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Conformal Fishnet Theory

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Collaborations with

Ö. Gürdogan	arXiv:1512.06704
J. Caetano and Ö. Gürdogan	arXiv:1612.05895
D. Grabner, N. Gromov, G. Korchemsky	arXiv:1711.04786
N. Gromov, G. Korchemsky, S. Negro, G. Sizov	arXiv:1706.04167
D. Chicherin, F. Loebbert, D. Mueller, D. Zhang	arXiv:1704.01967, arXiv:1708.00007
E. Olivucci	arXiv:1801.09844
N. Gromov, G. Korchemsky	arXiv:1808.02688
S. Derkachev, E. Olivucci	to appear
E. Olivucci, M. Preti	to appear

ENS

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Introduction

- Conformal Field Theories (CFT) are QFTs without mass scale, symmetric w.r.t.

- Strongly coupled CFTs describe many physical phenomena: phase transitions, quantum gravity (through AdS/CFT duality), Regge limit for high energy hadrons (BFKL) and, may be, the fundamental interactions from grand unification to quantum gravity scales. Supersymmetry is not really ubiquitous in the Nature!
- Not many such CFTs are explicitly defined in 3D (Ising, O(n)…), and almost none in 4D (Banks-Zaks,?)

Rychkov. Ratazzi, Simon-Duffin,… 2015

- Conformal bootstrap methods helped to drastically reduce the possible parameter space of CFTs and to identify the 3D Ising point. But this "experimental" method, based on numerics for crossing relations, does not define CFT microscopically

- Need for well defined in UV 4D nonsupersymmetric CFTs (possibly by Lagrangian)
- Possibly analysable at strong coupling, or even solvable (= integrable)
- We know so far only one such, supersymmetric 4D theory: N=4 SYM in planar, 't Hooft limit

Conformal correlation functions

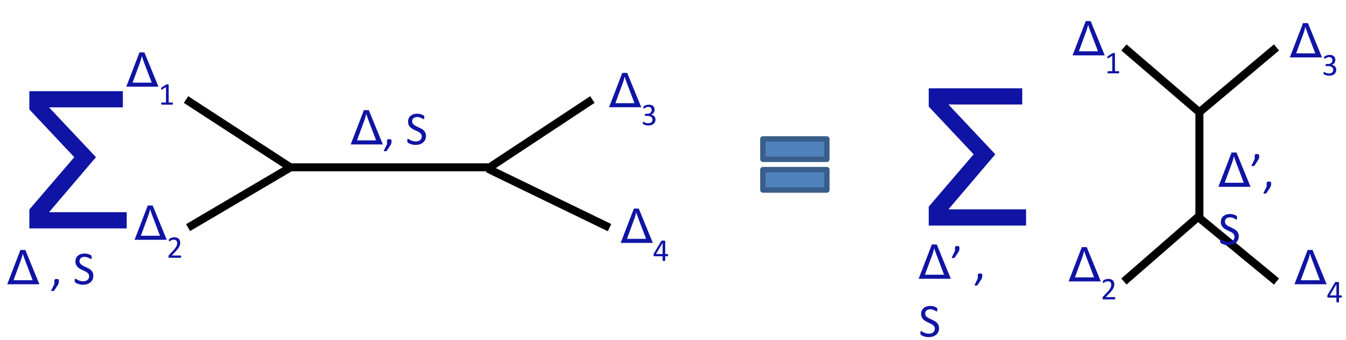
- 2-point correlation functions of conformal operators:

anomalous dimension

structure constant

$$\langle \mathcal{O}_{k_1}(x_1)\mathcal{O}_{k_2}(x_2)\mathcal{O}_{k_3}(x_3)\rangle = \frac{C_{k_1k_2k_3}}{|x_{12}|^{\Delta_{k_1}+\Delta_{k_2}-\Delta_{k_3}}|x_{23}|^{\Delta_{k_2}+\Delta_{k_3}-\Delta_{k_1}}|x_{31}|^{\Delta_{k_1}+\Delta_{k_3}-\Delta_{k_2}}}$$

- OPE for 4-point correlator in two different channels:



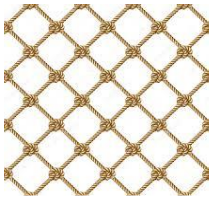
crossing

Outline

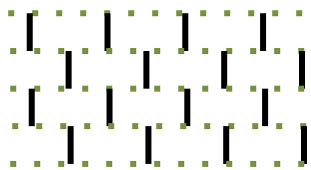
Gurdogan, V.K. 2015

- We proposed a family of integrable non-supersymmetric 4D CFTs from a double scaling limit of Super-YM with N=4 susy's: strong γ-deformation & weak coupling
- Dominated by specific, integrable (computable!) multi-loop 4D Feynman graphs:

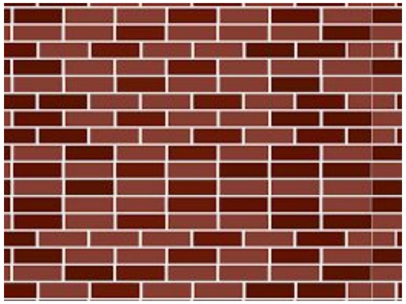
"fishnet" A. Zamolodchikov 1980
(massless, 4-scalar int.)



