
WIR SCHAFFEN WISSEN – HEUTE FÜR MORGEN

Sven Reiche :: Paul Scherrer Institut

Limitation of High-Efficient Taper Modelling

UCLA, April 2018

Outline

- Code Classification:
  - Steady-state vs Time-dependent
  - Slowly Varying Envelope Approximation & Period-Averaged Code
- Particle Oscillation in Radiation Field : Sideband Instability
- Beyond Saturation Power
- Missing Physics
- Realistic Models
- Optimization Problems
- Summary

Equation of Motions & Steady-State Simulations

- Electrons undergo a periodic oscillation, interacting with a radiation field, which slips over it.
- Resonant interaction with a ponderomotive wave:

$$(k + k_u)z - \omega t = const \quad \Rightarrow \quad \beta_z = \frac{k}{k + k_u}$$

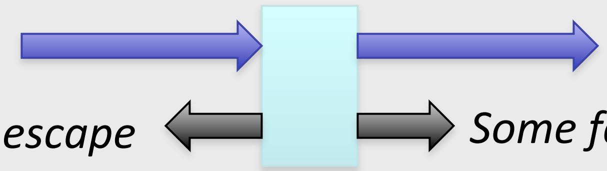
- "Resonance" is already a restriction in your model:
  - Relevant interaction occurs within a narrow bandwidth
  - There is a central wavelength with best performance

- Restricting to a single frequency component (only a single amplitude and phase) yields the simplest model:

Steady-State Model

Core Algorithm – Steady State Simulation

- Two step algorithm (Leap-frog Algorithm):
  - Advance radiation field (diffraction + emission by electrons)
  - Advance electrons (interaction with field and change in ponderomotive phase)



Electron Slice (one wavelength)

Field slips in

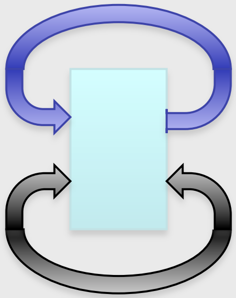
Field slips out

Some slow electrons escape

Some fast electrons escape

- In steady-state simulations:
  - Infinite long bunch with the same properties (no time-dependence)
  - Zero net flow of field and electrons of any slice

