

# Introduction to the Physics of Tapered Undulator FELs



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*Workshop on the Physics and  
Applications of High Efficiency FELs*

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# Outline

- What is undulator tapering and why it is important
- Brief review of basic 1982 KMR theory and suggestions
- Summary of particular tapered FEL experiments
- The recent rebirth of high gain, amplifier taper physics
  - experimental tapering returns: both SASE and seeded
  - theoretical optimization for TW-FELs
- Some detrapping concerns
- Best strategy for tapering self-seeded FELs?
- Desire for a well-diagnosed, well-benchmarked, short wavelength deep taper FEL expt.

# What is Undulator Tapering

- Undulators are characterized by their polarization (e.g., linear or circular), wavelength  $\lambda_u$ , and normalized undulator strength  $K$  or  $a_u$
- For an electron beam particle to be in exact resonance with the radiation wave

$$\lambda_s = \frac{\lambda_u}{2\gamma^2} \times (1 + a_u^2 + \gamma^2 \beta_{\perp}^2)$$

- Tapering involves systematically changing either  $K$  or (in principle at least)  $\lambda_u$  to account for electron energy loss
- Tapering has been studied theoretically and experimentally for nearly 4 decades

# Why Taper?

- Energy extraction efficiency --- defeat the smallness of  $\rho$ 
  - Std. result at saturation:  $P_{\text{RAD}} \approx 1.6 \rho P_{\text{BEAM}}$
  - XUV & X-ray wavelength region:  $\rho \lesssim 10^{-3}$
- For 3X or greater power increase, 50% undulator extension is *cheap!!!* (relative to *total* facility cost)
- Some other good reasons (but not covered here):
  - Reduction of spectral sideband contamination
  - Production of very intense, **ultrashort pulses** (e.g., XLEAP)
  - Reverse taper to change power/bunching ratio upstream of circularly-polarized “**afterburner**”
  - IFEL acceleration

# KMR Trapped Particle & Tapering Theory

*the urtext for FEL tapering physics ...*

1436

IEEE JOURNAL OF QUANTUM ELECTRONICS, VOL. QE-17, NO. 8, AUGUST 1981

## Free-Electron Lasers with Variable Parameter Wigglers

NORMAN M. KROLL, PHILIP L. MORTON, AND MARSHALL N. ROSENBLUTH

*(Invited Paper)*

# KMR Theory --- Decelerating Buckets...

- Hamiltonian analysis stimulated by analogy of FEL ponderomotive wells to RF acceleration buckets in linacs (P. Morton)
- Electrons could be both trapped and then stably decelerated via reducing  $K$  (ignoring effects of diffraction, betatron motion, spontaneous emission, *etc.*)

Standard FEL longitudinal equations:

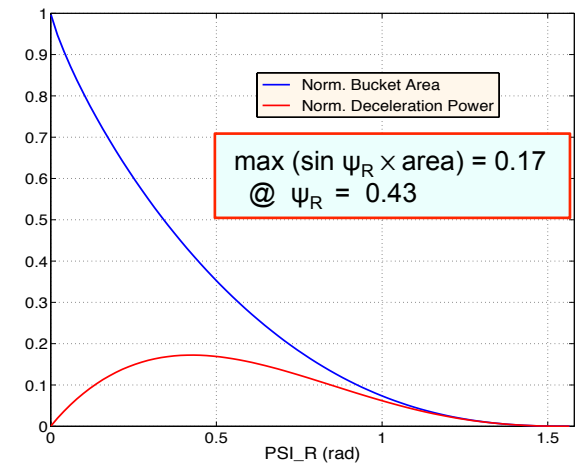
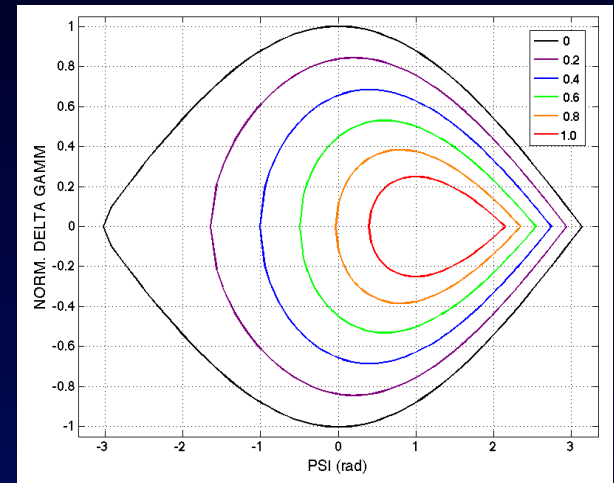
$$\frac{d\psi}{dz} = k_w - \frac{k_s}{2\gamma^2} (1 + a_w^2 + \gamma^2 \beta_1^2 - 2a_w a_s \cos\psi) + \frac{d\varphi}{dz}$$