"brick wall" Gurdogan, V.K. 2015
Caetano, Gurdogan, V.K. 2016
(Yukawa ψψφ interaction)



mix of both



Gromov, V.K. ,Leurent, Volin 2013

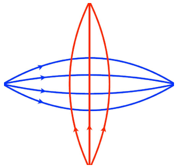
- Can be studied by N=4 SYM Quantum Spectral Curve or direct map to non-compact SU(2,2) Heisenberg spin chain

Chicherin, Derkachev, Isaev
Gurdogan, V.K. 2015
Gromov, V.K , Korchemsky, Negro, Sizov

- A possibility of analytic insight into non-perturbative structure of CFTs in 3d and 4d
- Generalization to any dimension. V.K., Olivucci
- Exact OPE data, 4-point correlators.
- Yangian symmetry of scalar scattering amplitudes
- Basso-Dixon 4D integrals : we found a 2D analogue

Chicherin, V.K., Loebbert, Mueller, Zhong (2017)

Grabner, Gromov, V.K., Korchemsky 2017
Gromov, V.K., Korchemsky 2018
V.K.,Olivuccci, Preti 2018
Gromov, V.K , Korchemsky



Basso, Dixon '17
Derkachev, V.K., Olivucci (to appear)

Gurdogan, V.K. 2015

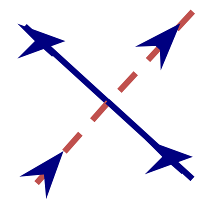
Bi-scalar, "Fishnet" CFT_4

- It is a particular, single coupling case of our general 3-coupling model

$$\mathcal{L}[\phi_1,\phi_2] = \frac{N_c}{2}\mathrm{tr}\,\left(\partial^\mu\phi_1^\dagger\partial_\mu\phi_1 + \partial^\mu\phi_2^\dagger\partial_\mu\phi_2 + 2\xi^2\,\phi_1^\dagger\phi_2^\dagger\phi_1\phi_2\right)\,.$$

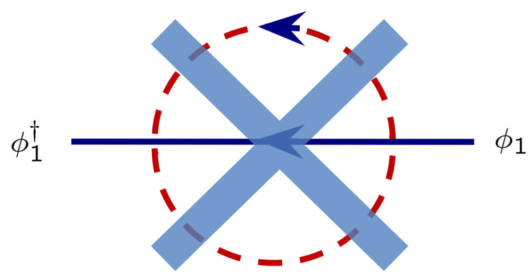
Zero dimensional analogue:
Kostov, Staudacher 1995

- Propagators



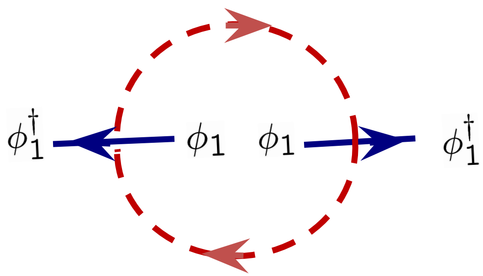
- Non-unitary logarithmic CFT, integrable at $N_c \to \infty$

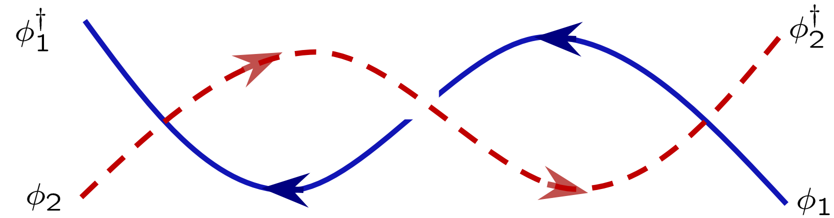
- Very limited number of planar graphs. No mass or vertex renormalization!



Double trace terms

Tseytlin, Zarembo
Dymarsky, Klebanov, Roiban
Witten
Sieg, Wilhelm, Fokken





$$\mathrm{tr}(\phi_1^\dagger \phi_2)\mathrm{tr}(\phi_2^\dagger \phi_1)$$

- ξ doesn't run in planar limit. Double-trace counter-terms do run:

Pomoni, Rastelli 2009

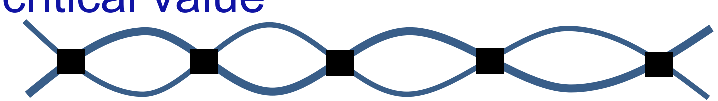
- Beta-function quadratic in double-trace coupling (agrees with general arguments)

Grabner, Gromov, V.K., Korchemsky '17

- A non-unitary CFT at any ξ, if we tune $(\text{trace})^2$ couplings (as γ-deformed N=4 SYM)

Sieg, Wilhelm 2016
Grabner, Gromov, V.K., Korchemsky '17

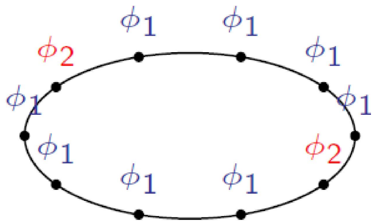
- The other double-trace coupling also tuned to the critical value to renormalize the only type of divergent graphs:



Caetano, Gurdogan, V.K , 2016

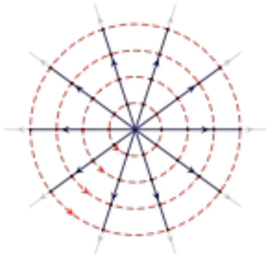
Operators, correlators, graphs…

- Operators with M magnons in the "vacuum" of



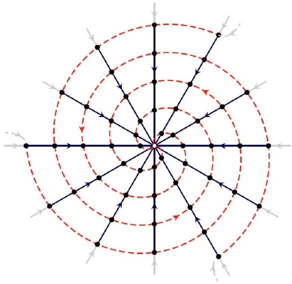
anomalous dimension

- Correlators
- BMN "vacuum" operator $\mathrm{tr}[\phi_1(x)]^L$ renormalized by "wheel" graphs, with L spokes and M frames:



- Multi magnon operators renormalized by multi-spiral (spider-web) graphs. Studied by Asymptotic Bethe ansatz

Caetano, Gurdogan, V.K , 2016



- Typical "fishnets" structure in the bulk of graphs. Integrable!
- Double wheels at any L and any multiple wheels at L=2,3 computed.
- Generalization to multi-wheels at any L possible (in work)

Grabner, Gromov, V.K., Korchemsky

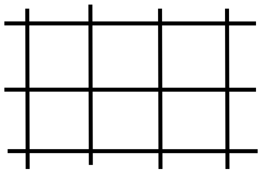
Ahn, Bajnok, Bombardelli, Nepomechie 2011
Gurdogan, V.K. 2015
Gromov, V.K , Korchemsky, Negro, Sizov

Bi-scalar fishnet CFT at any D

V.K., Olivucci 2018

- Bi-scalar CFT can be defined at any D by a non-local action

- D-dimensional “fishnet” graphs generalize 4D “fishnets”



- Anisotropic “fishnet” at D=2, $\omega=2s-1/2$, where s is the “on-site” spin

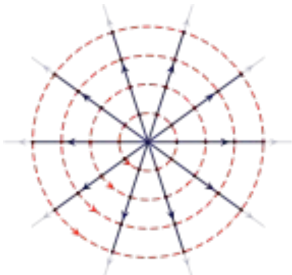
- D=2, s=1 $\leftrightarrow$ Lipatov’s reggeized gluon model (BFKL)

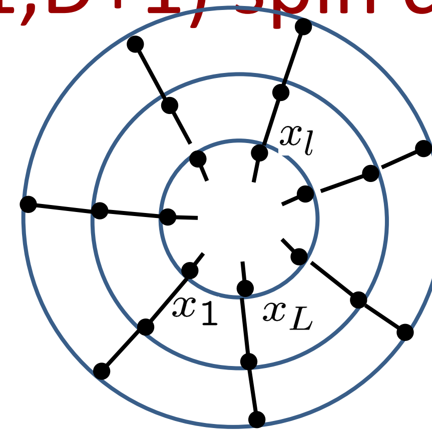
- Isotropic fishnet model at D=2, s=1/4 with propagators

- Integrable conformal SO(1,D) spin chain !

Wheel graphs and integrable SO(1,D+1) spin chain

- Operator generating a wheel (fishnet) graph



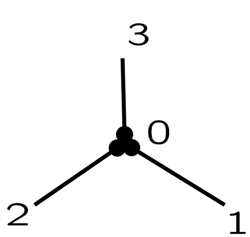


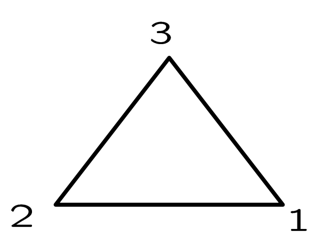
- Quantum integrability: graph-generating operator is a particular integral of motion of 1+1-dimensional integrable Heisenberg quantum spin chain with non-compact spins on conformal SO(1,D) group.

Gurdogan, V.K. 2015
Gromov, V.K , Korchemsky, Negro, Sizov
V.K., Olivucci

- Bi-scalar model is explicitly integrable to all orders!
- Integrability here is based on star-triangle relations

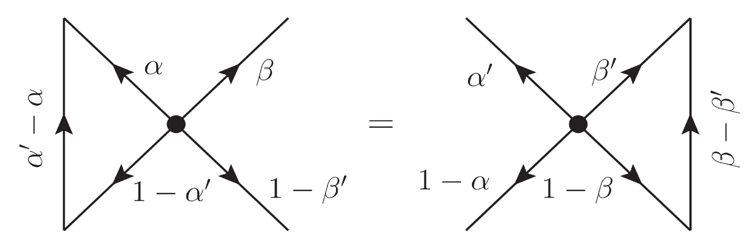
Zamolodchikov 1980
Derkachev, Manashov (2001-…)
Derkachev, Korchemsky, Manashev
Chicherin, Derkachev, Isaev 2012



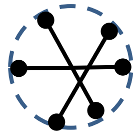


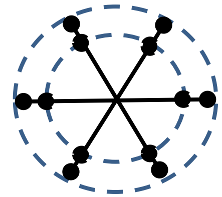
$$V(a,b,c) = \pi^{D/2}\frac{\Gamma(\frac{D}{2}-a)\Gamma(\frac{D}{2}-b)\Gamma(\frac{D}{2}-c)}{\Gamma(a)\Gamma(a)\Gamma(c)}$$

