

*Nedfest 2017: "From Asteroids to the Cosmic Microwave Background"*  
*At the UCLA Laskin Conference Center, August 25-26, 2017*

# *From Pioneer Anomaly to Modern Tests of Fundamental Gravitation*

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and*

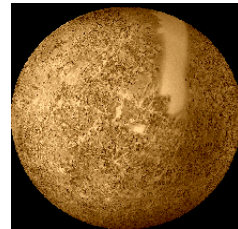
*UCLA Department of Physics and Astronomy  
475 Portola Plaza, Los Angeles, CA 90095 USA*



Discovery of Neptune: 1845

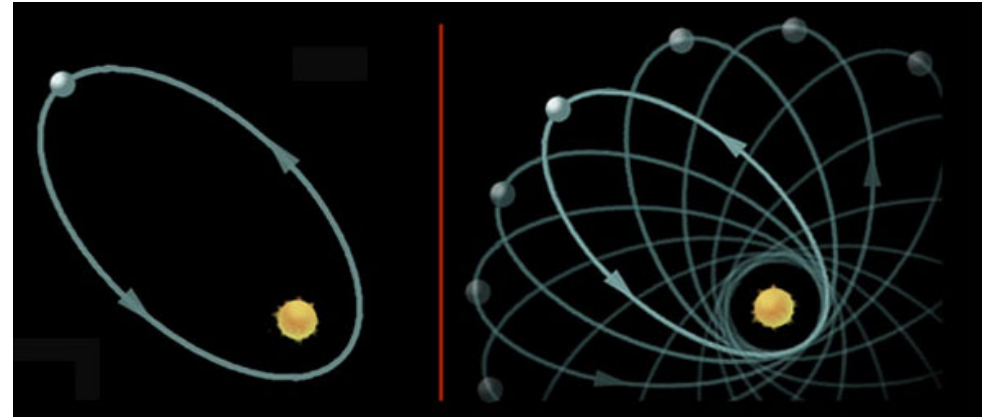


Urbain LeVerrier  
(1811-1877)



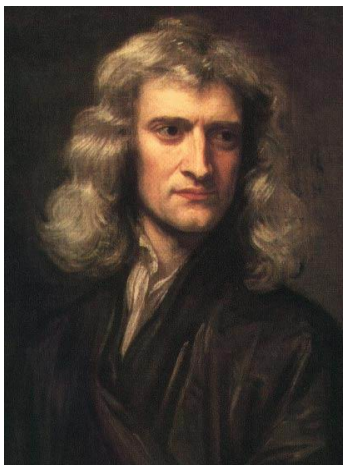
■ 1845: the search for Planet-X:

- Anomaly in the Uranus' orbit → Neptune
- Anomalous motion of Mercury → Vulcan



Newtonian Gravity

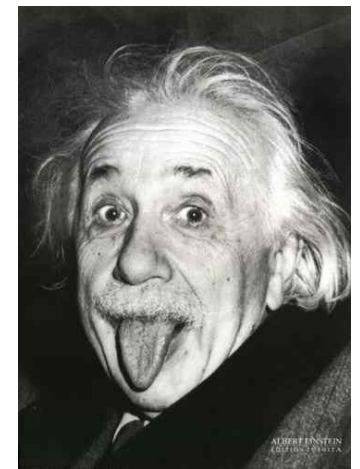
General Relativity



Sir Isaac Newton  
(1643-1727)

- Anomalous precession of Mercury's perihelion :
  - 43 arcsec/cy can not be explained by Newton's gravity
- Before publishing GR, in 1915, Einstein computed the expected perihelion precession of Mercury
  - When he got out 43 arcsec/cy – a new era just began!!

Almost in one year LeVerrier both confirmed the Newton's theory (Neptune) & cast doubt on it (Mercury's' anomaly).



Albert Einstein  
(1879-1955)

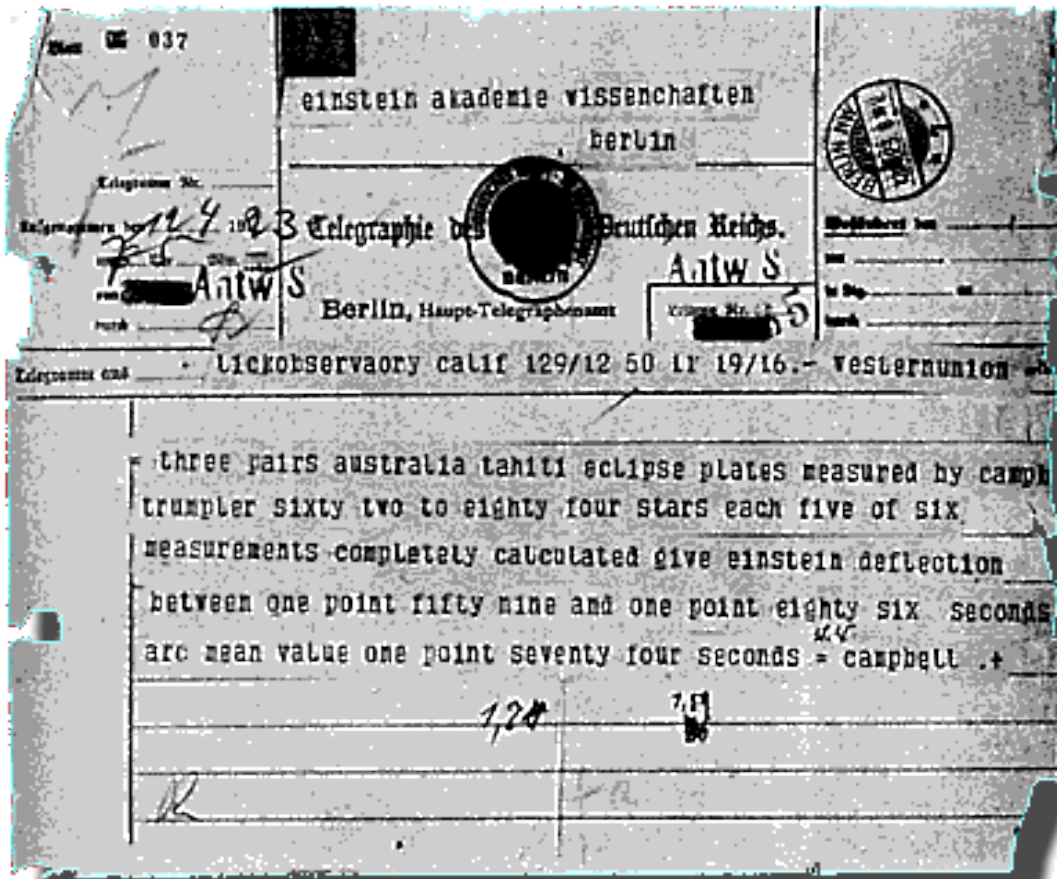
# The First Test of General Theory of Relativity

## Gravitational Deflection of Light:

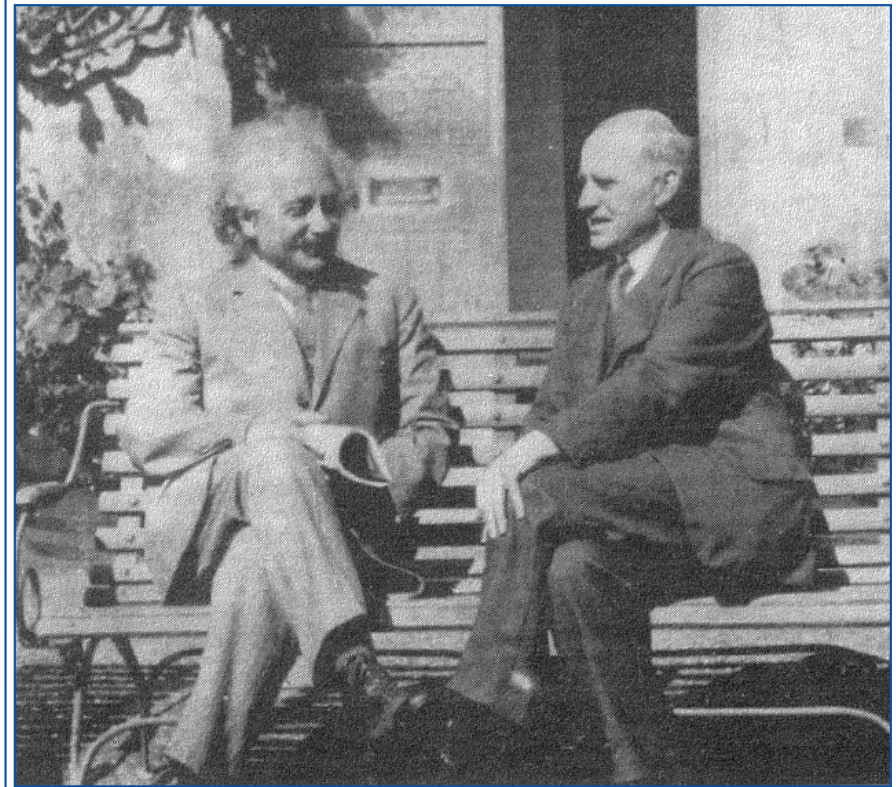
$$\theta_{gr}(b) = \frac{2(1+\gamma)GM_{\odot}}{bc^2} \simeq 8 \times 10^{-6} \left( \frac{1+\gamma}{2} \right) \left( \frac{R_{\odot}}{b} \right)$$

Solar Eclipse 1919:  
possible outcomes

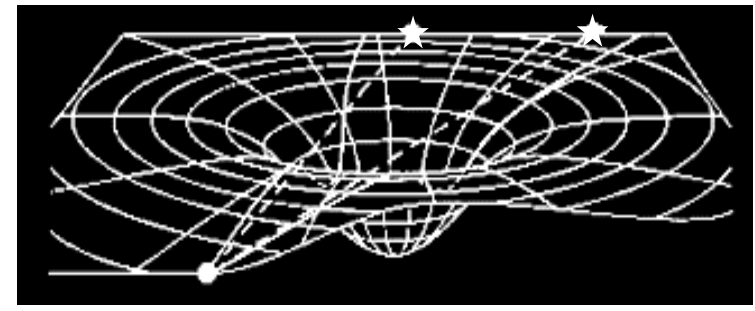
- Deflection = 0;
- Newton = 0.87 arcsec;
- Einstein = 2 x Newton = 1.75 arcsec



