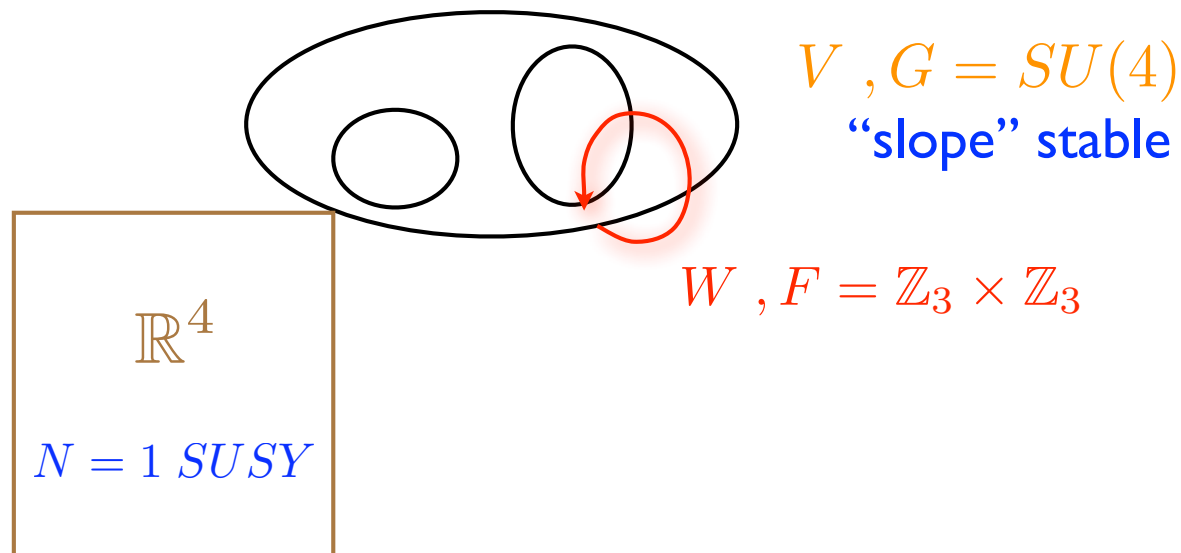
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Brief Review of a Realistic String Vacuum:

SU(4) Heterotic Compactification:

$X, D = 6$ “Schoen” CY



\mathbb{R}^4 Theory Gauge Group:

$$G = SU(4) \Rightarrow E_8 \rightarrow Spin(10)$$

Choose the $\mathbb{Z}_3 \times \mathbb{Z}_3$ Wilson lines to be

$$\chi_{T_{3R}} = e^{iY_{T_{3R}} \frac{2\pi}{3}}, \quad \chi_{B-L} = e^{iY_{B-L} \frac{2\pi}{3}}$$

where

$$Y_{B-L} = 2(H_1 + H_2 + H_3) = 3(B - L)$$
$$Y_{T_{3R}} = H_4 + H_5 = 2(Y - \frac{1}{2}(B - L)) = 2T_{3R}$$

arise “naturally” and is called the “**canonical basis**”. \Rightarrow

$$Spin(10) \rightarrow SU(3)_C \times SU(2)_L \times U(1)_{T_{3R}} \times U(1)_{B-L}$$

\mathbb{R}^4 Theory Spectrum:

$$n_r = (h^1(X, U_R(V)) \otimes \mathbf{R})^{\mathbb{Z}_3 \times \mathbb{Z}_3} \Rightarrow \text{3 families of quarks/leptons}$$

$$Q = (U, D)^T = (3, 2, 0, \frac{1}{3}), \quad u = (\bar{3}, 1, -\frac{1}{2}, -\frac{1}{3}), \quad d = (\bar{3}, 1, \frac{1}{2}, -\frac{1}{3})$$
$$L = (N, E)^T = (1, 2, 0, -1), \quad \underline{\nu = (1, 1, -\frac{1}{2}, 1)}, \quad e = (1, 1, \frac{1}{2}, 1)$$

and **1** pair of Higgs-Higgs conjugate fields

$$H = (1, 2, \frac{1}{2}, 0), \quad \bar{H} = (1, 2, -\frac{1}{2}, 0)$$

under $SU(3)_C \times SU(2)_L \times U(1)_{T_{3R}} \times U(1)_{B-L}$.

Wilson Line Breaking:

$\pi_1(X/(\mathbb{Z}_3 \times \mathbb{Z}_3)) = \mathbb{Z}_3 \times \mathbb{Z}_3 \Rightarrow$ 2 independent classes of non-contractible curves. \Rightarrow each Wilson line has a mass scale $M_{\chi_{T_{3R}}}$, $M_{\chi_{B-L}}$. At a generic region of moduli space

$$M_{\chi_{T_{3R}}} \simeq M_{\chi_{B-L}} (\simeq M_U)$$

which we henceforth assume. We find that

$$M_U = 3.15 \times 10^{16} \text{ GeV}$$

At this scale, we statistically scatter the 24 soft supersymmetry parameters in the range $(\frac{M}{f}, Mf)$ where $M = 2.7 \text{ TeV}$, $f = 3.3$.