- Crossing relation



Wheel graphs at any L and dimension of $\mathrm{tr}[\phi_1(x)]^L$





Broadherst 1980

$$\gamma^{(2)}(L) = \frac{2^3}{\Gamma^2(L)}\Bigg\{-\sum_{j_1=0}^{L-1}\frac{(2L-2)!}{L-j_1}\binom{L+j_1-1}{j_1}\zeta_{2L-3}\zeta_{\substack{L+j_1-2,\\ L-j_1}} + \frac{\Gamma^2(L)}{8}\binom{4L-2}{2L-1}\zeta_{4L-5}$$

$$+\sum_{\substack{j_1,j_2>0\\ j_1+j_2<2L-3}}\frac{\Gamma(2L-j_1-j_2-1)\Gamma(L+j_1)\Gamma(L+j_2)}{\Gamma(j_1+1)\Gamma(j_2+1)\Gamma(L-j_1)\Gamma(L-j_2)}\Bigg[\sum_{k=1}^{L+j_1-2}\Bigg(\binom{L+k+j_2-4}{k-1}\Big(2\zeta_{\substack{L+j_1-k-1,\\ L+j_2+k-3,\\ 2L-j_1-j_2-1}} + \zeta_{\substack{L+j_1-k-1,\\ 3L-j_1+k-4}}\Big)$$

$$-\binom{L+k+j_2-2}{k}\Big(2\zeta_{\substack{L+j_1-k-1,\\ L+j_2+k-2,\\ 2L-j_1-j_2-2}} - 2\zeta_{\substack{L+j_1-k-1,\\ L+j_2+k-1,\\ 2L-j_1-j_2-3}}\Big)\Bigg) + 2\zeta_{\substack{2L+j_1+j_2-3,\\ 2L-j_1-j_2-2}} + 2\zeta_{\substack{2L+j_2-1,\\ L+j_1-1,\\ 2L-j_1-j_2-3}}\ \zeta_{\substack{L+j_1-2,\\ 3L-j_1-3}} - \zeta_{\substack{2L+j_1+j_2-2,\\ 2L-j_1-j_2-3}}\Bigg]$$

$$+\frac{\Gamma^2(2L-1)}{2\Gamma^2(L)}\Bigg[\sum_{k=1}^{2L-4}\Bigg(\binom{2L+k-4}{k}\Big(3\zeta_{\substack{2L-k-3,\\ 2L+k-2}} + 2\zeta_{\substack{2L-k-3,\\ 2L+k-4,\\ 2}}\Big) - 2\Bigg(\binom{2L+k-4}{k-1} + \binom{2L+k-4}{k+1}\Bigg)\zeta_{\substack{2L-k-3,\\ 2L+k-3,\\ 1}}\Bigg)$$

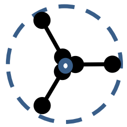
$$+2\zeta_{\substack{2L-3,\\ 2L-4,\\ 2}} - 4(L-3)\zeta_{\substack{2L-3,\\ 2L-3,\\ 1}} + \zeta_{\substack{2L-4,\\ 2L-1}} - 4\zeta_{\substack{2L-2,\\ 2L-3}} + 4\zeta_{\substack{4L-6,\\ 1}} + 2\zeta_{\substack{3L-4,\\ 2L-2,\\ 1}} + \frac{2L+1}{L}\zeta_{2L-3}\zeta_{2L-2} - 5\zeta_{4L-5} - \zeta^2_{2L-3}\Bigg]\Bigg\}$$

Gurdogan, V.K. 2015

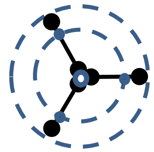
Ahn, Bajnok, Bombardelli, Nepomechie 2013

L=3 BMN vacuum and all-loop wheel graphs

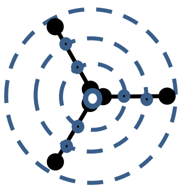
Gromov, V.K , Korchemsky, Negro, Sizov (2017)