Campbell's telegram to Einstein, 1923



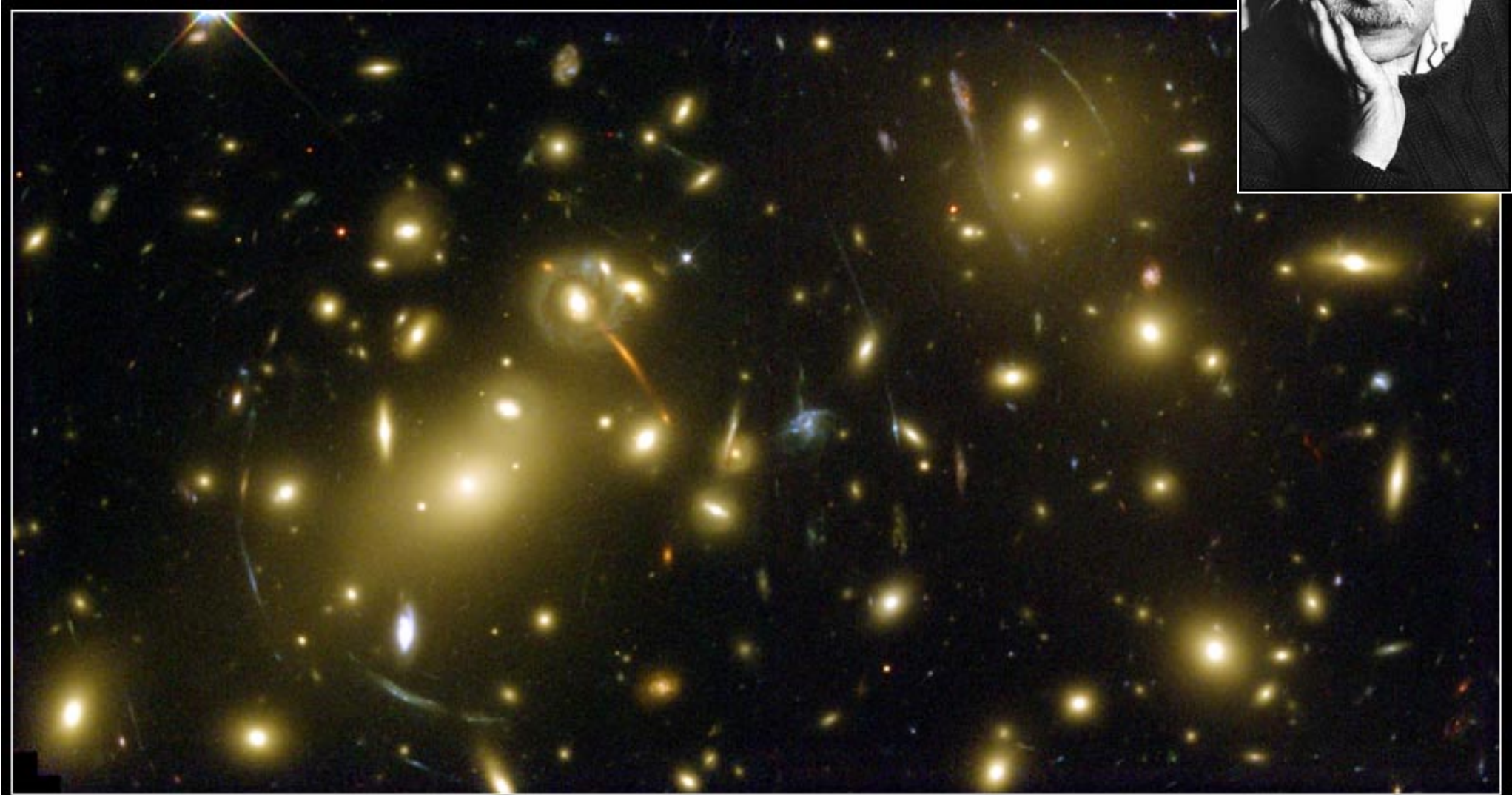
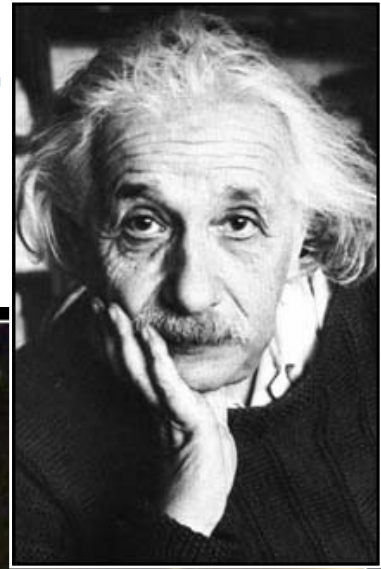
Einstein and Eddington, Cambridge, 1930





TESTS OF RELATIVISTIC GRAVITY IN SPACE

# Gravitational Deflection of Light is a Well-Known Effect Today



**Galaxy Cluster Abell 2218**

**HST • WFPC2**

NASA, A. Fruchter and the ERO Team (STScI) • STScI-PRC00-08

*not a complete list...*

|                           |                 |                                       |                            |                            |
|---------------------------|-----------------|---------------------------------------|----------------------------|----------------------------|
| Newton 1686               | Poincaré 1890   |                                       |                            |                            |
| Einstein 1912             | Nordstrøm 1912  | Nordstrøm 1913                        | Einstein & Fokker 1914     | Einstein 1915              |
| Whitehead 1922            | Cartan 1923     | Kaluza & Klein 1932                   | Fierz & Pauli 1939         | Birkhoff 1943              |
| Milne 1948                | Thiry 1948      | Papapetrou 1954                       | Jordan 1955                | Littlewood & Bergmann 1956 |
| Brans & Dicke 1961        | Yilmaz 1962     | Whitrow & Morduch 1965                | Kustaanheimo & Nuotio 1967 |                            |
| Page & Tupper 1968        | Bergmann 1968   | Deser & Laurent 1968                  | Nordtvedt 1970             | Wagoner 1970               |
| Bollini et al. 1970       | Rosen 1971      | Will & Nordtvedt 1972                 | Ni 1972                    | Hellings & Nordtvedt 1972  |
| Ni 1973                   | Yilmaz 1973     | Lightman & Lee 1973                   | Lee, Lightman & Ni 1974    | Rosen 1975                 |
| Belinfante & Swihart 1975 | Lee et al. 1976 | Bekenstein 1977                       | Barker 1978                | Rastall 1979               |
| Coleman 1983              | Hehl 1997       | Overlooked (20 <sup>th</sup> century) |                            |                            |

### Theory must be:

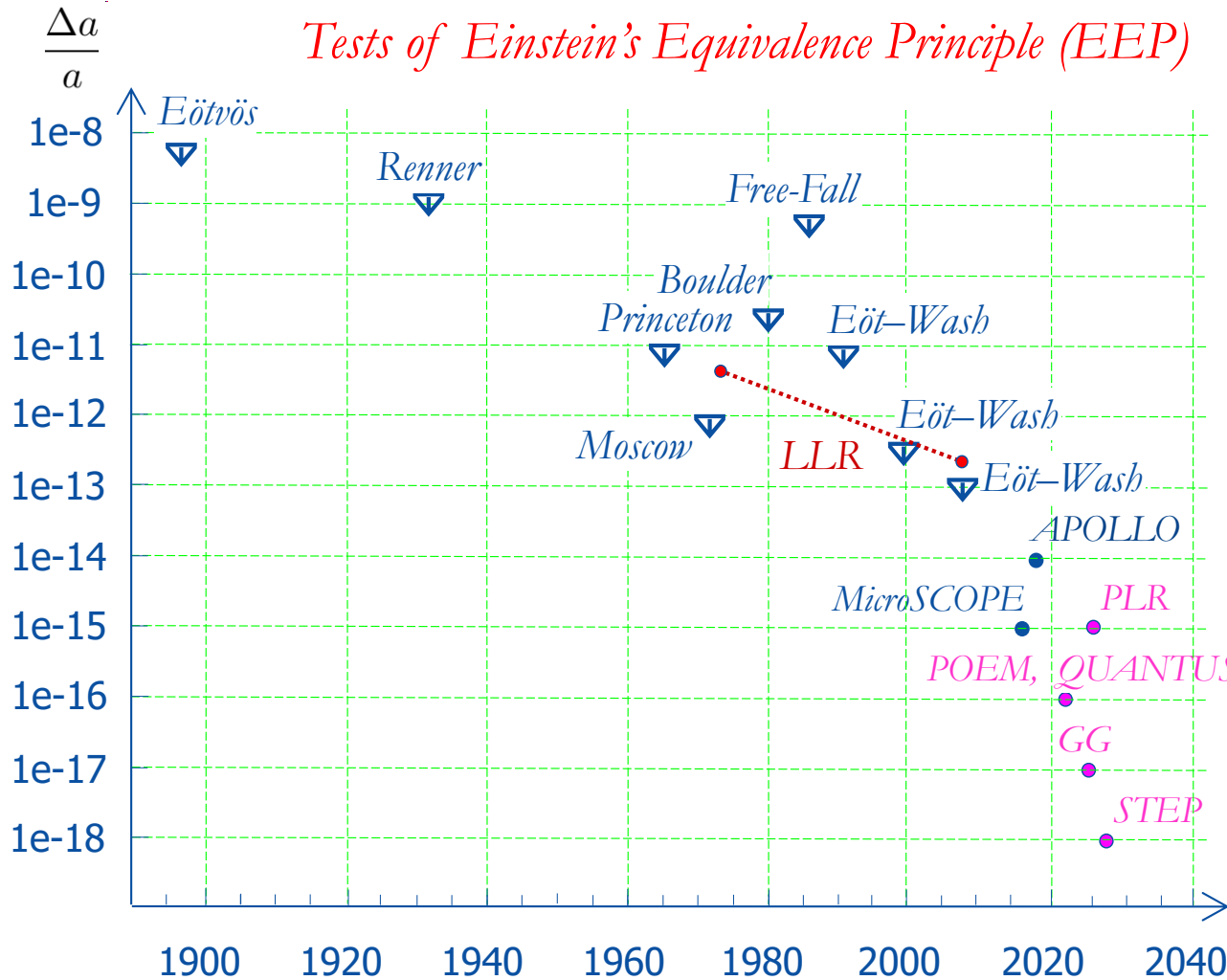
- Some authors proposed more than one theory, e.g. Einstein, Ni, Lee, Nordtvedt, Papapetrou, Yilmaz, etc.
- Some theories were variations of others
- Some were proposed in the 1910s/20s; many theories were in the 1960s/70s
- Overlooked: this is not a complete list!
- **Complete:** not a law, but a theory. Derive experimental results from first principles
- **Self-consistent:** get same results no matter which mathematics or models are used
- **Relativistic:** Non-gravitational laws are those of Special Relativity
- **Newtonian:** Reduces to Newton's equation in the limit of low gravity and low velocities