The results are subjected to all present **phenomenological constraints**-- namely

A) B-L symmetry is radiatively broken at $M_{B-L} > 2.5 \text{ TeV}$

B) EW symmetry is radiatively broken at $M_Z = 91.2 \text{ GeV}$

C) The Higgs mass is given by $M_{H^0} = 125.36 \pm 0.82 \text{ GeV}$

In addition, we will **enforce** that all sparticle masses exceed their present experimental bounds. These are given by

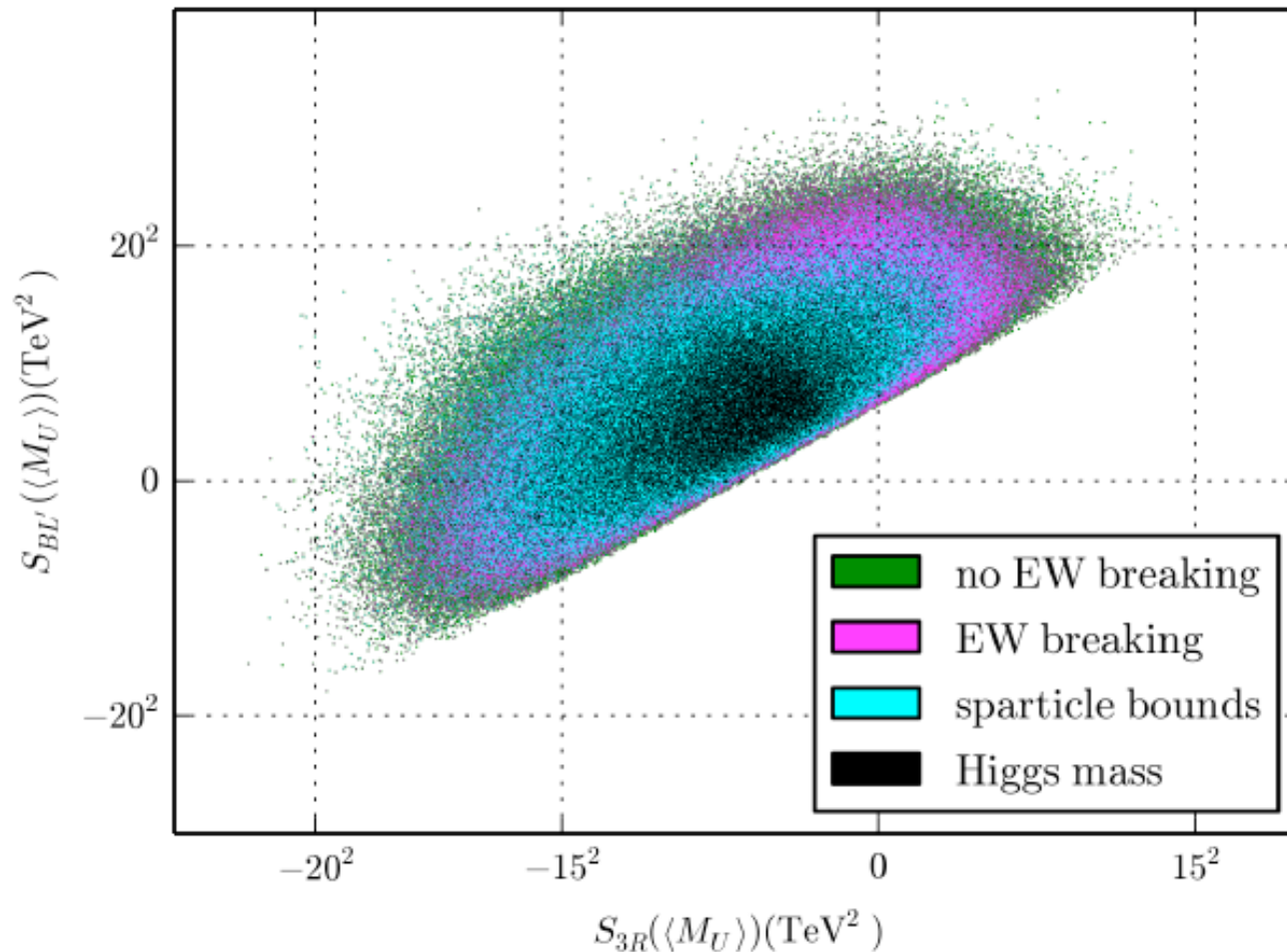
Particle(s)	Lower Bound
Left-handed sneutrinos	45.6 GeV
Charginos, sleptons	100 GeV
Squarks, except for stop or sbottom LSP's	1000 GeV
Stop LSP (admixture)	450 GeV
Stop LSP (right-handed)	400 GeV
Sbottom LSP	500 GeV
Gluino	1300 GeV
Z_R	2500 GeV

We find that most of the RGE scaling behaviour is dominated by the two parameters

$$S_{BL'} = \text{Tr} (2m_{\tilde{Q}}^2 - m_{\tilde{u}^c}^2 - m_{\tilde{d}^c}^2 - 2m_{\tilde{L}}^2 + m_{\tilde{\nu}^c}^2 + m_{\tilde{e}^c}^2) ,$$

$$S_{3R} = m_{H_u}^2 - m_{H_d}^2 + \text{Tr} \left(-\frac{3}{2}m_{\tilde{u}^c}^2 + \frac{3}{2}m_{\tilde{d}^c}^2 - \frac{1}{2}m_{\tilde{\nu}^c}^2 + \frac{1}{2}m_{\tilde{e}^c}^2 \right)$$

⇒ we can plot our results in a two-dimensional space. We find that out of **10 million** random initial points in SUSY breaking parameter space, all points that break B-L symmetry with $M_{B-L} > 2.5 \text{ TeV}$ are



Of these, there are **44,884** “valid” black points that satisfy **all phenomenological requirements**.

Sneutrino-Higgs Inflation:

Once again consider the B-L MSSM, but now as a possible framework for inflationary cosmology satisfying all Planck2015 bounds. To do this, we must couple the theory to N=1 supergravity.

Recall that the B-L MSSM arises as the observable sector of a class of vacua of heterotic M-theory. The generic coupling of such a theory to N=1 supergravity was carried out in

A. Lukas, B. A. Ovrut and D. Waldram, “On the four-dimensional effective action of strongly coupled heterotic string theory,” Nucl. Phys. B 532, 43 (1998)

The results are easily applied to the B-L MSSM. To begin, we first assume that--with the exception of the two complex “universal” geometrical moduli--all other geometric and vector bundle moduli are stabilized and sufficiently heavy to be ignored at low energy.