→ field and particles are fed back into the same slice

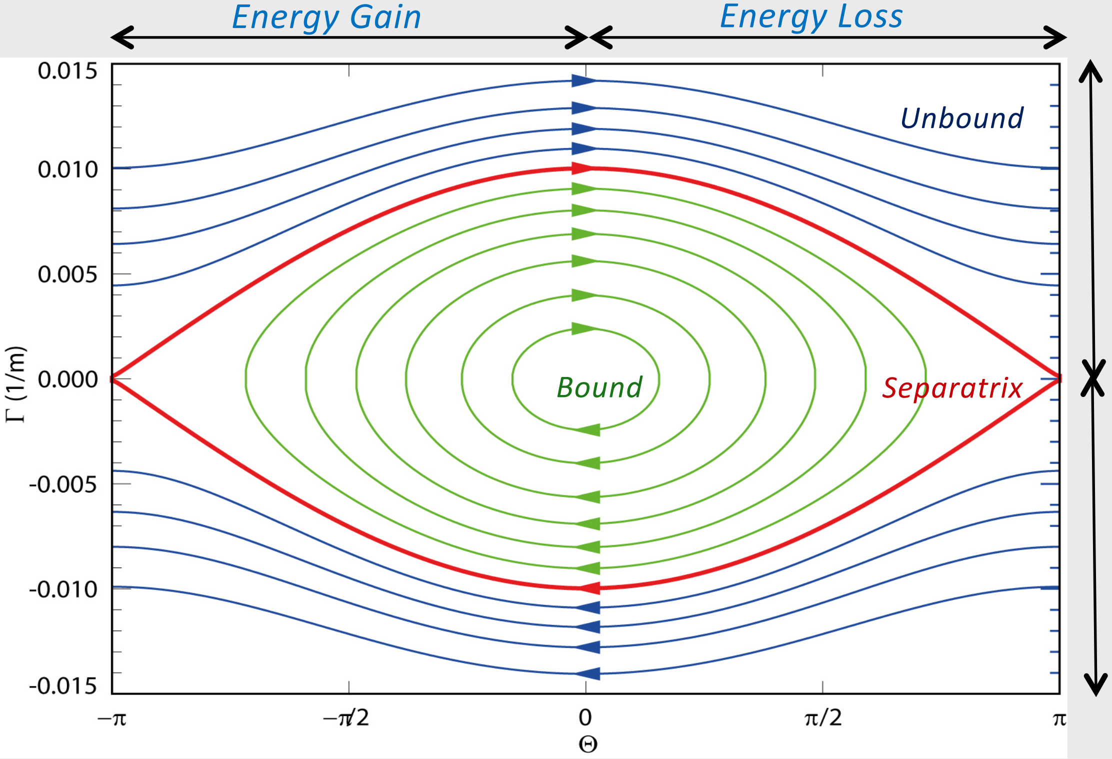


Tracking of only on radiation field and one electron slice

Self-fed Amplifier

Limitation of Steady-State Model

- The instantaneous longitudinal motion of the electrons is a rotation in phase space



Pendulum Equations

$$\frac{d}{dz}\Theta = \Gamma$$

$$\frac{d}{dz}\Gamma = -k_s^2 \sin\Theta$$

$$k_s = \sqrt{2k_u \frac{eE_0K}{\overline{\beta}_z\gamma_r^2 mc^2}}$$

- The rotation frequency mixes the resonance condition of the FEL

$$k_r \Rightarrow k_r \pm k_s$$

Limitation of Steady-State Model

- Towards saturation the synchtrotron oscillation shifts the resonance condition out of the bandwidth

$$\frac{k_s}{k_u} \approx \rho$$

- At saturation the synchrotron wavelength approaches the gain length.
- Due to the change in the resonance condition (e.g. splitting of frequencies) a single frequency code does not represent well the post saturation dynamics.
- Nevertheless it can be tried to follow the resonance condition to get valuable input on rate of extraction from the electron beam to the radiation field.

Kroll, Morton, and Rosenbluth - Model

Slowly Varying Envelope Approximation

- To track simultaneously multiple frequencies, the radiation field is expanded around the central frequency and a slow variation in amplitude and phase of the envelope
- Numerically the Maxwell equation is simplified:

$$\left[\nabla^2 - \frac{\partial^2}{c^2\partial t^2}\right]\vec{A} = -\mu_0\vec{J}$$

$$\vec{A} = \vec{e}_x A_0 e^{ikz-i\omega t}$$

$$\left[\nabla_\perp^2 + 2ik\left(\frac{\partial}{\partial z} + \frac{\partial}{c\partial t}\right)\right]A_0(x,y,z,t) = -\mu_0 J_x e^{-ikz+i\omega t}$$

Paraxial (eikonal) Representation

Drop this term for steady-state model

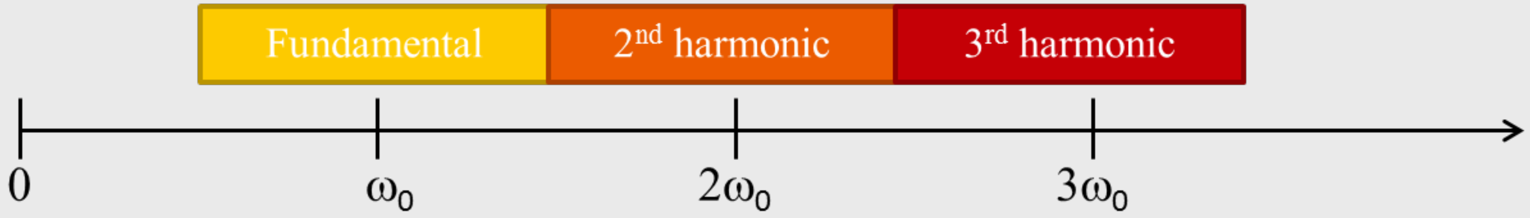
- Parabolic PDE are numerically easier to solve than hyperbolic

Using the Resonant Behavior….

- Slowly Varying Envelope Approximation becomes useful in conjunction of another approximation:

Period-Average Equation of Motion

- Consequences:
  - Envelope sampling with a frequency of the resonant frequency or lower to be consistent with Nyquist theorem.
  - No “rapid” motion on the scale of the period for larger integration step sizes
  - Higher Harmonics are modelled with independent frequencies bands.
  - Introduction of coupling coefficients (e.g. due to longitudinal oscillation)