$$\frac{d\gamma}{dz} = -\frac{a_w e_s}{\gamma} \sin\psi .$$

To keep a resonant “design” electron at constant  $\psi_R$ , balance gamma loss by reduction in  $a_w$  ( $\equiv K_{RMS}$ ); eikonal phase derivative can be important

KMR often remembered for a **constant**  $\psi_R$  (z) approach  
Resultant bucket area is a strong function of  $\psi_R$

It is convenient for discussion and probably desirable as a design characteristic to choose  $\gamma_r$  so that  $\psi_r$  is constant. Then



# KMR Theory - Variable $\psi_R$ ?

☞ KMR were no dummies – they knew  $\psi_R$  could be varied: mum one. Again, in anticipation of the amplifier case, we note that an increase of  $a_s$  with  $z$  can eliminate the detrapping associated with an increase in  $\psi_r$  so that an increase of  $\psi_r$  with  $z$  should have some advantages for an amplifier.

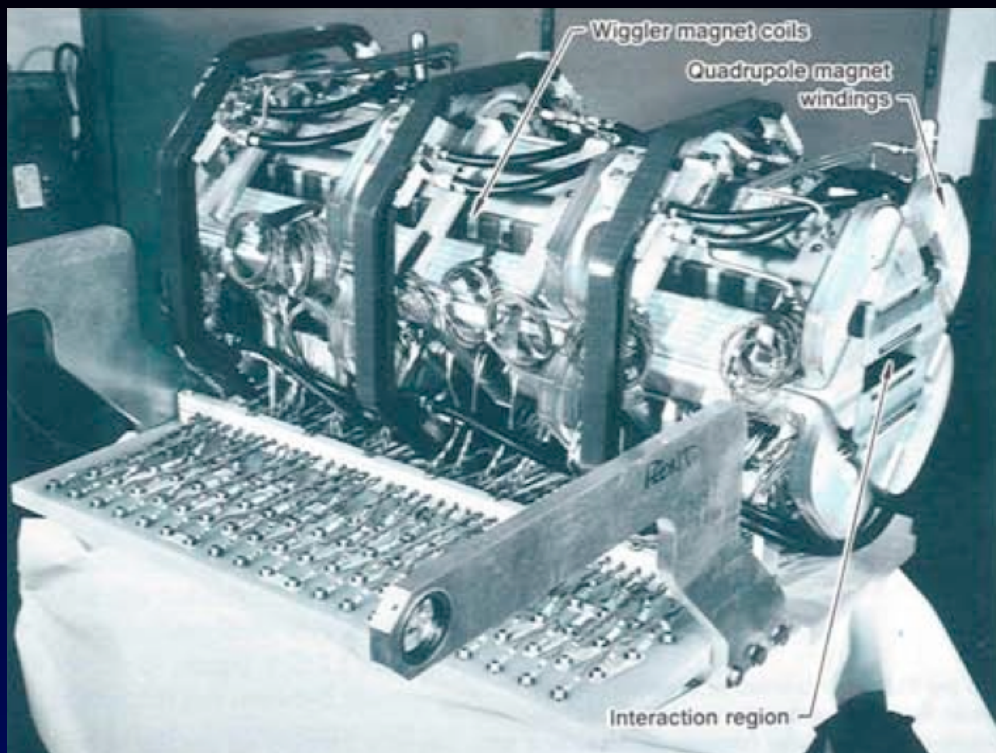
- “Fine-tuning”  $\psi_R$  and  $K$  in saturation region can improve trapping --- this is now well-appreciated by many workers
- FRED/GINGER “self-design” algorithm (mid-1980’s):  
$$\psi_R = \psi_R^0 + g(z - z_0) \text{ for } z_0 \leq z \leq z_M,$$
  