The real parts of the two universal moduli are the “breathing modes” of the CY and S_1/Z_2 orbifold, $a(x)$ and $c(x)$ respectively, defined in the 11-dimensional metric by

$$ds^2 = g_{\mu\nu} dx^\mu dx^\nu + e^{2a(x)} \Omega_{AB} dx^A dx^B + e^{2c(x)} (dx^{11})^2$$

In the $M_P \rightarrow \infty$ limit, these moduli also decouple from ordinary matter and can be ignored. However, for finite M_P this is no longer the case, and must be included when coupling the B-L MSSM to supergravity. Specifically, the Kahler potential is altered to

$$K = -\ln(S + \bar{S}) - 3 \ln(T + \bar{T} - \sum_i \frac{|C_i|^2}{M_P^2})$$

where the sum is over all complex scalar matter fields C_i and

$$S = e^{6a} + i\sqrt{2}\sigma, \quad T = e^{2\hat{c}} + i\sqrt{2}\chi + \frac{1}{2} \sum_i \frac{|C_i|^2}{M_P^2}$$

where $\hat{c} = c + 2a$.

Similarly, to lowest order, the gauge kinetic function is given by

$$f = S$$

Both the Kahler potential and gauge kinetic function are identical to those found in the weakly coupled heterotic string and are consistent with the requisite “new minimal” supergravity multiplet.

Putting K and f , as well as the superpotential W , into the above formalism gives the Lagrangian for the B-L MSSM coupled to $N=1$ supergravity. Henceforth, we set $M_P = 1$ and assume that both S and T are stabilized at fixed VEV's. By scaling

$$C'_i = \left(\frac{3}{T + \bar{T}}\right)^{1/2} C_i, \quad g'_a = \left(\frac{2}{S + \bar{S}}\right)^{1/2} g_a, \quad \text{for } a = 3, 2, 3R, BL'$$

both S and T can be eliminated from the Lagrangian. \Rightarrow

$$K = -3 \ln\left(1 - \sum_i \frac{|C_i|^2}{3}\right), \quad f = 1$$

Recalling that the **matter kinetic energy** terms are

$$-K_{i\bar{j}}\partial_\mu C^i\partial^\mu C^{\bar{j}}$$

\Rightarrow for small values of the fields there is **no mixing** and the KE is **canonically normalized**. However, this is **no longer true** for fields approaching the Planck scale.

Now consider the remaining parts of the Lagrangian. The purely **gravitational action** is simply

$$-\frac{1}{2}\int_{M_4}\sqrt{-g}R$$

That is

- The pure gravitational action is canonical. We do not require any “non-canonical” coupling of matter to the curvature tensor R .*

The **matter potential energy** breaks into supersymmetric **F**-terms and **D**-terms given by

$$V_F = e^K (K^{i\bar{j}} D_i W \overline{D_{\bar{j}} W} - 3|W|^2) , \quad V_D = \frac{1}{2} \sum_a D_a^2$$

where W is the B-L MSSM superpotential and $D_a, a = 3, 2, 3R, BL'$ are the functions

$$D_a^r = -g_a \frac{\partial K}{\partial C_i} [T_{(a)}^r]_i{}^j C_j = \frac{g_a}{(1 - \frac{1}{3} \sum_i |C_i|^2)} \mathcal{D}_{(a)}^r , \quad \mathcal{D}_{(a)}^r = -\overline{C}^i [T_{(a)}^r]_i{}^j C_j$$

In addition, there is a **soft SUSY breaking potential**

$$V_{soft} = (m_{Q_f}^2 |Q_f|^2 + m_{u_{R,f}}^2 |u_{R,f}|^2 + m_{d_{R,f}}^2 |d_{R,f}|^2 + m_{L_f}^2 |L_f|^2 + m_{\nu_{R,f}}^2 |\nu_{R,f}|^2 \\ + m_{e_{R,f}}^2 |e_{R,f}|^2) + m_{H_u}^2 |H_u|^2 + m_{H_d}^2 |H_d|^2 + (bH_u H_d + h.c.) + \dots$$

Given these potentials, we want to search for a “stable” valley down which a scalar field can “slow roll” and, hence, produce inflation.

To enhance the stability of the valley, we demand that it be D-flat; that is

$$V_D = 0$$

We will restrict ourselves to fields not charged under $SU(3)_C$ or $U(1)_{EM}$. \Rightarrow We are led naturally to the field configuration

$$H_u^0 = \nu_{R,3} = \nu_{L,3}$$

with all other fields set to zero. It is helpful to define three new fields ϕ_i , $i = 1, 2, 3$ using

$$H_u^0 = \frac{1}{\sqrt{3}} (\phi_1 - \phi_2 - \phi_3),$$

$$\nu_{L,3} = \frac{1}{\sqrt{3}} \phi_1 + \left(\frac{1}{2} + \frac{1}{2\sqrt{3}} \right) \phi_2 + \left(\frac{1}{2\sqrt{3}} - \frac{1}{2} \right) \phi_3$$

$$\nu_{R,3} = \frac{1}{\sqrt{3}} \phi_1 + \left(\frac{1}{2\sqrt{3}} - \frac{1}{2} \right) \phi_2 + \left(\frac{1}{2} + \frac{1}{2\sqrt{3}} \right) \phi_3$$