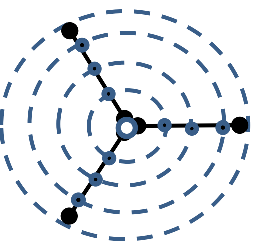


Broadherst 1980



Ahn, Bajnok, Bombardelli, Nepomechie 2013
E.Panzer, 2015



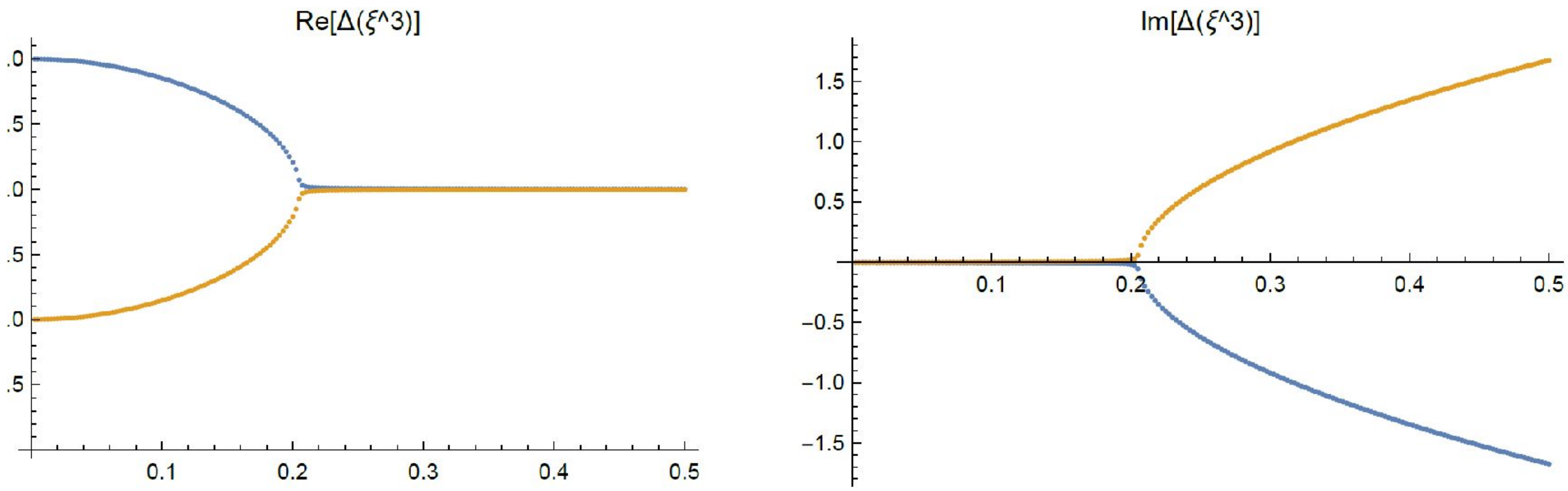


$$
\begin{aligned}
\Delta - 3 = \; & -12\xi^{6}\zeta(3) + \xi^{12}\left(189\zeta(7) - 144\zeta(3)^{2}\right) \\
& + \xi^{18}\Big(-1944\zeta(8,2,1) - 3024\zeta(3)^{3} - 3024\zeta(5)\zeta(3)^{2} + \frac{198\pi^{8}\zeta(3)}{175} + 6804\zeta(7)\zeta(3) \\
& + \frac{612\pi^{6}\zeta(5)}{35} + 270\pi^{4}\zeta(7) + 5994\pi^{2}\zeta(9) - \frac{925911\zeta(11)}{8}\Big) + \\
& \xi^{24}\Big(\frac{10368}{5}\pi^{4}\zeta(8,2,1) + 5184\pi^{2}\zeta(9,3,1) + 51840\pi^{2}\zeta(10,2,1) - 148716\zeta(11,3,1) \\
& - 1061910\zeta(12,2,1) + 62208\zeta(10,2,1,1,1) - 93312\zeta(3)\zeta(8,2,1) - 288\zeta(3)^{5} \\
& + 72\gamma\pi^{2}\zeta(3)^{4} - 77760\zeta(3)^{4} - \frac{80756\pi^{6}\zeta(3)^{3}}{945} - 145152\zeta(5)\zeta(3)^{3} - \frac{29}{270}\gamma\pi^{8}\zeta(3)^{2} \\
& + \frac{9504\pi^{8}\zeta(3)^{2}}{175} - 879\pi^{4}\zeta(5)\zeta(3)^{2} - 2025\pi^{2}\zeta(7)\zeta(3)^{2} + 244944\zeta(7)\zeta(3)^{2} \\
& + 186588\zeta(9)\zeta(3)^{2} + \frac{2910394\pi^{12}\zeta(3)}{2627625} - 2592\pi^{2}\zeta(5)^{2}\zeta(3) + \frac{29376}{35}\pi^{6}\zeta(5)\zeta(3) \\
& + 12960\pi^{4}\zeta(7)\zeta(3) + 298404\zeta(5)\zeta(7)\zeta(3) + 287712\pi^{2}\zeta(9)\zeta(3) \\
& - 5555466\zeta(11)\zeta(3) + 57672\zeta(5)^{3} - 71442\zeta(7)^{2} + \frac{13953\pi^{10}\zeta(5)}{1925} + \frac{7293\pi^{8}\zeta(7)}{175} - \frac{19959\pi^{6}\zeta(9}{5} \\
& + \frac{119979\pi^{4}\zeta(11)}{2} + \frac{10738413\pi^{2}\zeta(13)}{2} - \frac{4607294013\zeta(15)}{80}\Big) + O\left(\xi^{25}\right)
\end{aligned}
$$

Riemann (multi)-zeta numbers

- Generalization to any number of spokes L is in work
  (Baxter equation is available)

Numerics for L=3 "BMN vacuum"



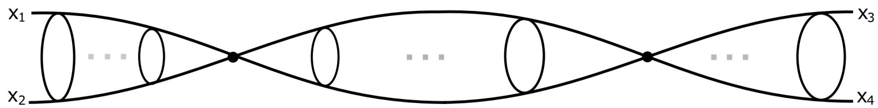
- Around ξ^3=0.2 dimension becomes imaginary: phase transition, finite convergence radius.
- What happens to the string dual?

Grabner, Gromov, V.K., Korchemsky 2017
V.K., Olivucci 2018

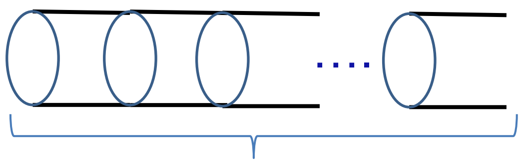
Zero-magnon 4-point correlator and exact OPE data

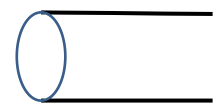
- Exact all-loop calculation of a 4-point correlator (only from conformal symmetry!)

cross-ratios



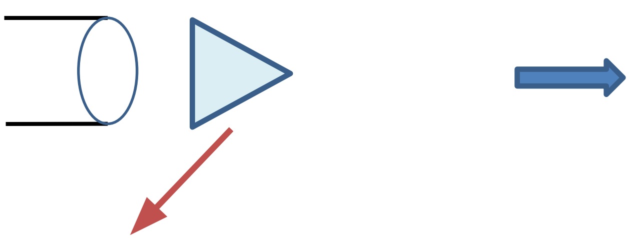
Defined by “wheel” graphs, generated by powers of graph-building operator