Bandwidth Limitation of Eikonal Model

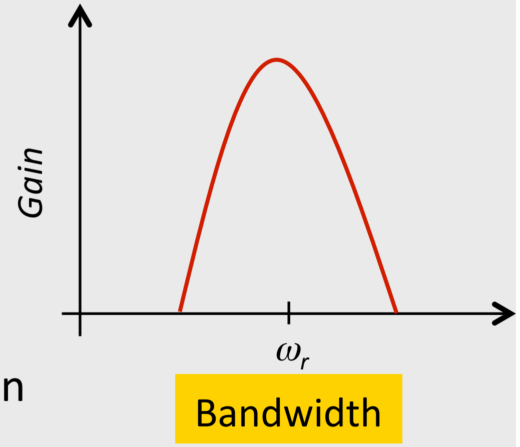
- Typically Leap-Frog SVEA Solver are still “steady-state” but needs to push sufficiently enough radiation slices though one electron slice.

But it has some limitations (example):

- Electron resonant to frequency component at edge of resolved bandwidth should allow amplification.
- Electron position and radiation phase slips by 180 degree over one undulator period but stay in constant phase relation to each other
- However solver pushes electron independently, thus sampling different phases for higher order solver

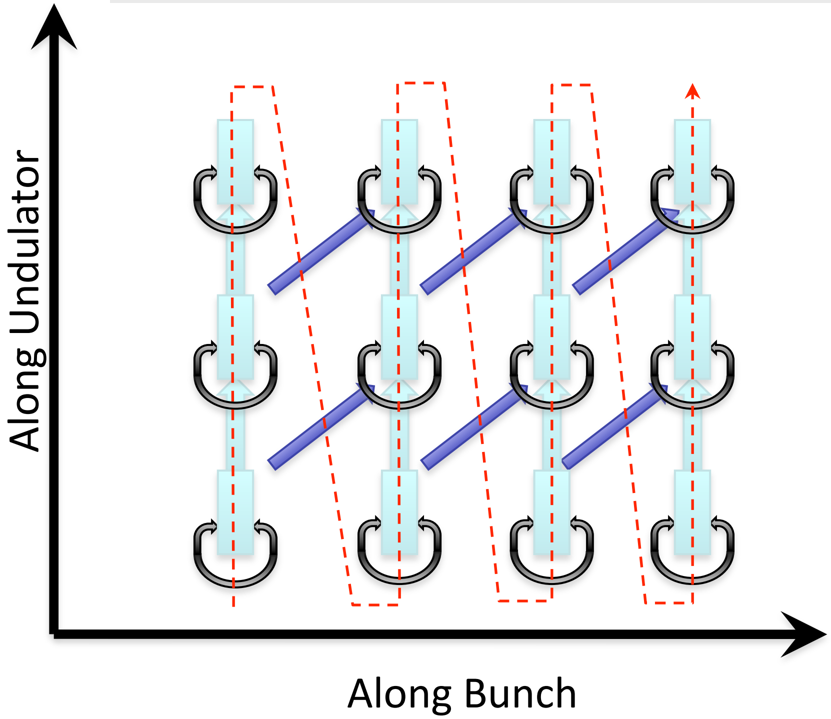
Effective Gain at bandwidth limit is reduced to zero.
Good region is actually smaller.

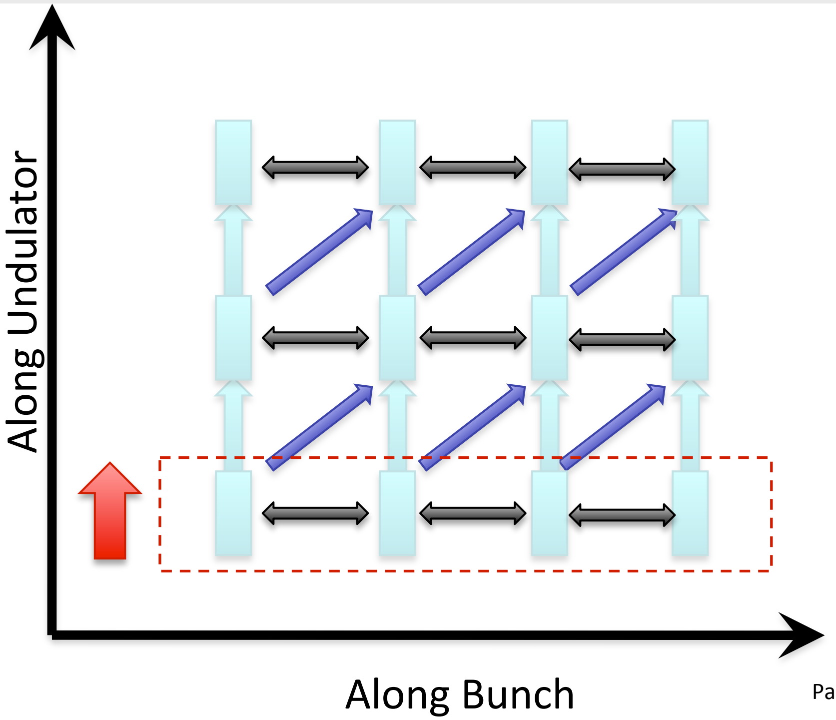
- Improvement possible:
  - Interpolation in Runge-Kutta solver
  - Sub-period integration step with linear field interpolation