The ϕ_1 field corresponds to the D-flat direction while ϕ_2, ϕ_3 are orthogonal directions with positive potential. Note that

$$\phi_1 = \frac{1}{\sqrt{3}} (H_u^0 + \nu_{L,3} + \nu_{R,3})$$

with the associated soft mass squared given by

$$m^2 = \frac{1}{3} (m_{H_u^0}^2 + m_{\nu_{L,3}}^2 + m_{\nu_{R,3}}^2)$$

Setting all fields to zero except $\phi_1 \Rightarrow V_D = 0$ and the Lagrangian becomes

$$\mathcal{L} = -\frac{1}{(1 - \frac{1}{3}|\phi_1|^2)^2} \partial_\mu \bar{\phi}_1 \partial^\mu \phi_1 - V_F(\phi_1) - V_{soft}(\phi_1)$$

where

$$V_F(\phi_1) = \frac{3|\phi_1|^2 (\mu^2 + Y_{\nu 3}^2 |\phi_1|^2)}{(3 - |\phi_1|^2)^2}, \quad V_{soft}(\phi_1) = m^2 |\phi_1|^2$$

$Y_{\nu 3} \sim \mathcal{O}(10^{-12})$ is the **third family sneutrino Yukawa coupling** and μ is the **supersymmetric Higgs parameter**. Note that \mathcal{L} is globally U(1) invariant. Henceforth, restrict to the real field $\phi_1 = \phi_1^* \Rightarrow$

- *The inflaton is a linear combination of the real parts of H_w^0 , $\nu_{L,3}$ and $\nu_{R,3}$ and, hence, is composed of fields already appearing in the $B - L$ MSSM.*

To **canonically normalize** the kinetic energy term, define a new real scalar field ψ by

$$\phi_1 = \sqrt{3} \tanh \left(\frac{\psi}{\sqrt{6}} \right)$$

The Lagrangian now becomes

$$\mathcal{L} = -\frac{1}{2}\partial_\mu\psi\partial^\mu\psi - V_F(\psi) - V_{soft}(\psi)$$

where $V_F(\psi)$ is determined from the above and

$$V_{soft}(\psi) = 3m^2 \tanh^2\left(\frac{\psi}{\sqrt{6}}\right)$$

Primordial Parameters:

For an arbitrary potential V define

$$\epsilon = \frac{1}{2} \left(\frac{V'}{V} \right)^2, \quad \eta = \frac{V''}{V}$$

In an **interval of slow-roll inflation**, these parameters must satisfy

$$\epsilon, |\eta| \ll 1$$

We define the **smallest** value of the field where inflation ends to be ψ_{end} . The value of the field which precedes ψ_{end} by exactly 60 e-foldings is denoted ψ_* .

The “spectral index” and the “scalar to tensor” ratio are defined by

$$n_s \simeq 1 + 2\eta_* - 6\epsilon_* , \quad r \simeq 16\epsilon_*$$

respectively. The label “_{*}” denotes quantities evaluated at ψ_* .

In addition, the Planck2015 normalization of the CMB fluctuations requires that

$$V_*^{1/4} = 1.88 \left(\frac{r}{0.10} \right)^{1/4} \times 10^{16} \text{ GeV}$$

where we have restored dimensionful units for clarity.

Let us now evaluate these quantities for the B-L MSSM potential $V = V_F + V_{soft}$. Begin by considering V_{soft} alone. We find that

$$\psi_{end} = 1.21 , \quad \psi_* = 6.23$$

and

$$n_s \simeq 0.967 , \quad r \simeq 0.00326$$

which are consistent with the Planck2015 data.

Inserting this value for r above gives

$$V_*^{1/4} = 7.97 \times 10^{15} \text{ GeV} \implies m = 1.55 \times 10^{13} \text{ GeV}$$

\implies

- *In order to be consistent with the Planck2015 cosmological data, supersymmetry must be broken at a high scale of $\mathcal{O}(10^{13} \text{ GeV})$.*

Note that for $M_{\chi_{T_3R}} \simeq M_{\chi_{B-L}} (\simeq M_U)$ the analysis presented earlier is valid for **arbitrarily high values** of soft SUSY breaking. Also note that although ψ must be **trans-Planckian** at the start of inflation

- *The physical fields H_w^0 , $\nu_{3,R}$ and $\nu_{3,L}$ are all sub-Planckian during the entire inflationary epoch.*

The form of our potential V_{soft} has arisen in **other contexts**. However

- *Our V_{soft} potential arises entirely from the associated soft supersymmetry breaking quadratic term, rescaled to canonically normalize the kinetic energy.*

Now consider the **F-term potential** V_F . The Yukawa coupling $Y_{\nu 3} \sim \mathcal{O}(10^{-12})$ is small and that term in V_F can be ignored.

However, to keep from ruining the slow-roll behavior of V_{soft} , we must choose the μ parameter to be at least **three order of magnitude** smaller than m . For specificity, take

$$\mu = 1.20 \times 10^{10} \text{ GeV}$$

Combining both potentials, we find

$$\psi_{end} \simeq 1.21, \quad \psi_* \simeq 6.25$$

and

$$n_s \simeq 0.969, \quad r \simeq 0.00334$$

again both **consistent with the Planck2015 data**. Also

$$V_*^{1/4} = 8.07 \times 10^{15} \text{ GeV} \Rightarrow m = 1.58 \times 10^{13} \text{ GeV}$$

The potentials V_{soft} and V_F are individually plotted in Figure 1.