- Summing up these graphs, we get

- Due to conformal symmetry, diagonalized by 3-point correlation function with spin



Grabner, Gromov, V.K., Korchemsky 2017
Olivucci, V.K. 2018

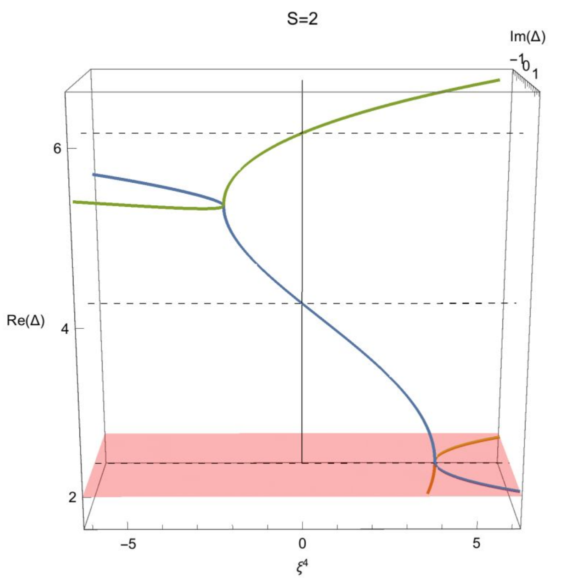
4-point correlator: exact results

- We obtain integral representation for 4-point correlation function

conformal group measure conformal block

- Integration by residues at poles (physical dimensions):
  gives the exact OPE over exchange operators

  - Dimensions :



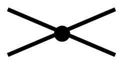
Twist=0,2,4 are related by monodromies

- explicit OPE coefficients (structure constants) for exchange operators, e.g. for D=4

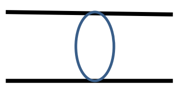
Grabner, Gromov, V.K., Korchemsky 2017

Perturbation theory for 4-point function at 4D

- Weak-coupling expansion, using exact OPE data:



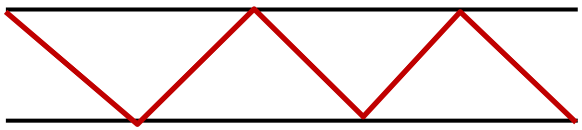
single-valued harmonic polylogarithms (SVHP)

 + 

All-loop one- and two- magnon 4-point correlator

- 4-point function with one-magnon exchange operators:

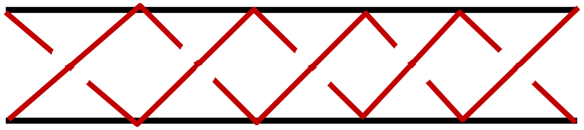
- "Spiral" graphs:



- Dimensions and structure constants :

- 4-point function with one-magnon exchange operators :

"double-spiral" graphs:



- Dimensions (4D) of exchange operators (Konishi-like)

- Structure constants are also available

Full 3-coupling CFT: 4-point correlators

- Lagrangian of full 3-coupling theory from double scaling limit of gamma-twisted N=4 SYM theory: Gurdogan, V.K. 2015

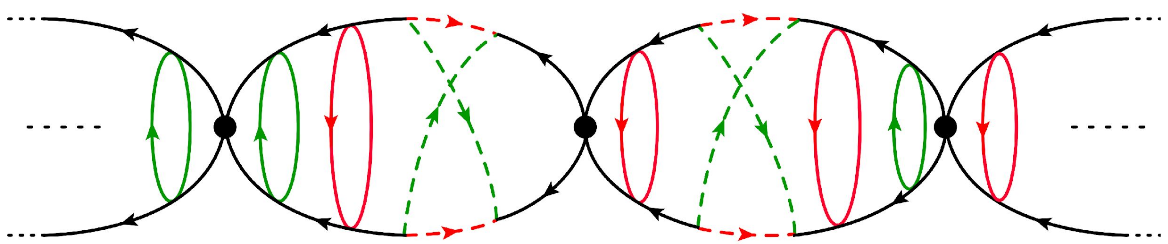
$$\mathcal{L} = N_c \mathrm{tr}[-\frac{1}{2}\partial^\mu \phi_i^\dagger \partial_\mu \phi^i + i\bar{\psi}_A^{\dot\alpha} \partial_{\dot\alpha}^{\alpha} \psi_\alpha^A] + \mathcal{L}_{\text{int}}$$

$$\mathcal{L}_{\text{int}} = N_c \,\mathrm{tr}[\xi_1^2\, \phi_2^\dagger \phi_3^\dagger \phi_2 \phi_3 + \xi_2^2\, \phi_3^\dagger \phi_1^\dagger \phi_3 \phi_1 + \xi_3^2\, \phi_1^\dagger \phi_2^\dagger \phi_1 \phi_2 +$$

$$+ i\sqrt{\xi_2\xi_3}(\psi^3\phi^1\psi^2 + \bar\psi_3 \phi_1^\dagger \bar\psi_2) + i\sqrt{\xi_1\xi_3}(\psi^1\phi^2\psi^3 + \bar\psi_1 \phi_2^\dagger \bar\psi_3) + i\sqrt{\xi_1\xi_2}(\psi^2\phi^3\psi^1 + \bar\psi_2 \phi_3^\dagger \bar\psi_1)\,].$$