(Quasi) - Time-Dependent Code

- Historically most time-dependent codes are/were “quasi”-time dependent, where the code crawls sequentially through the electron bunch
- With growing computer resources, bunch can be pushed through undulator collectively in fully time-dependent simulations
- Next level of consistency would be true 3D grid solver of radiation field (quasi-PIC) and/or non “period-averaged “code if CPU resources permit.

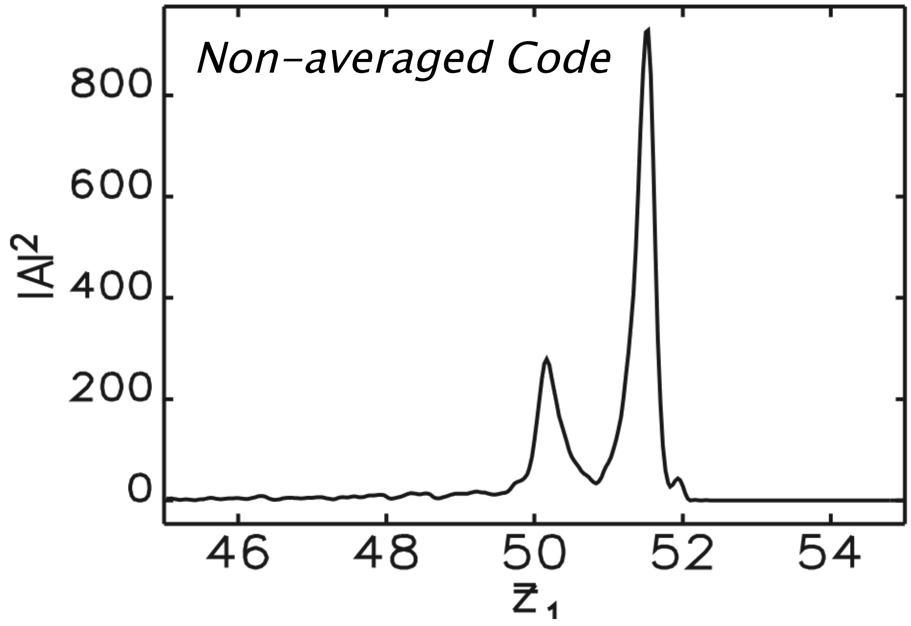


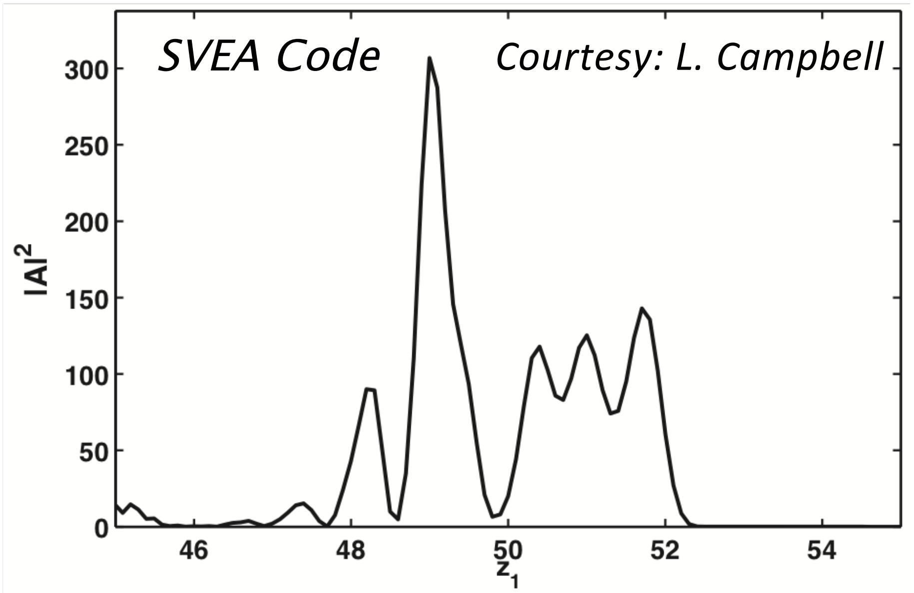


When does this Bandwidth Problem Matter?