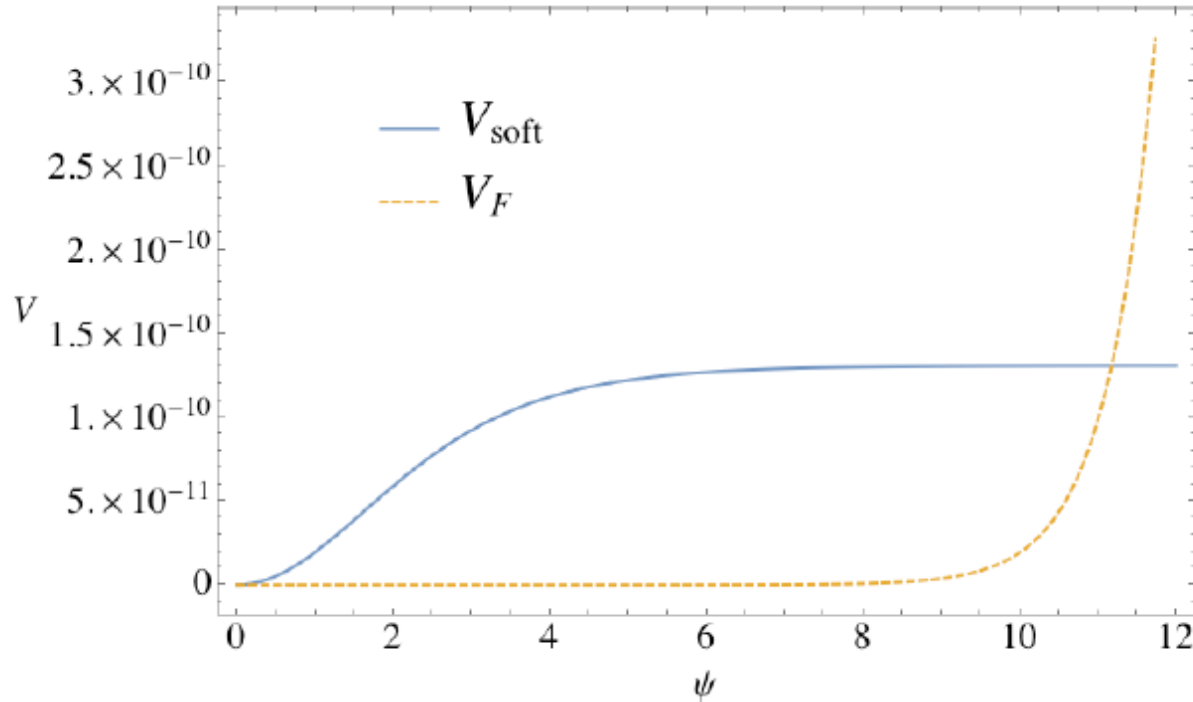


FIG. 1. The blue line is a plot of V_{soft} for the soft mass value $m = 1.58 \times 10^{13}$ GeV. The orange line is a graph of V_F for the parameters $Y_{\nu 3} \sim 10^{-12}$ and $\mu = 1.20 \times 10^{10}$ GeV

Note that the F-term potential acts as a natural “cut-off” of the inflation potential V_{soft} for values of $\psi \gtrsim 8$. This gives a supersymmetric realization of the “Inflation without Selfreproduction” mechanism introduced by Mukhanov.

The complete potential is plotted in Figure 2.

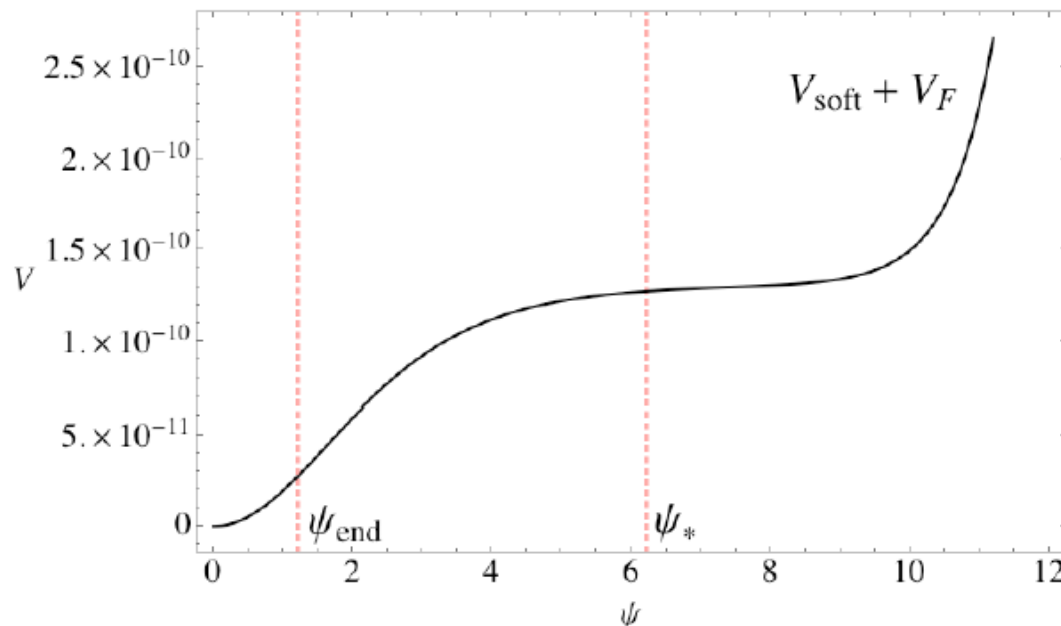


FIG. 2. The black line is a graph of the potential $V_{soft} + V_F$ for the parameters $m = 1.58 \times 10^{13}$ GeV, $Y_{\nu 3} \sim 10^{-12}$ and $\mu = 1.20 \times 10^{10}$ GeV. For these values of the parameters, the vertical red dashed lines mark $\psi_{end} \simeq 1.21$ and $\psi_* \simeq 6.25$ respectively.

We have now demonstrated that the B-L MSSM coupled to N=1 supergravity can **naturally produce a period of slow-roll inflation**.