### *Theories that fail already*

|                           |                      |                                       |                                       |                            |  |
|---------------------------|----------------------|---------------------------------------|---------------------------------------|----------------------------|--|
| <b>Newton 1686</b>        | <b>Poincaré 1890</b> |                                       |                                       |                            |  |
| Einstein 1912             | Nordstrøm 1912       | Nordstrøm 1913                        | Einstein & Fokker 1914                | Einstein 1915              |  |
| Whitehead 1922            | Cartan 1923          | Kaluza & Klein 1932                   | <b>Fierz &amp; Pauli 1939</b>         | <b>Birkhoff 1943</b>       |  |
| <b>Milne 1948</b>         | Thiry 1948           | Papapetrou 1954                       | Jordan 1955                           | Littlewood & Bergmann 1956 |  |
| Brans & Dicke 1961        | Yilmaz 1962          | <b>Whitrow &amp; Morduch 1965</b>     | <b>Kustaanheimo &amp; Nuotio 1967</b> |                            |  |
| Page & Tupper 1968        | Bergmann 1968        | Deser & Laurent 1968                  | Nordtvedt 1970                        | Wagoner 1970               |  |
| Bollini et al. 1970       | Rosen 1971           | Will & Nordtvedt 1972                 | Ni 1972                               | Hellings & Nordtvedt 1972  |  |
| Ni 1973                   | Yilmaz 1973          | Lightman & Lee 1973                   | Lee, Lightman & Ni 1974               | Rosen 1975                 |  |
| Belinfante & Swihart 1975 | Lee et al. 1976      | Bekenstein 1977                       | Barker 1978                           | Rastall 1979               |  |
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- **Newton (1686)** – non-relativistic: implicit action at a distance - incompatible with special relativity
- **Poincare (1890)** and conformally flat theory of **Whitrow-Morduch (1965)** - incomplete: do not mesh well with non-gravitational physics (i.e., with electromagnetism of Maxwell)
- **Fierz & Pauli (1939)** [ "spin-2 field theory" ] – was inconsistent: field equations  $\Rightarrow$  all gravitating bodies move along straight lines, equation of motion  $\Rightarrow$  gravity deflects bodies
- **Birkhoff (1943)** – not Newtonian: demands *speed of sound = speed of light*.
- **Milne (1948)** – incomplete - no gravitational red-shift prediction
- **Kustaanheimo-Nuotio (1967)** – inconsistent: grav. redshift for photons, but not for light waves.

## Tests of Einstein's Equivalence Principle (EEP)



**Uniqueness of the Free Fall**  
 (⇒ Weak Equivalence Principle):

$$\vec{F} = m_I \vec{a} = m_G \vec{g}$$

$$\Rightarrow m_I = m_G$$

**All bodies fall with the same acceleration**

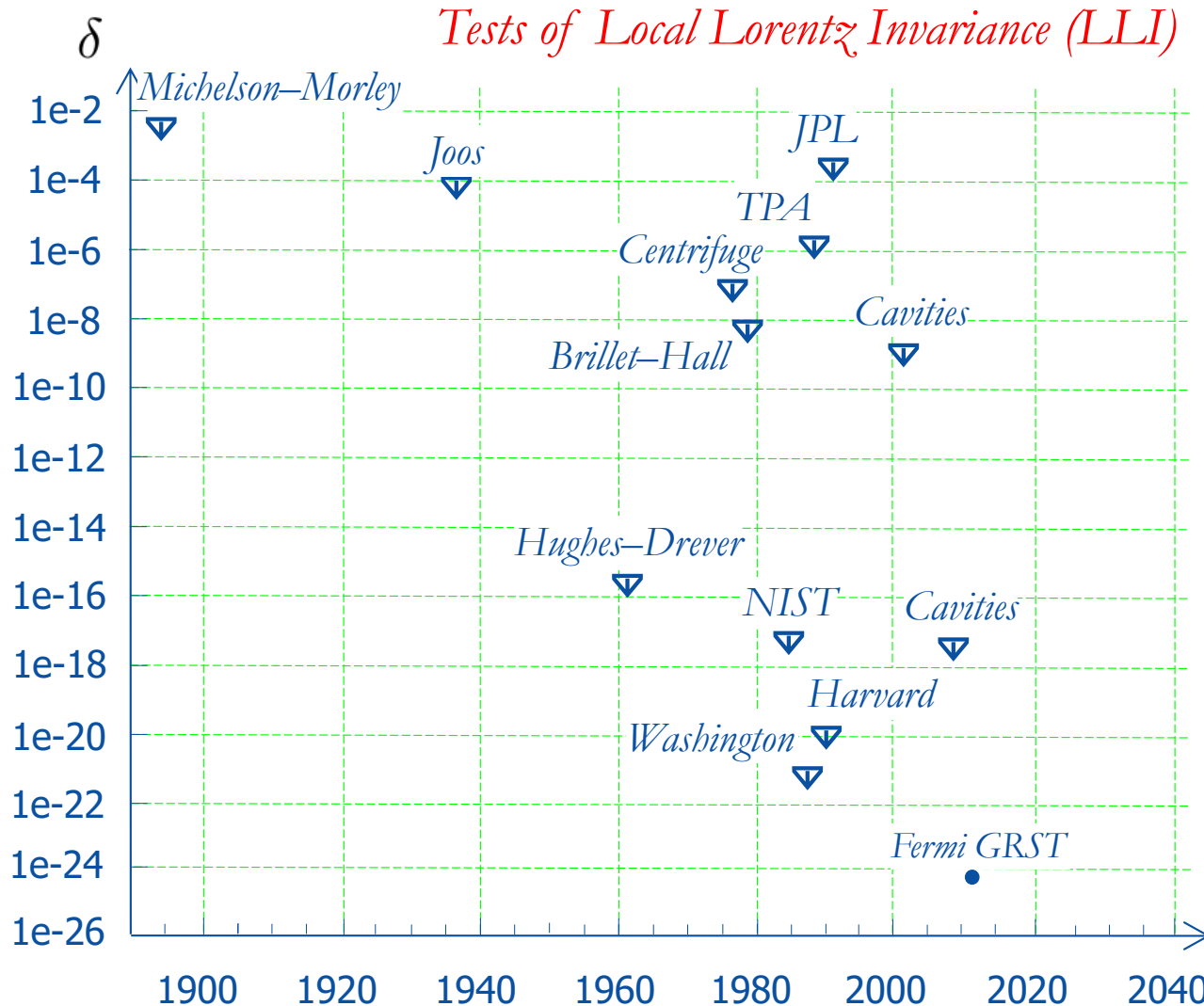
Define the test parameter that signifies a violation of the **WEP**

$$\frac{\Delta a}{a} = \frac{(a_1 - a_2)}{\frac{1}{2}(a_1 + a_2)} = \left[ \frac{m_G}{m_I} \right]_1 - \left[ \frac{m_G}{m_I} \right]_2$$

Let  $\Omega$  is the gravitational binding energy of a test body, then the test parameter that signifies a violation of the **SEP** is

- ● funded projects
  - ● proposed projects
  - *LLR, APOLLO, and PLR* are testing the Strong Equivalence Principle (SEP)
- $$\left[ \frac{m_G}{m_I} \right]_{\text{SEP}} = 1 + \eta \left( \frac{\Omega}{mc^2} \right)$$
- $$\frac{\Delta a}{a} = (4\beta - \gamma - 3) \left\{ \left[ \frac{\Omega}{mc^2} \right]_1 - \left[ \frac{\Omega}{mc^2} \right]_2 \right\}$$

## Tests of Local Lorentz Invariance (LLI)



- Michelson-Morley, Joos, Brillet-Hall: round-trip propagation
- Centrifuge, TPA, JPL: one-way signal propagation
- The rest are the Hughes-Drever experiments

$$\delta \equiv \frac{c}{c_0} - 1$$

### Local Lorentz Invariance:

- The outcome of a (small-scale) experiment does not depend on the orientation and the velocity of the (inertial) laboratory.
- Frameworks by Kostelecky et al., Jacobson et al.