- Consider again zero-magnon 4-point function

- Dominated by two types of wheels and a single type of fermionic loops

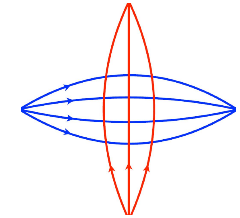


V.K., Olivucci, Preti 2018

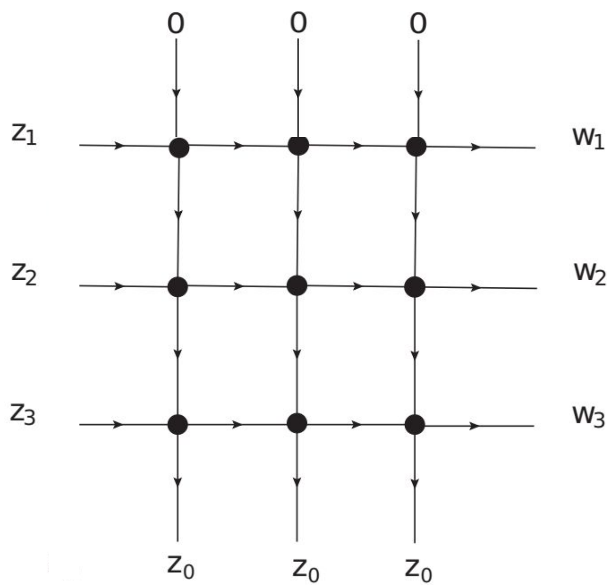
- Anomalous dimensions and OPE structure coefficients of exchange operators explicitly computed by Bethe-Salpeter method and conformal symmetry in terms of hypergeometric functions

Derkachev, V.K., Olivucci 2018

Basso-Dixon formula in 2D



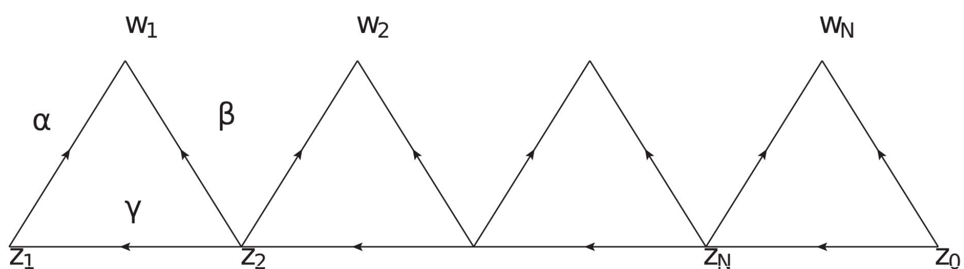
in 4D: Basso, Dixon 2017

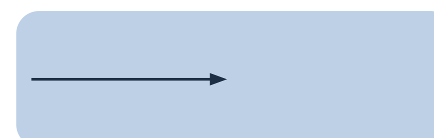


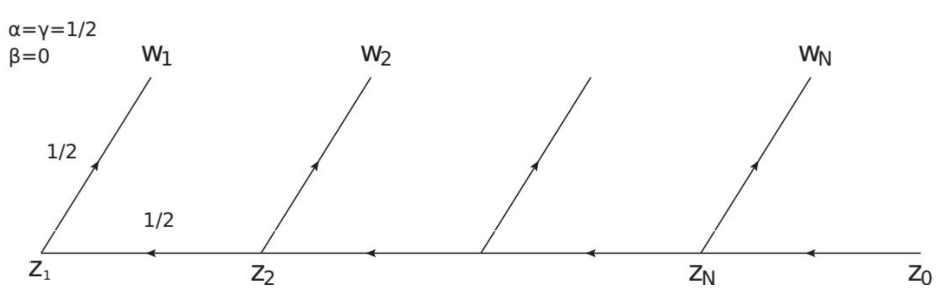
Remove from BD graph the upper row of propagators (due to conformality) and add left column (not integrated)