However, to do so we had to **raise** the soft SUSY breaking scale to

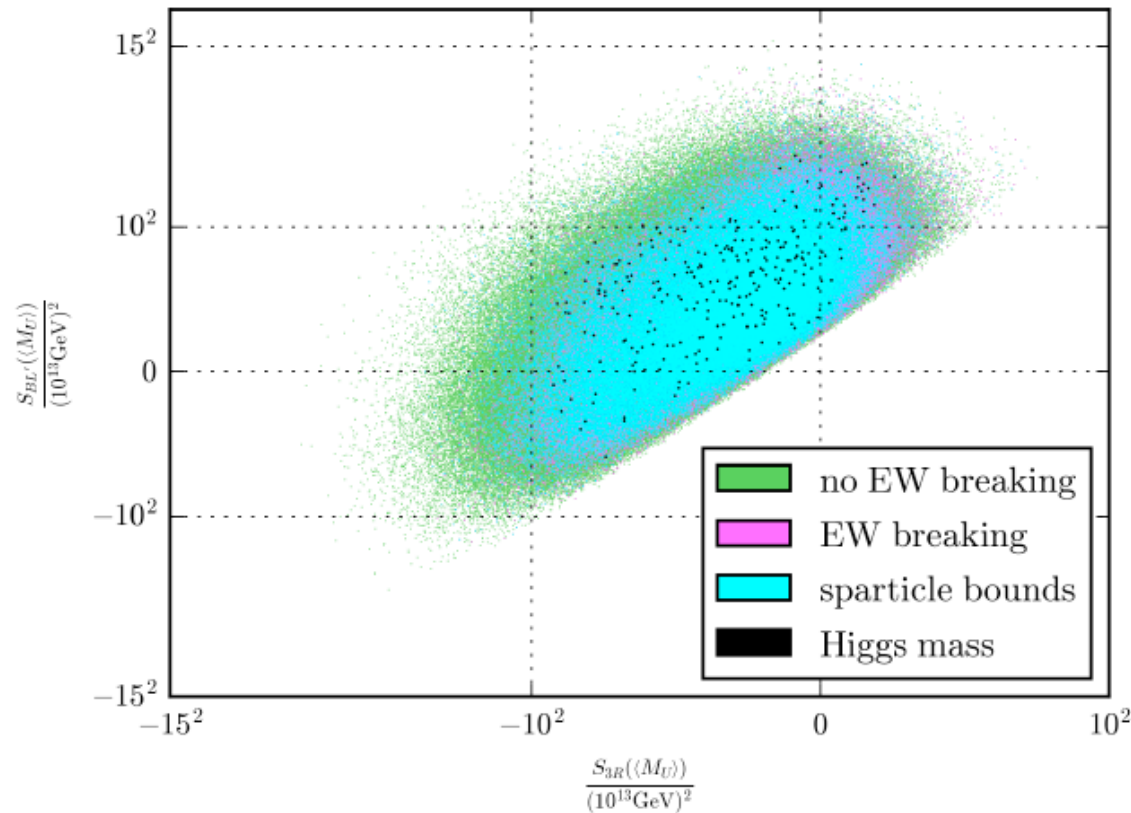
$$m = 1.58 \times 10^{13} \text{ GeV} \gg \mathcal{O}(1 - 10 \text{ TeV})$$

Is this still consistent with all low energy phenomenology?

Redoing our earlier analysis, but now taking

$$\left(\frac{M}{f}, Mf\right) \text{ where } M = m = 1.58 \times 10^{13} \text{ GeV}, f = 3.3$$

we find



Each black “valid” point is consistent with all low energy phenomenology as well as being a theory of inflation satisfying all Planck2015 constraints.

Perturbative Reheating:

It was shown in several papers by J. Ellis and K. Olive that within the context of Higgs and Sneutrino inflation in no-scale $N=1$ supergravity perturbative reheating can predominate over “preheating” for a significant range of input parameters. Here, we assume that is the case and focus exclusively on perturbative reheating.

Classical Behavior of ψ and H :

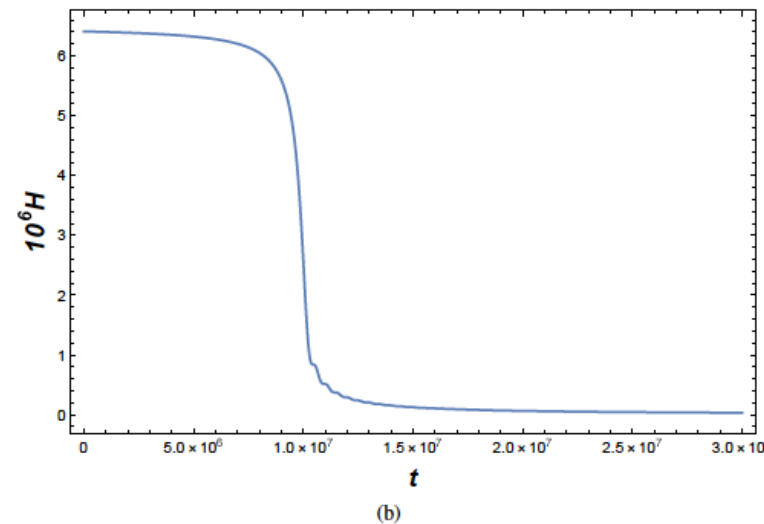
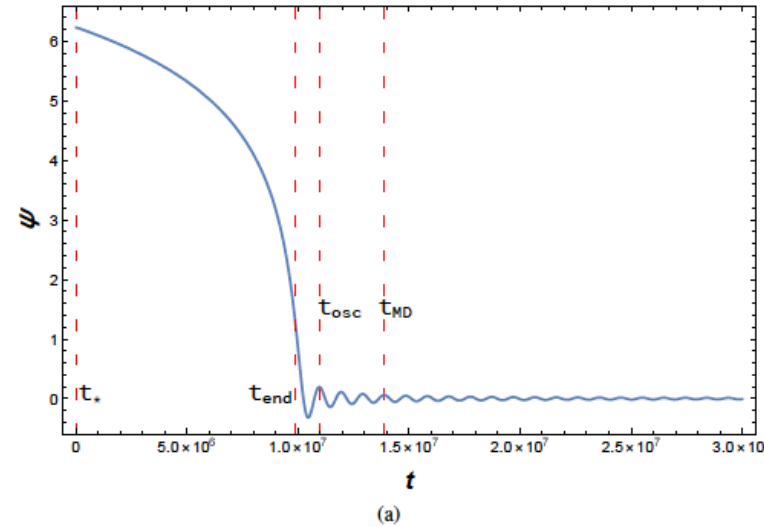
The relevant equations of motion in the inflationary and post-inflationary regimes are