### Future experiments:

- Clock comparisons
- Clocks vs microwave cavities
- Time of flight of high energy photons
- Birefringence in vacuum
- Neutrino oscillations
- Threshold effects in particle physics

### Test of one-way speed of light:

- Important to fundamental physics, cosmology, astronomy and astrophysics



Laboratory tests of **Lorentz Invariance**: search for preferred-frame effects

|                    |            |                                |
|--------------------|------------|--------------------------------|
| frame1 : $S(T, X)$ | e.g. CMB   | $v_{sol} \approx 377$ km/s     |
| frame2 : $s(t, x)$ | laboratory | $RA, dec = (11.2, -6.4^\circ)$ |

### Mansouri & Sexl, 1977

|  |  |                      |
|--|--|----------------------|
| $dT = \frac{1}{a}(dt + \frac{v}{c^2}dx)$                 | $a = 1 + \alpha \frac{v^2}{c^2} + \mathcal{O}(c^{-4})$ | time dilation        |
| $dX = \frac{1}{b}dx + \frac{v}{a}(dt + \frac{v}{c^2}dx)$ | $b = 1 + \beta \frac{v^2}{c^2} + \mathcal{O}(c^{-4})$  | length $\parallel v$ |
| $dY = \frac{1}{d}dy, dZ = \frac{1}{d}dz$                 | $d = 1 + \delta \frac{v^2}{c^2} + \mathcal{O}(c^{-4})$ | length $\perp v$     |

Deviations from the 2-way (round-trip) speed of light:  $\frac{c}{c'} \sim 1 + \left( \beta - \delta - \frac{1}{2} \right) \frac{v^2}{c^2} \sin^2 \theta + (\alpha - \beta + 1) \frac{v^2}{c^2}$

SR:  $\alpha = -1/2, \beta = 1/2, \delta = 0$

**Clock comparison experiments:**

$$P_{MM} = \left(\frac{1}{2} - \beta + \delta\right) \quad \text{Michelson-Morley: orientation dependence}$$

$$P_{KT} = (\beta - \alpha - 1) \quad \text{Kennedy-Thorndike: velocity dependence}$$

$$P_{IS} = \left|\alpha + \frac{1}{2}\right| \quad \text{Ives-Stillwell: contraction, dilation}$$

**Precision tests of Lorentz Invariance:**

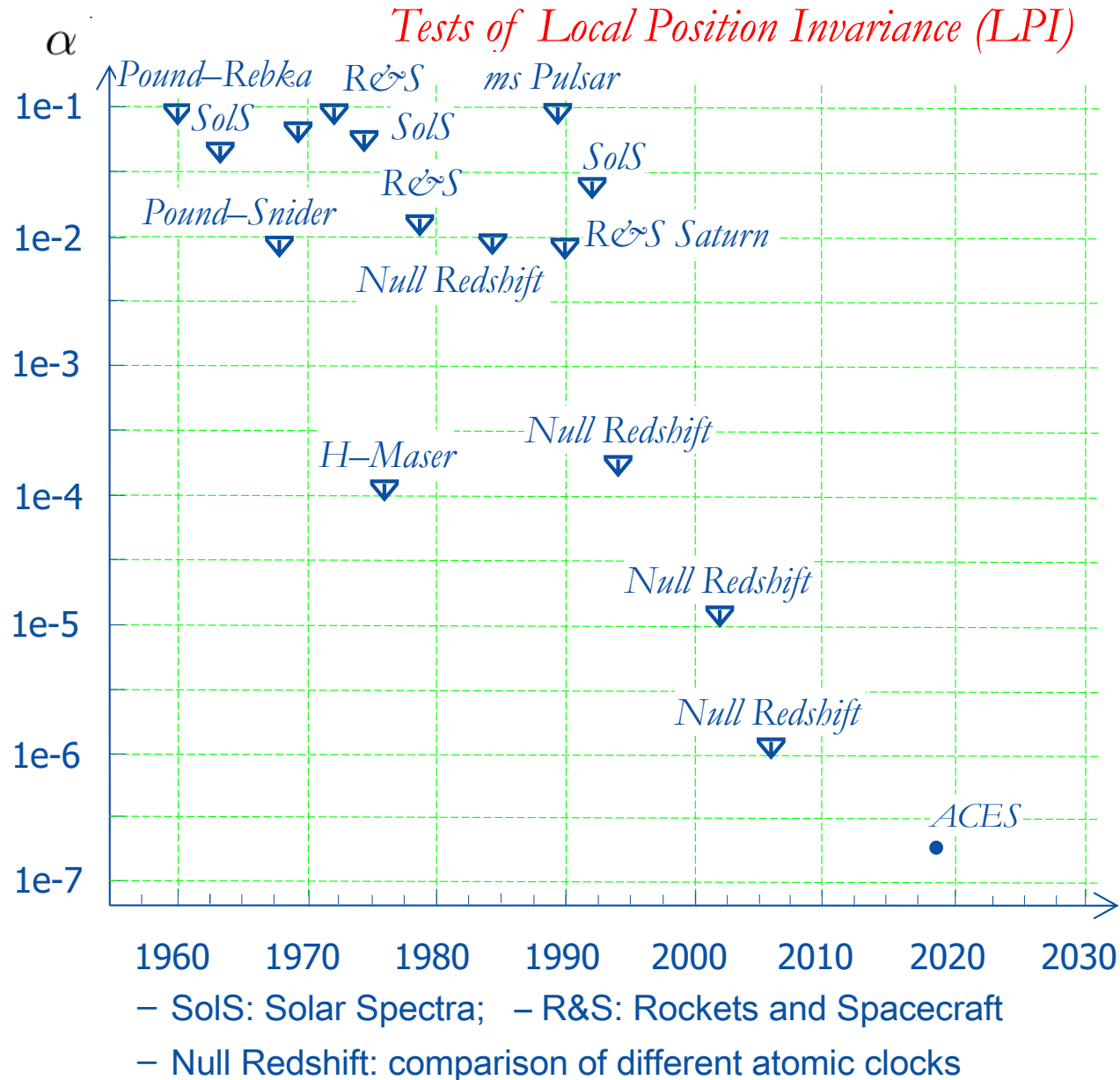
$$P_{MM} = (-4 \pm 8) \times 10^{-12} \quad \text{Herrmann et al, PRD 80 (2009) 105011}$$

$$P_{KT} = (4.8 \pm 3.7) \times 10^{-8} \quad \text{Toobar et al, PRD 81 (2010) 022003}$$

$$P_{IS} \leq 8.4 \times 10^{-8} \quad \text{Reinhardt et al, Nature Physics 3 (2007) 861}$$

**Tests of isotropy of the speed of light:**

$$\Delta c_{\theta}/c \lesssim 1 \times 10^{-17} \quad \text{Herrmann et al, PRD 80 (2009) 105011}$$



*Gravitational redshift:*

$$\frac{\Delta\nu}{\nu} = (1 + \alpha) \frac{\Delta U}{c^2}$$

**Local Position Invariance:**

- The outcome of any local non-gravitational experiment is independent of where & when in the universe it is performed

Splits into:

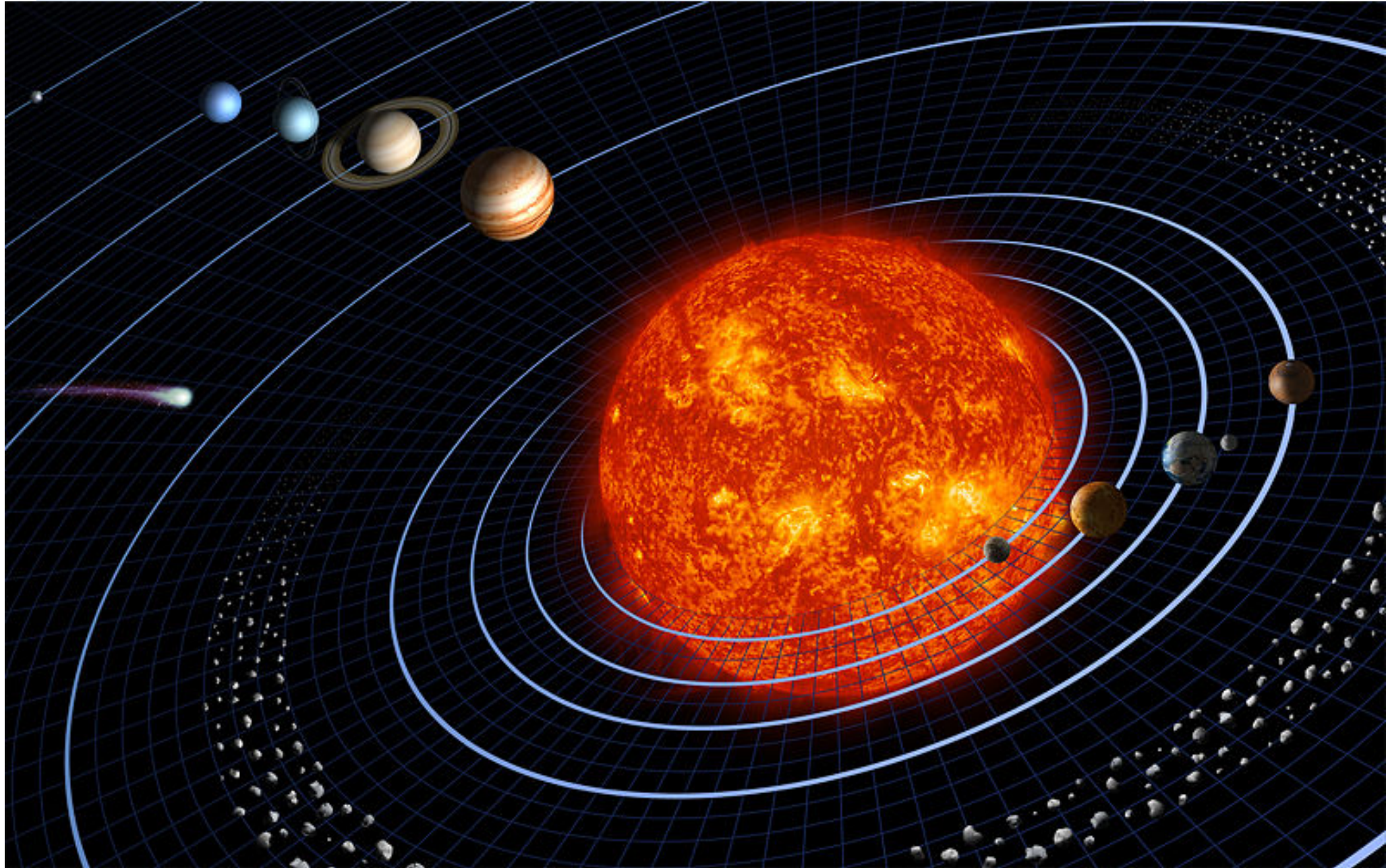
- spatial invariance
- temporal invariance
- Current best result is by Ashby et al., Phys. Rev. Lett. 98, 070802 (2007)
 
$$|\alpha| < 1.4 \times 10^{-6}$$
- A BEC test was attempted by Müller, Peters, and Chu, Nature 463, 926 (2010).