- Superradiance is a mechanism to amplify radiation way beyond saturation limit and is a pure time-dependent effect:
  - Peak power grows quadratically
  - Pulse length shrinks with square root
- Pulse length reduction yields a broader spectrum, but outer frequency components can be damped numerically.
- Superradiant regime is numerically stopped.

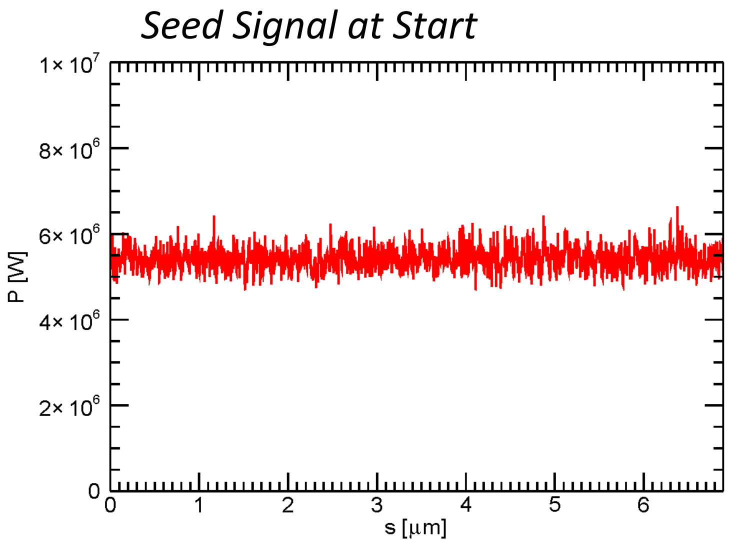
$$\frac{k_s}{k_u} \geq \rho$$

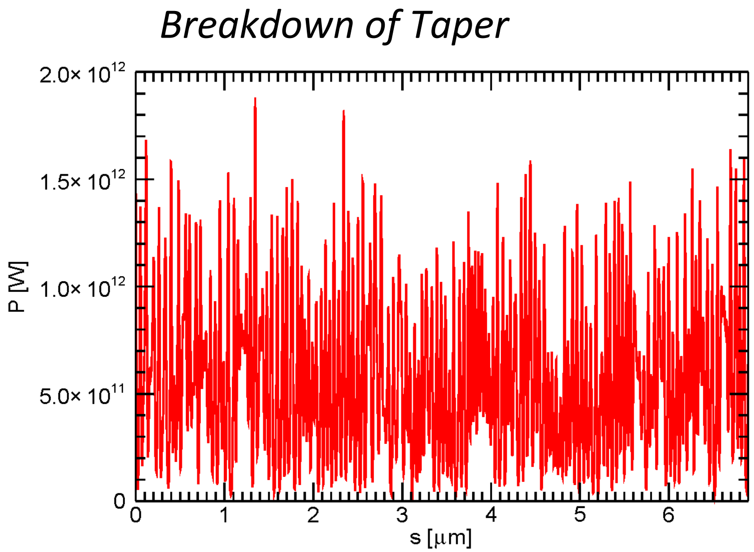




The Sideband Instability

- Any ripple in the power profile (or inderectly in electron beam parameters, such as mean energy and energy spread) will be amplified by the mechanism of superradiance.
- Once the longitudinal power variation becomes too large, particles are detrapped.
- Numerically, the sample rate should be sufficient (in particular for Angstrom-FELs) to avoid numerical broadening of the pulse.