$$\begin{aligned}3H^2 &= \frac{1}{2}\dot{\psi}^2 + V(\psi) \\ \dot{H} &= -\frac{1}{2}\dot{\psi}^2, \\ \ddot{\psi} + 3H\dot{\psi} + V_{,\psi} &= 0,\end{aligned}$$

where $M_P = 1$ and, since $V_F \ll V_{soft}$ we can take $V = V_{soft} \Rightarrow$

$$V(\psi) = 3m^2 \tanh^2\left(\frac{\psi}{\sqrt{6}}\right) = \frac{1}{2}m^2\psi^2 \left[1 - \left(\frac{\psi}{3}\right)^2 + \left(\frac{17\psi^4}{1620}\right) + \dots\right]$$

Solving these equations numerically, we find



The numerical solutions for $\psi(t)$ and $H(t)$, where we have set $M_P = 1$. Note that $t_* = 0$ and $t_{end} \simeq 9.89 \times 10^6$ mark the beginning and end of the inflationary period. The times $t > t_{end}$ correspond to the post inflationary epoch. As defined in the text, $t_{osc} \simeq 1.096 \times 10^7$ marks the time at which the potential energy is well approximated by $V = \frac{1}{2}m\psi^2$ and $t_{MD} \simeq 1.387 \times 10^7$ is the time at which our analytic solutions for ψ and H become valid.

Behavior of ψ and H during Particle Decay:

The relevant equations of motion in the post-inflationary regime are modified to

$$\begin{aligned} 3H^2 &= \frac{1}{2}\dot{\psi}^2 + V(\psi) + \sum_i \rho_i, \\ \dot{H} &= -\frac{1}{2}\dot{\psi}^2 - \frac{1}{2} \sum_i (\rho_i + p_i), \\ \ddot{\psi} + \left(3H + \sum_i \Gamma_{d,i} \right) \dot{\psi} + V'(\psi) &= 0, \\ \dot{\rho}_i + 3(1 + \omega_i(t)) H \rho_i - \Gamma_{d,i} \dot{\psi}^2 &= 0, \end{aligned}$$

where $\Gamma_{d,i}$ is the decay rate of ψ into the i -th matter species, and ρ_i and p_i are the energy density and pressure respectively of the i -th species in the decay products. The quantities ρ_i and p_i are related by the relation $p_i = \omega_i(t)\rho_i$, where $\omega_i = 0$ and $1/3$ respectively for matter and radiation. The initial conditions for ψ and H are set by their classical values at the end of the inflationary epoch.

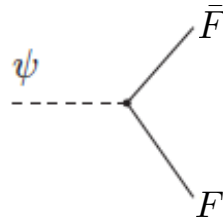
Expansion of ψ :

We expand ψ around its root mean square value. That is

$$\psi = \sqrt{\langle \psi^2 \rangle} + \delta\psi \quad (\equiv \psi)$$

Decay Species:

a) **up-quarks**- Let F denote an up-quark. Then



$$\Rightarrow \Gamma_d(\psi \rightarrow F\bar{F}) = \frac{y_{\psi F}^2 m_\psi}{8\pi} \left[1 - 4 \left(\frac{m_F}{m_\psi} \right)^2 \right]^{\frac{3}{2}},$$

where

$$m_F = y_{\psi F} \sqrt{\langle \psi^2 \rangle}, \quad m_\psi = m = 6.49 \times 10^{-6}.$$

Note that

$$\Gamma_d \longrightarrow \frac{y_{\psi F}^2 m_\psi}{8\pi} \equiv \Gamma_d^{\max} = \text{constant}$$

b) **charginos**

c) **neutralinos**

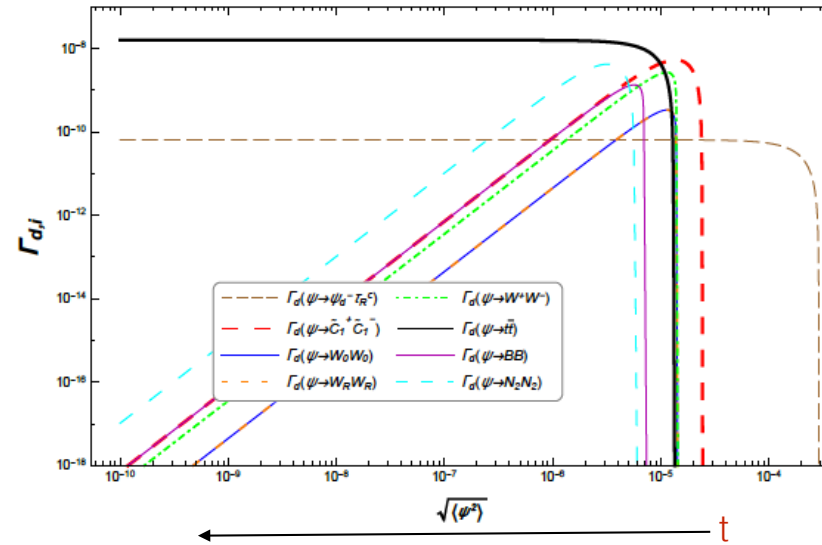
d) **gauge bosons**- For example, consider W_R . Then

$$\Gamma_d(\psi \rightarrow W_{R\mu} W_R^\mu) = \frac{g_2^4 \langle \psi^2 \rangle}{1152\pi m_\psi} \left[1 - 4 \frac{m_{W_R}^2}{m_\psi^2} \right]^{1/2}$$