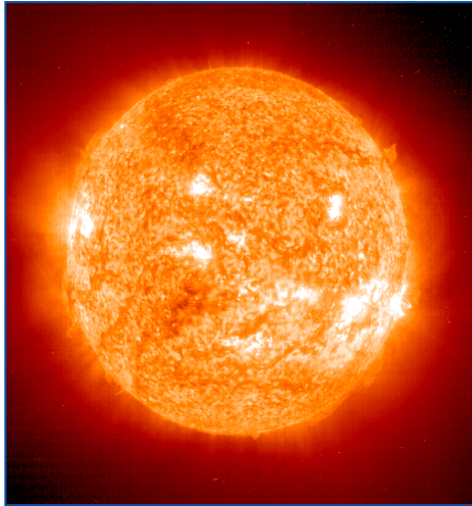
TESTS OF RELATIVISTIC GRAVITY IN SPACE



# Our solar system and tests of gravity



# Laboratory for Relativistic Gravity Experiments: Our Solar System

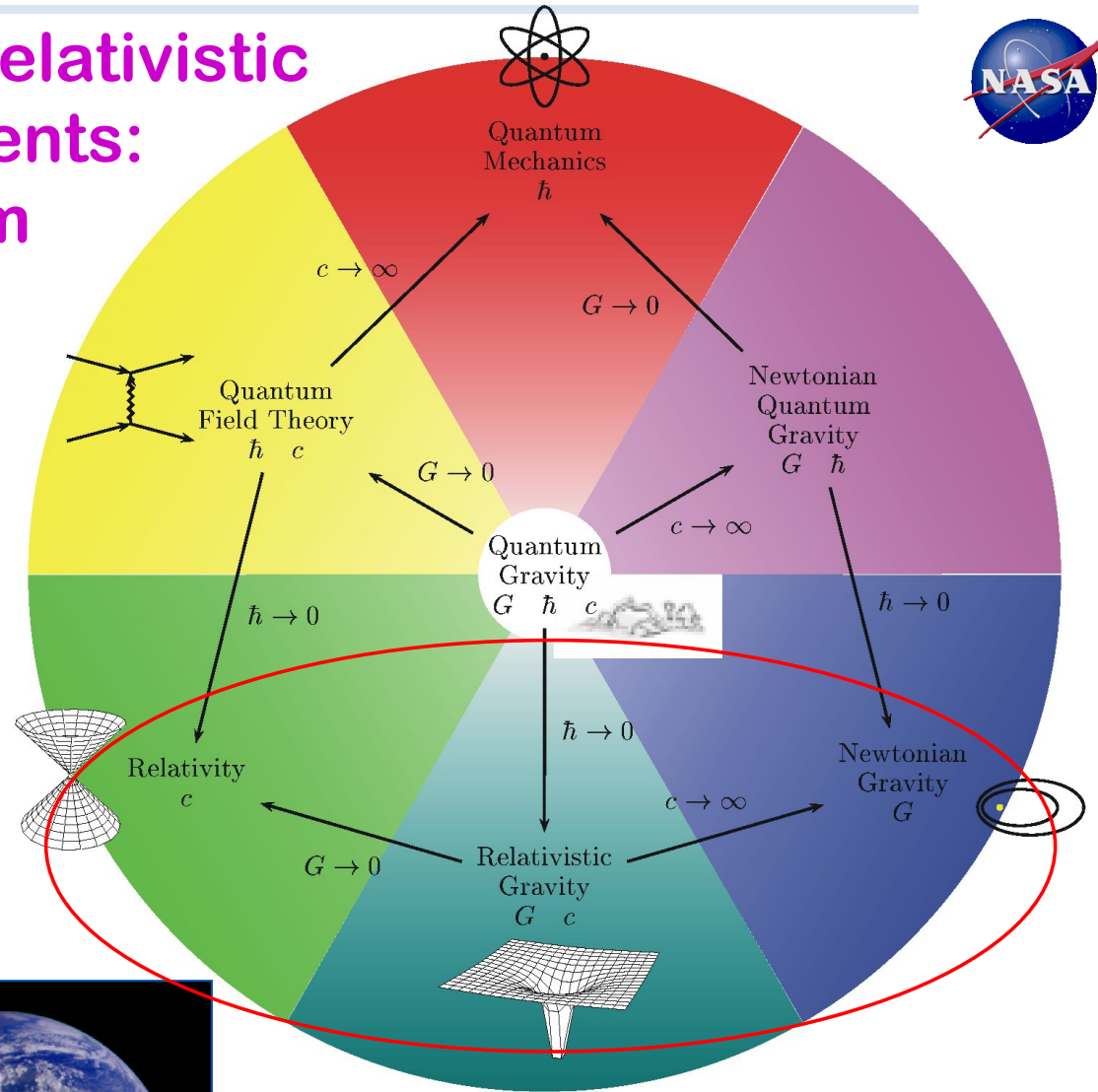


Strongest gravity potential

$$\frac{GM_{Sun}}{c^2 R_{Sun}} \sim 10^{-6}$$



$$\frac{GM_{\oplus}}{c^2 R_{\oplus}} \sim 10^{-9}$$



Most accessible region for gravity tests in space:

- ISS, LLR, SLR, free-fliers

Technology is available to conduct tests in the immediate solar proximity

# Parameterized Post-Newtonian (PPN) formalism

PPN Formalism: Eddington, Fock, Chandrasekhar, Dicke, Nordtvedt, Thorne, Will,...

$$\begin{aligned}
 g_{00} &= 1 - \frac{2}{c^2} \sum_{j \neq i} \frac{\mu_j}{r_{ij}} + \frac{2\beta}{c^4} \left[ \sum_{j \neq i} \frac{\mu_j}{r_{ij}} \right]^2 - \frac{1+2\gamma}{c^4} \sum_{j \neq i} \frac{\mu_j \dot{r}_j^2}{r_{ij}} + \\
 &\quad + \frac{2(2\beta-1)}{c^4} \sum_{j \neq i} \frac{\mu_j}{r_{ij}} \sum_{k \neq j} \frac{\mu_k}{r_{jk}} - \frac{1}{c^4} \sum_{j \neq i} \mu_j \frac{\partial^2 r_{ij}}{\partial t^2} + \mathcal{O}(c^{-5}) \\
 g_{0\alpha} &= \frac{2(1+\gamma)}{c^3} \sum_{j \neq i} \frac{\mu_j \dot{\mathbf{r}}_j^\alpha}{r_{ij}} + \mathcal{O}(c^{-5}) \\
 g_{\alpha\beta} &= -\delta_{\alpha\beta} \left( 1 + \frac{2\gamma}{c^2} \sum_{j \neq i} \frac{\mu_j}{r_{ij}} + \frac{3\delta}{2c^4} \left[ \sum_{j \neq i} \frac{\mu_j}{r_{ij}} \right]^2 \right) + \mathcal{O}(c^{-5})
 \end{aligned}$$

- Assumption: Local Lorentz Invariance (LLI) and local position invariance (LPI) hold, thus, preferred frame parameters  $\alpha_1, \alpha_2, \alpha_3$  are not included...
- General case, there are 10 PPN parameters:  $\gamma, \beta, \zeta, \alpha_1, \alpha_2, \alpha_3, \xi_1, \xi_2, \xi_3, \xi_4$
- $\gamma$  are  $\beta$  the Eddington's parameterized post-Newtonian (PPN) parameters:

General relativity:  $\gamma = \beta = 1$

Brans-Dicke theory:  $\gamma = \frac{1+\omega}{2+\omega}, \beta = 1$

- $\delta$  is the post-PPN parameter – important for next generation of light propagation tests.

# PPN Equations of Motion (a part of the model)

$$\begin{aligned}
 \ddot{\mathbf{r}}_i = & \sum_{j \neq i} \frac{Gm_j(\mathbf{r}_j - \mathbf{r}_i)}{r_{ij}^3} \left\{ \left[ \frac{m_G}{m_I} \right]_i - \frac{2(\beta + \gamma)}{c^2} \sum_{l \neq i} \frac{Gm_l}{r_{il}} - \frac{2\beta - 1}{c^2} \sum_{k \neq j} \frac{Gm_k}{r_{jk}} + \right. \\
 & + \gamma \left( \frac{\dot{r}_i}{c} \right)^2 + (1 + \gamma) \left( \frac{\dot{r}_j}{c} \right)^2 - \frac{2(1 + \gamma)}{c^2} \dot{\mathbf{r}}_i \dot{\mathbf{r}}_j + \frac{\dot{G} \cdot t}{G} - \\
 & - \frac{3}{2c^2} \left[ \frac{(\mathbf{r}_i - \mathbf{r}_j) \dot{\mathbf{r}}_j}{r_{ij}} \right]^2 + \frac{1}{2c^2} (\mathbf{r}_j - \mathbf{r}_i) \ddot{\mathbf{r}}_j \left. \right\} + \\
 & + \frac{1}{c^2} \sum_{j \neq i} \frac{Gm_j}{r_{ij}^3} \left\{ [\mathbf{r}_i - \mathbf{r}_j] \cdot [(2 + 2\gamma)\dot{\mathbf{r}}_i - (1 + 2\gamma)\dot{\mathbf{r}}_j] \right\} (\dot{\mathbf{r}}_i - \dot{\mathbf{r}}_j) + \\
 & + \frac{3 + 4\gamma}{2c^2} \sum_{j \neq i} \frac{Gm_j \ddot{\mathbf{r}}_j}{r_{ij}} + \sum_{m=1}^3 \frac{Gm_m(\mathbf{r}_m - \mathbf{r}_i)}{r_{im}^3} + \sum_{c,s,m} \mathbf{F}_{\text{asteroids}}
 \end{aligned}$$