Seed Signal at Start


Breakdown of Taper

Missing Physics – Beyond Superradiance

- For very strong fields, the drop of some terms are no longer justified, e.g. particle energy

$$\frac{d}{dz}\gamma = \frac{kA_0K}{\gamma}\left[\cos((k+k_u)z-\omega t)+\cos((k-k_u)z-\omega t)\right]$$

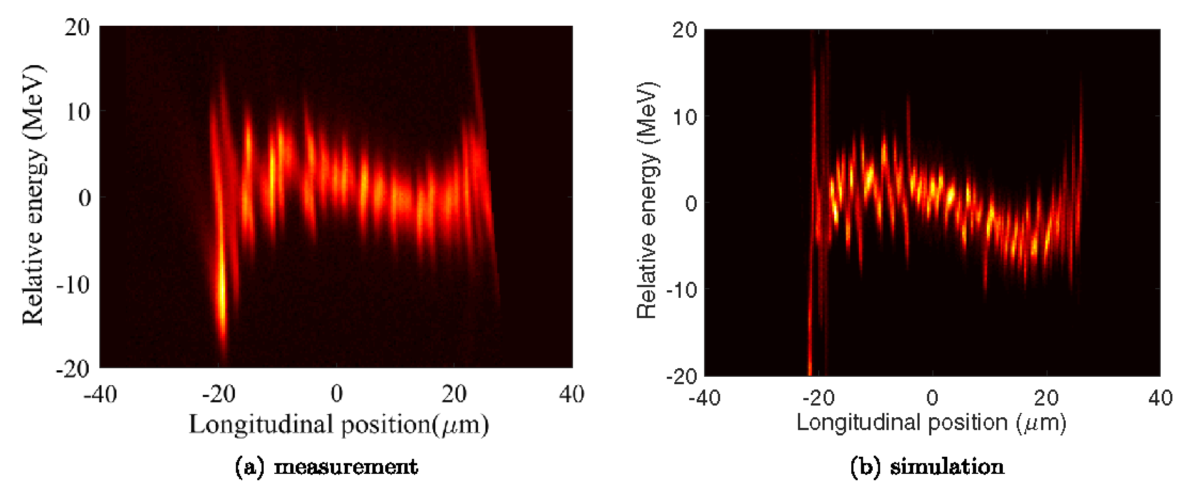
Dropped ~cos(2k_uz)

- Condition is $k_s \approx k_u$
- This roughly corresponds to a factor 1/ρ^2 of the saturation power level.
  - For LCLS like parameter: in the order of 100’s of PW!
  - For IFEL in the visible with powerful laser: TW is in reach
- In addition the wiggling in the laser field adds additional terms in the equation of motion, in particular a slow-down effect since the normalized vector potential of the field becomes comparable with that of the undulator field.

At this point the period-average model breaks down (not necessarily the SVEA!) and a more “basic” code has to be used.

Realistic Model

- Electron Beam
  - Tapering depends very heavily on the electron beam parameters
  - Very difficult to predict a realistic beam with start-end simulation
  - Slight variation in current, energy or energy spread can seed the sideband instability (e.g. residual from self-seeding)



(a) measurement **(b) simulation**

[J. Qiang et al, PRAB 20 (2017) 054402

- Undulator
  - Often a simplified model.
  - To resolve entrance and exit taper, a non-”period average” is needed.

Optimization

- Numerically taper optimization is very tedious:
  - Problem is prone to many local minima, e.g. a phase shifter can push beam into strong emission phase but degrading the bunching factor significantly.
  - It has a huge set of parameters to be optimized unless the model is simplified, e.g. KRM model
  - Sideband instability should require a time-dependent model
  - Single-shot peak performance vs robustness has different optimization sets: e.g. tapering after self-seeding has a fluctuation in the input power, resulting in a fluctuation of the saturation length.
  - Still it has been done: Y. Jiao *et al.*, Phys. Rev. ST Accel. Beams 15, 050704 (2012).

Optimization Example

- Problem: Hard X-ray self-seeded FEL with taper towards TW
- Parameter-Set: About 20 K-values of modules, 20 quadrupoles, 20 phase-shifters

Genetic Algorithm

- 60 Genes
- Population of about 200 members
- At least 100 Generation

Assuming time-dependent simulations of about 50 CPU hours per member

- 1.000.000 CPU hours:
  - About 42 days non-stop on a 1000 core Cluster
  - Cost: 400.000 CHF (according to Swiss National Foundation Pricing)
  - Result has little scientific merit (since is depends strongly on assumed beam parameters)

Summary

- Numerical Limitations based on the algorithm of the code and its approximations

- Models with decreasing limitation
  - Steady-state model
  - Time-dependent Eikonal Leap-Frog Model
  - Interpolated Eikonal Model
  - Non-Period Average Code

- Taper sensitive to sideband instability, requiring at least time-dependent code

- Full optimization difficult:
  - Reduced parameter set for optimization (e.g. KRM-Model)
  - CPU expensive generic (e.g. genetic) optimizer

- Effort and cost can easily exceed scientific merit and is limited by resources not by code itself.