Note that

$$\Gamma_d \longrightarrow 0$$

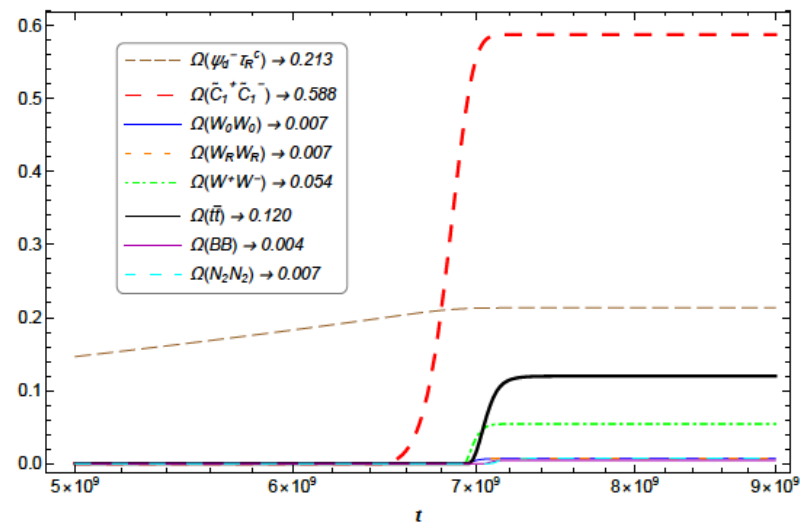
Plotted as a function of $\sqrt{\langle\psi^2\rangle}$, we find



The **fractional energy density** for the i -th decay species is given by

$$\Omega_i(t) = \frac{\rho_i(t)}{\rho_{total}},$$

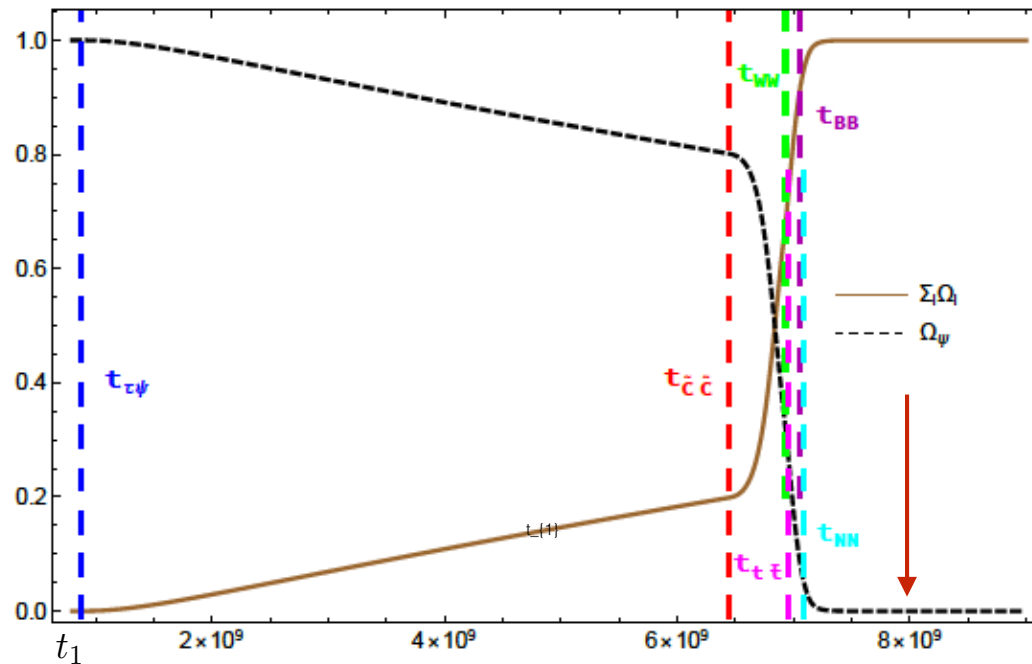
where $\rho_{total} = 3H^2$. Plotted as a function of t



Defining

$$\Omega_\psi = \rho_\psi / \rho_{total} \text{ with } \rho_\psi = \frac{1}{2}\dot{\psi}^2 + V(\psi)$$

we find



We plot the evolutions of Ω_ψ and $\sum_i \Omega_i$ from t_1 to sometime after the end of reheating. The time at which each specie is turned on is marked with a vertical line, where $t_{\tau\psi*} = 8.78 \times 10^8$, $t_{\bar{c}\bar{c}*} = 6.45 \times 10^9$, $t_{WW*} = 6.93 \times 10^9$, $t_{t\bar{t}*} = 6.95 \times 10^9$, $t_{BB*} = 7.06 \times 10^9$ and $t_{NN*} = 7.08 \times 10^9$.

Complete reheating has occurred at the time when

$$\sum_i \Omega_i \rightarrow 1 \Rightarrow \Omega_\psi \rightarrow 0$$

It follows that

$$t_R \simeq 8 \times 10^9$$

Finally, we have shown that the plasma is in thermal equilibrium at t_R and that the reheat temperature is

$$T_R \simeq 1.13 \times 10^{13} \text{ GeV}$$