Possible EP violation

Possible temporal dependence of  $G$

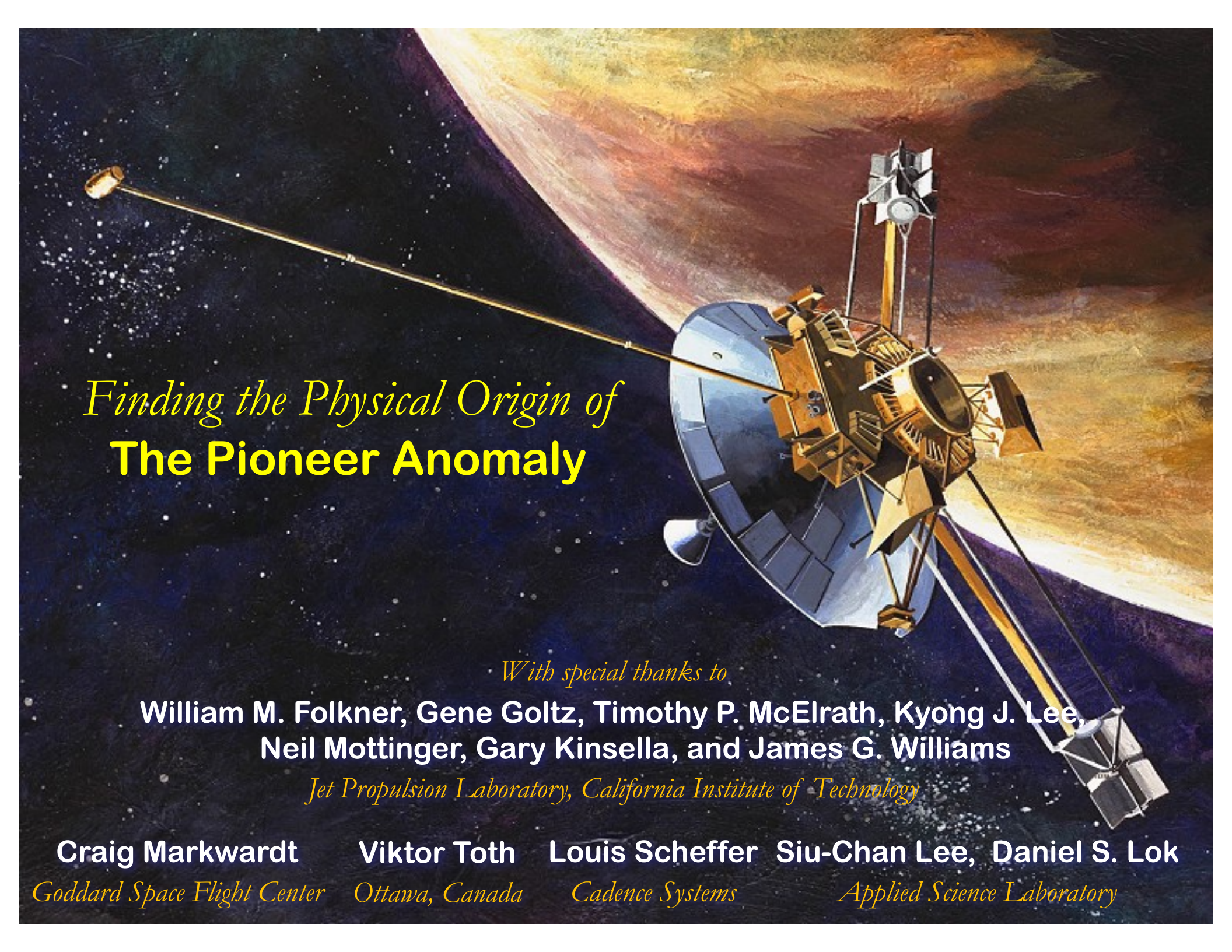
$$\left[ \frac{m_G}{m_I} \right]_{\text{SEP}} = 1 + \eta \left( \frac{\Omega}{mc^2} \right)$$

$$\eta = 4\beta - \gamma - 3$$

$$\Omega_i = -\frac{G}{2} \int_i d^3x \rho_i U_i = -\frac{G}{2} \int_i d^3x d^3x' \frac{\rho_i(\mathbf{r}) \rho_i(\mathbf{r}')}{|\mathbf{r} - \mathbf{r}'|}$$

- In general theory of relativity  $\beta = \gamma = 1$ , thus  $\eta = 0$  (this is not the case for scalar-tensor theories of gravity, for instance, where these parameters can have different values).

$$t_2 - t_1 = \frac{r_{12}}{c} + (1 + \gamma) \sum_i \frac{\mu_i}{c^3} \ln \left[ \frac{r_1^i + r_2^i + r_{12}^i + \frac{(1+\gamma)\mu_i}{c^2}}{r_1^i + r_2^i - r_{12}^i + \frac{(1+\gamma)\mu_i}{c^2}} \right] + \mathcal{O}(c^{-5})$$



*Finding the Physical Origin of*  
**The Pioneer Anomaly**

*With special thanks to*

**William M. Folkner, Gene Goltz, Timothy P. McElrath, Kyong J. Lee,  
Neil Mottinger, Gary Kinsella, and James G. Williams**

*Jet Propulsion Laboratory, California Institute of Technology*

**Craig Markwardt**

**Viktor Toth**

**Louis Scheffer**

**Siu-Chan Lee, Daniel S. Lok**

*Goddard Space Flight Center*

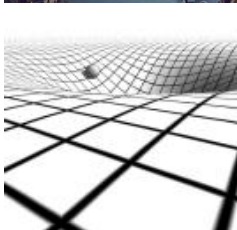
*Ottawa, Canada*

*Cadence Systems*

*Applied Science Laboratory*



# The Pioneer Anomaly: 1998-2012

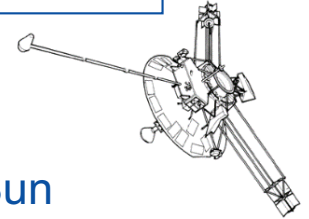


Phys. Rev. Let. 81 (1998) 2858

■ Anomalous acceleration of Pioneers 10 & 11:

$$a_P = (8.74 \pm 1.33) \times 10^{-10} \text{ m/s}^2$$

- A constant acceleration of both Pioneers *towards* the Sun
- **No mechanism** or **theory** to unambiguously explain the effect
- Most likely cause is on-board systematics, yet to be found (1998)



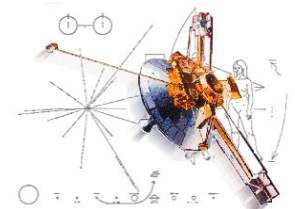
Phys. Rev. D 65 (2002) 082004

■ Pioneer anomaly at face value:

- Pioneers 10/11 conducted the largest-scale-ever test of gravity in the solar system... that failed to confirm Newton's law of gravitation...
- In fact, the Pioneer anomaly is an apparent violation of the Newton's gravity  $1/r^2$  law in regions farther than 25 AU from the Sun....

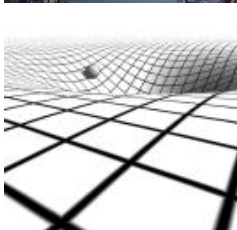
## Possible Origin of the "Dark Force"?

- New Physics: interesting ideas suggested...  $a_P \simeq cH$
- We focus on **conventional physics**, as the cause:
  - Gas leaks, drag force, **thermal recoil force**, etc...



Ned, of course, new the right answer already in ~2001:  
<http://www.astro.ucla.edu/~wright/PioneerAA.html>

# The Pioneer Anomaly: 1998-2012

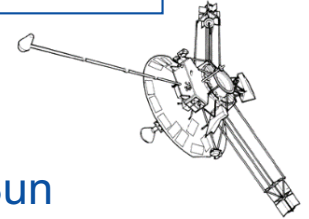


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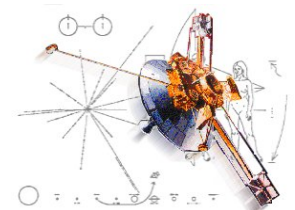
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**2012: The Pioneer anomaly is of the thermal origin!**

PRL 108, 241101 (2012)

- Beware of the PA when driving at night with high-beams on...

# 40+ Years of Solar System Gravity Tests

## Techniques for Gravity Tests:

### Radar Ranging:

- Planets: Mercury, Venus, Mars
- s/c: Mariners, Vikings, Pioneers, Cassini, Mars Global Surveyor, Mars Orbiter, etc.
- VLBI, GPS, etc.

### Laser:

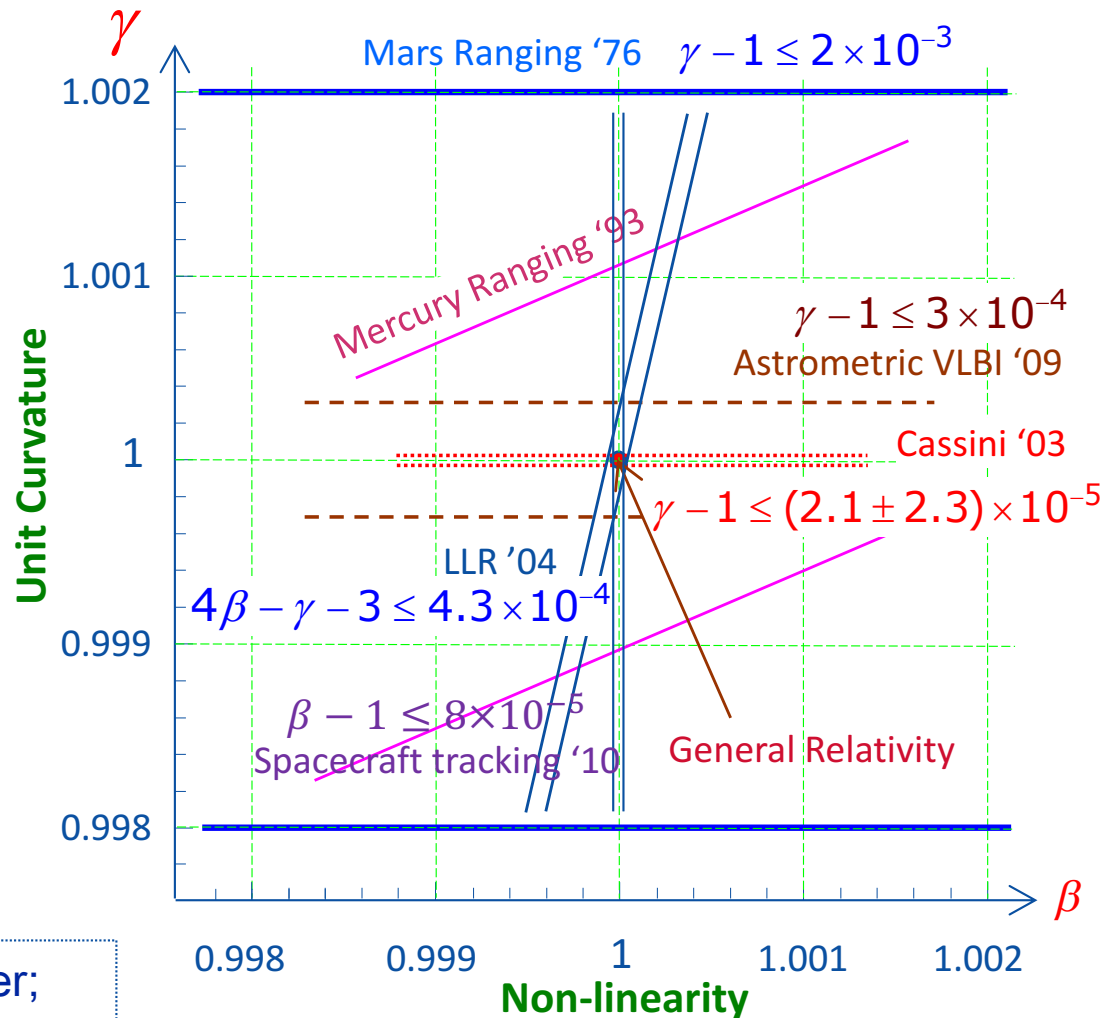
- SLR, LLR, interplanetary, etc.