Graph-building operator arises in a special limit of a more general object

$\alpha+\beta+\gamma=1$





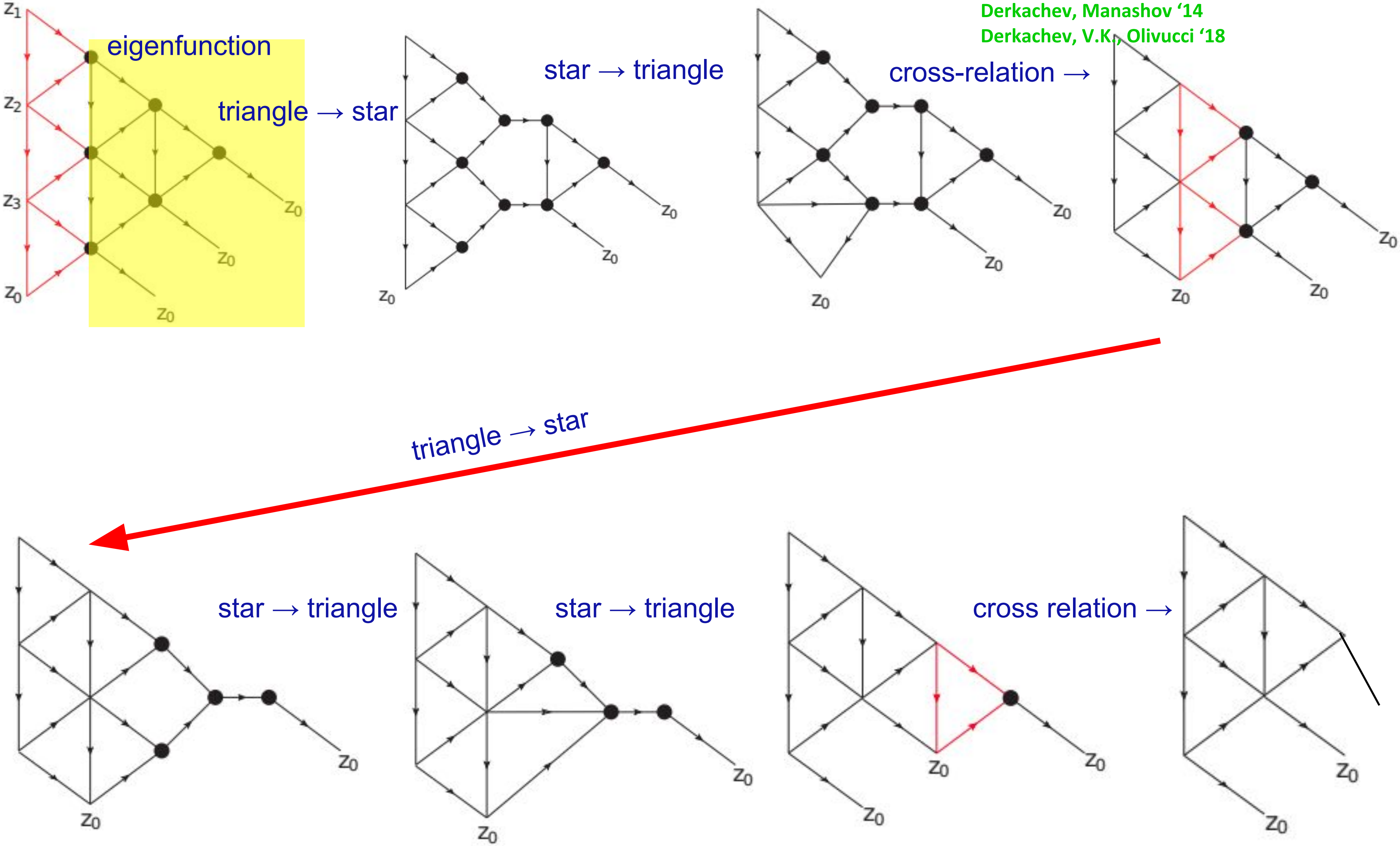


We have to diagonalize it.
Use the SOV method

Sklyanin
Derkachev, Korchemsky, Manashev 2001, 2003
Derkachev, Manashov

Diagonalization of graph-building operator

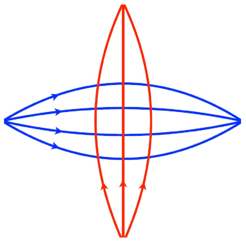
Derkachev, Korchemsky, Manashov 01'-03'
Derkachev, Manashov '14
Derkachev, V.K., Olivucci '18



We reproduced the original eigenfunction!

Derkachev, V.K., Olivucci 2018

Explicit Basso-Dixon formula in 2D



- Represented through N×N determinant of explicit functions

  of size of fishnet L×N, of conformal ratio

  and anisotropy parameter (irrep of SL(2,C))

  where

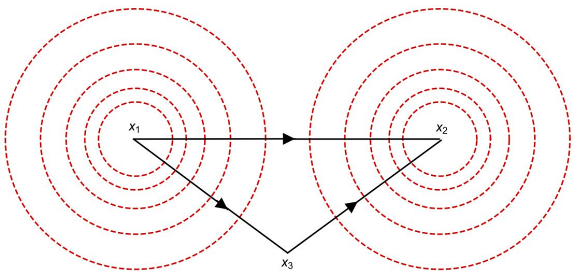
  and the last factor is represented through the hypergeometric functions:

- The homogeneous fishnet BS formula is recovered at
- Might be interesting application to BFKL limit

Other recent results

- Large fishnet as a sigma model on AdS_5 Basso, Zhang 2018
- Strong coupling asymptotics of fishnet four-point functions Gromov, V.K., Korchemsky '18
- More complicated 3-point functions, glued from wheels Gromov, V.K., Korchemsky '18 (similar to computations in SYK model)

Klebanov, Tarnopolsky + … '17-'18
Gross, Rosenhaus '17
Liu, Perlamutter, Rosenhaus, Simons-Duffin, '18



Problems to solve

- Weak coupling expansions of 4-point functions in cross channel Grabner, Gromov, V.K., Korchemsky (in work)
- Wheels and spider-webs of any L, M in 4D (or any D) from QSC and spin chain
- Wheels in 2D from Sklyanin SOV in SU(2,L) spin chain Derkachev, V.K., Olivucci (in work)
- Finite T, Hagedorn transition, "black hole" regimes for fishnet CFT A-la Harmark, Wilhelm 17'
- More complicated 3- and 4-point functions
- Systematic solution for spectra of non-compact integrable spin chains
- Operators of L=2 in full gamma-deformed N=4 SYM
- AdS dual of fishnet?
- From fishnet integrability to full N=4 SYM integrability

Thank you!