## Dedicated Gravity Missions:

- LLR (1969 - on-going!!)**
- GP-A, '76; LAGEOS, '76, '92; GP-B, '04; LARES, '12; MicroSCOPE, '16, ACES, '18; eLISA, 2030+(?)

## New Engineering Discipline – Applied General Relativity:

- Daily life: GPS, geodesy, time transfer;
- Precision measurements: deep-space navigation &  $\mu$ as-astrometry (ESA's Gaia).



A factor of 100 in 40 years is impressive, but is not enough for the near future!

# The Current Values of the PPN Parameters (2017)

| Parameter  | What is measured relative to General Relativity?  | Current value        | Effects                      | Experiments                                    |
|------------|---|----------------------|------------------------------|--|
| $\gamma-1$ | Measure of space curvature produced by unit mass  | $2.3 \times 10^{-5}$ | Time delay, light deflection | Cassini tracking                               |
| $\beta-1$  | Measure of non-linearity in gravitational superposition   | $8.0 \times 10^{-5}$ | Perihelion shift             | Solar system planetary and spacecraft tracking |
| $\xi$      | Measure of existence of preferred location effects  | $4 \times 10^{-9}$   | Spin precession              | Millisecond pulsars                            |
| $\alpha_1$ | Measure the existence of preferred frame effects  | $1 \times 10^{-4}$   | Orbit polarization           | Lunar laser ranging                            |
|            |   | $4 \times 10^{-5}$   | Orbit polarization           | PSR J1738+0333                                 |
| $\alpha_2$ |   | $4 \times 10^{-7}$   | Spin precession              | Sun axis' alignment w/ ecliptic                |
|            |   | $2 \times 10^{-9}$   | Spin precession              | Millisecond pulsars                            |
| $\alpha_3$ |   | $4 \times 10^{-20}$  | Self-acceleration            | Pulsar spin-down statistics                    |
| $\zeta_1$  | Measure (plus $\alpha_3$ ) of the failure of conservation laws of energy, momentum and angular momentum | $2 \times 10^{-2}$   | –                            | Combined PPN bounds                            |
| $\zeta_2$  |   | $4 \times 10^{-5}$   | Binary pulsar acceleration   | Pulsar: PSR 1913+16                            |
| $\zeta_3$  |   | $1 \times 10^{-8}$   | Newton's 3rd law             | Lunar acceleration                             |
| $\zeta_4$  |   | $6 \times 10^{-3}$   | –                            | Kreuzer experiment                             |

### *“Aesthetics-Based” Conclusion for 20<sup>th</sup> Century*

Newton 1686 Poincaré 1890

Einstein 1912 Nordstrøm 1912 Nordstrøm 1913 Einstein & Fokker 1914 **Einstein 1915**

Whitehead 1922 Cartan 1923 Kaluza & Klein 1932 Fierz & Pauli 1939 Birkhoff 1943

Milne 1948 Thiry 1948 Papapetrou 1954 Jordan 1955 Littlewood & Bergmann 1956

Brans & Dicke 1961 Yilmaz 1962 Whitrow & Morduch 1965 Kustaanheimo & Nuotio 1967

Page & Tupper 1968 Bergmann 1968 Deser & Laurent 1968 Nordtvedt 1970 Wagoner 1970

Bollini et al. 1970 Rosen 1971 Will & Nordtvedt 1972 Ni 1972 Hellings & Nordtvedt 1972

Ni 1973 Yilmaz 1973 Lightman & Lee 1973 Lee, Lightman & Ni 1974 Rosen 1975

Belinfante & Swihart 1975 Lee et al. 1976 Bekenstein 1977 Barker 1978 Rastall 1979

Coleman 1983 Hehl 1997 Overlooked (20<sup>th</sup> century)

- “Among all bodies of physical law none has ever been found that is simpler and more beautiful than Einstein's geometric theory of gravity”
  - Misner, Thorne and Wheeler, 1973
- “[...] Unfortunately, any finite number of effects can be fitted by a sufficiently complicated theory. [...] Aesthetic or philosophical motives will therefore continue to play a part in the widespread faith in Einstein's theory, even if all tests verify its predictions.”
  - Malcolm MacCallum, 1976

*First decade of 21<sup>st</sup> century... they are back!*

|  |  |                                       |                               |                            |  |
|--|--|---------------------------------------|-------------------------------|----------------------------|--|
| Newton 1686                                      | Poincaré 1890                              |                                       |                               |                            |  |
| Einstein 1912                                    | Nordstrøm 1912                             | Nordstrøm 1913                        | Einstein & Fokker 1914        | <b>Einstein 1915</b>       |  |
| Whitehead 1922                                   | Cartan 1923                                | <b>Kaluza &amp; Klein 1932</b>        | Fierz & Pauli 1939            | Birkhoff 1943              |  |
| Milne 1948                                       | Thiry 1948                                 | Papapetrou 1954                       | Jordan 1955                   | Littlewood & Bergmann 1956 |  |
| <b>Brans &amp; Dicke 1961</b>                    | Yilmaz 1962                                | Whitrow & Morduch 1965                | Kustaanheimo & Nuotio 1967    |                            |  |
| Page & Tupper 1968                               | Bergmann 1968                              | Deser & Laurent 1968                  | Nordtvedt 1970                | Wagoner 1970               |  |
| Bollini et al. 1970                              | Rosen 1971                                 | Will & Nordtvedt 1972                 | Ni 1972                       | Hellings & Nordtvedt 1972  |  |
| Ni 1973  | Yilmaz 1973                                | Lightman & Lee 1973                   | Lee, Lightman & Ni 1974       | Rosen 1975                 |  |
| Belinfante & Swihart 1975                        | Lee et al. 1976                            | Bekenstein 1977                       | Barker 1978                   | Rastall 1979               |  |
| Coleman 1983                                     | Hehl 1997                                  | Overlooked (20 <sup>th</sup> century) | <b>Scalar-Tensor Theories</b> |                            |  |
| <b>Arkani-Hamed, Dimopoulos &amp; Dvali 2000</b> | <b>Dvali, Gabadadze &amp; Poratti 2003</b> | <b>Strings theory?</b>                |                               |                            |  |
| <b>Bekenstein 2004</b>                           | <b>Moffat 2005</b>                         | <b>Multiple f(R) models 2003-10</b>   | <b>Bi-Metric Theories</b>     |                            |  |

### Need for new theory of gravity:

- Classical GR description breaks down in regimes with large curvature
- If gravity is to be quantized, GR will have to be modified or extended

### Other challenges:

- Dark Matter
- Dark Energy

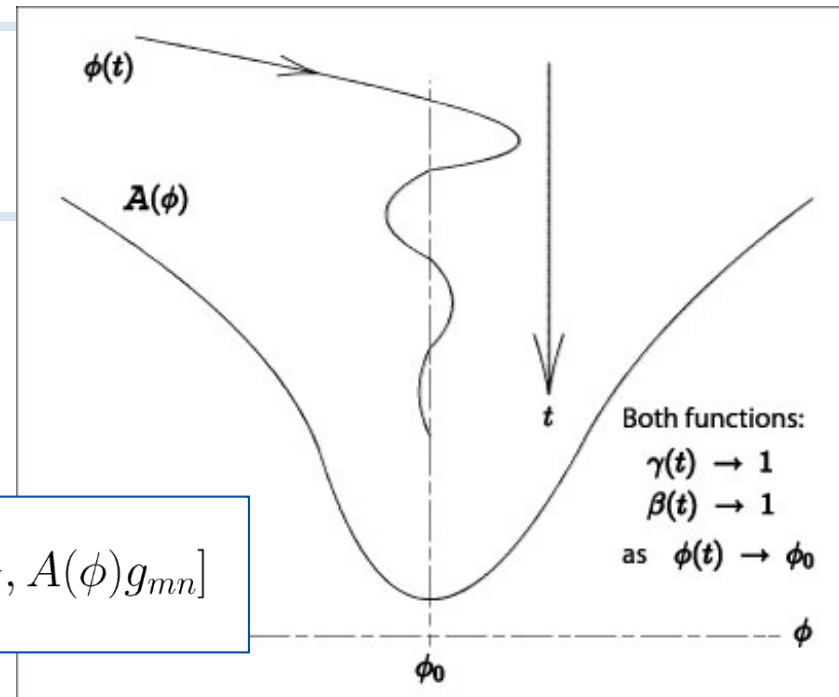
### Motivations for new tests of GR:

- GR is a fundamental theory
- Alternative theories & models
- New ideas & techniques require comprehensive investigations

## Long-range massless [or low-mass] scalar:

The low-energy limit of the String Theory in 'Einstein Frame' (Damour-Nordtvedt-Polyakov 1993) suggests:

$$S = -\frac{1}{16\pi G} \int dx^4 \sqrt{-g} \left( R - 2g^{mn} \nabla_m \phi \nabla_n \phi \right) + S_M[\psi_M, A(\phi) g_{mn}]$$



Expansion  $A(\phi)$  around background value  $\phi_0$  of the scalar leads:

$$\ln A(\varphi) = \ln A(\varphi_0) + \alpha_0(\varphi - \varphi_0) + \frac{1}{2}k_0(\varphi - \varphi_0)^2 + \mathcal{O}(\Delta\varphi^3)$$

Slope  $\alpha_0$  measures the coupling strength of interaction between matter and the scalar.

$$\gamma - 1 = \frac{-2\alpha_0^2}{1 + \alpha_0^2} \simeq -2\alpha_0^2$$

$$\beta - 1 = \frac{1}{2} \frac{\alpha_0^2 k_0}{(1 + \alpha_0^2)^2} \simeq \frac{1}{2} \alpha_0^2 k_0 \simeq \frac{1}{4} (1 - \gamma) k_0$$

Scenario for cosmological evolution of the scalar (Damour, Piazza & Veneziano 2002):

$$\gamma - 1 \sim 7.3 \times 10^{-7} \left( \frac{H_0}{\Omega_0^3} \right)^{\frac{1}{2}}$$

$\Rightarrow$

$$\gamma - 1 \sim 10^{-5} - 10^{-7}$$

**The unit curvature PPN parameter  $\gamma$  is the most important quantity to test**

### Modifications of Einstein Gravity

$$S = \frac{1}{16\pi G} \int d^4x \sqrt{-g} f(R) + S_m[g_{\mu\nu}, \psi]$$

Carroll et al, PRD 70 (2004) 043528

...

### Modification of PPN Gravity

$$\gamma - 1 = - \frac{f''(R)^2}{f'(R) + 2f''(R)^2},$$

$$\beta - 1 = \frac{1}{4} \frac{f'(R) \cdot f''(R)}{2f'(R) + 3f''(R)^2} \frac{d\gamma}{dR}.$$

Analogy between scalar-tensor and higher-order gravity

Constraints on ...  $f(R)$  from solar system experiments...

...tight restrictions on the form of the gravitational Lagrangian

Need for cosmological “PPN formalism”



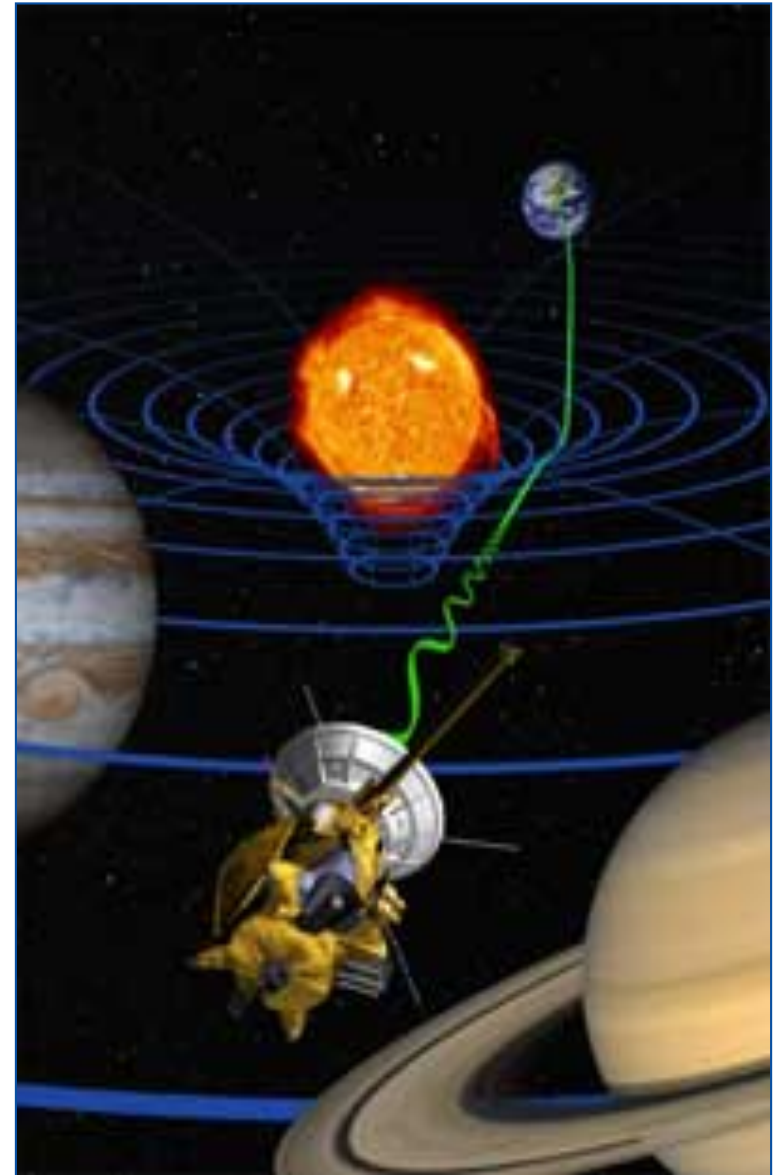
## Cassini 2003: Where Do We Go From Here?

### Cassini Conjunction Experiment:

- Spacecraft—Earth separation > 1 billion km
- Doppler/Range: X~7.14GHz & Ka~34.1GHz
- Result:  $\gamma = 1 + (2.1 \pm 2.3) \times 10^{-5}$

### Possible with Existing Technologies?!

- VLBI [current  $\gamma = 3 \times 10^{-4}$ ]: limited to  $\sim 1 \times 10^{-4}$ :
  - uncertainty in the radio source coordinates
- LLR [current  $\eta = 4 \times 10^{-4}$ ]: in 5 years  $\sim 3 \times 10^{-5}$ :
  - mm accuracies [APOLLO] & modeling efforts
- $\mu$ -wave ranging to a lander on Mars  $\sim 6 \times 10^{-6}$
- GRACE-FO in Earth's orbit (2017):  $\sim 5 \times 10^{-6}$
- tracking of BepiColombo s/c at Mercury  $\sim 2 \times 10^{-6}$
- Optical astrometry [current  $\gamma = 3 \times 10^{-3}$ ]:
  - ESA's Gaia mission (2013)  $\sim 1 \times 10^{-6}$  (2018?)

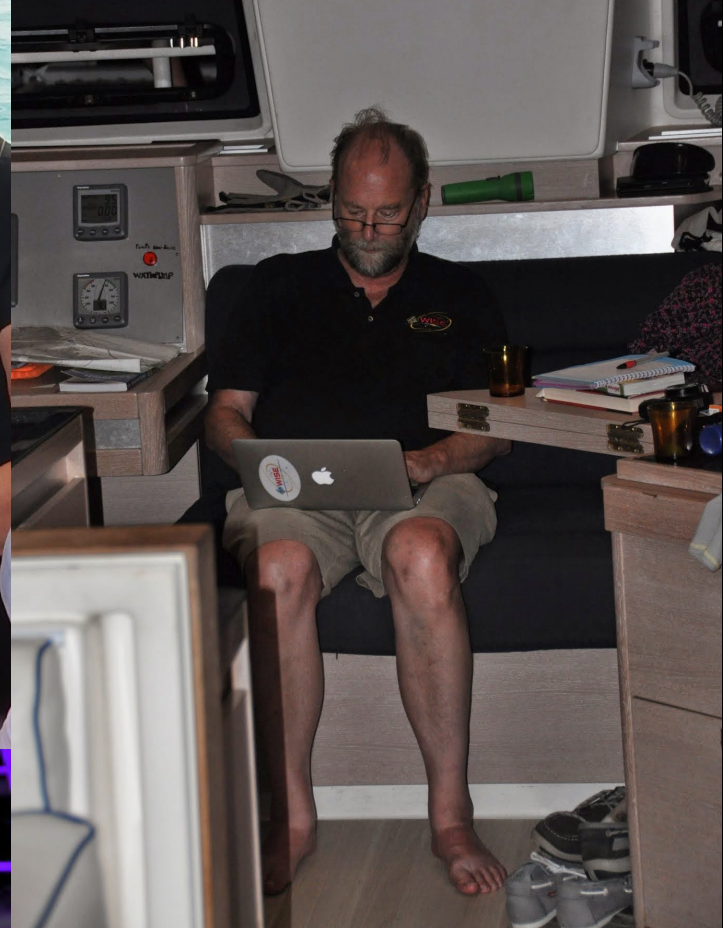


One needs a **dedicated mission** to explore accuracies better than  $10^{-6}$  for both PPN parameters  $\gamma$  (and  $\beta$ ). Interplanetary laser ranging is a possibility.

## Conclusions

- Recent technological progress: [arXiv:0902.3004 \[gr-qc\]](https://arxiv.org/abs/0902.3004)
  - Resulted in new instruments with unique performance
  - Could lead to major improvements in the tests of relativistic gravity
  - Already led to a number of recently proposed gravitational experiments
- Challenges for solar system tests of gravity:
  - Dedicated space-based experiments are very expensive – the science must worth the cost... – *EP,  $G$ -dot and PPN  $\gamma$  tests are most relevant.*
  - Motivation for the tests in a weak gravity field is a challenge: there is no strong expectation to see deviations from GR in the solar system (we are looking for anomalies...) – *access to strong(er) gravity regime is needed!*
  - GR is very hard to modify, embed, extend or augment (whatever your favorite verb is...) – *thus, perhaps, those anomalies are important...*
  - PPN formalism becomes less relevant for modern gravity research...
  - Looking to Cosmos for help? There is none: Little or no correspondence between cosmological tests and physical principles in the foundation of tests of PPN gravity – *EP, LLI, LPI, energy-momentum conservation, etc...*

Thank You, Ned!



Happy Birthday!