then constant with  $z$ ;  $z_0$  typically  $z_{sat} - 2L_G$
- Many FEL codes including GENESIS, FAST, etc., allow one to pre-specify a given  $K(z)$

# 1984-6 ELF Expts. @ Livermore

- Joint LBNL – LLNL experiment to study physics of **high gain, high energy extraction FELs**
- Initial expt. @ 35 GHz (8 mm) in over-moded waveguide seeded with low power ( $\sim 50$  kW;  $P_{\text{SAT}} \sim 100$  MW) magnetron
- Findings included:
  - importance of **good matching** to undulator transport
  - high gain and **saturation** for untapered undulator
  - confirmation of “**launching losses**” (factor of  $1/9$  in power coupling)
  - SASE studies: **expt. stimulated K-J Kim classic SASE paper**
  - very high energy extraction efficiency ( $>35\%$ ) for **tapered undulator** (starting with  $P \ll P_{\text{SAT}}$ )



# ELF: Undulator & Untapered 35 GHz Results



1-m section of ELF Undulator  
Designed by **K. Halbach** LBNL  
fully electromagnetic  
every 2 periods individually controllable

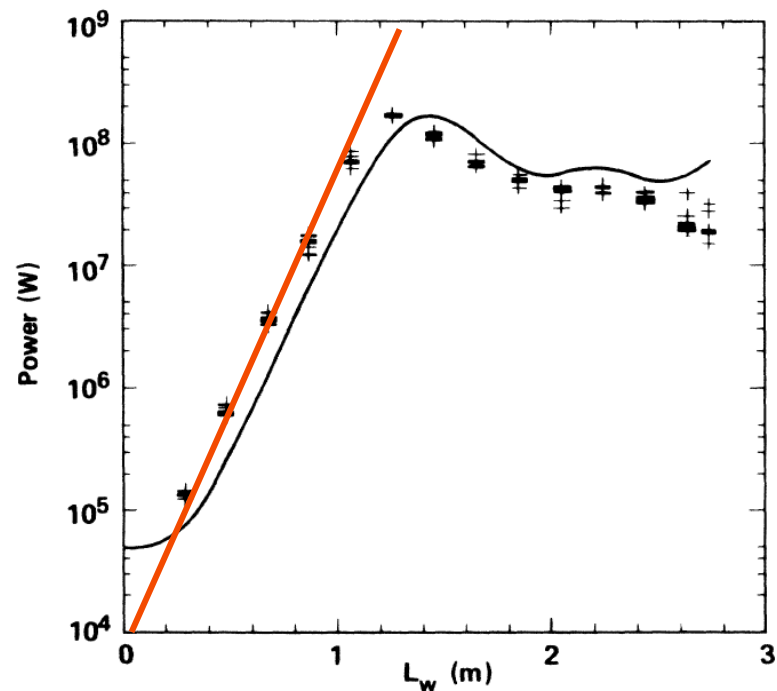


FIG. 1. Amplified signal output as a function of wiggler length for uniform (flat) wiggler. Crosses indicate experimental values and the solid line is the result of numerical evaluation.

Orzechowski et al., PRL 57, 2172 (1986)

- ➔ FRED code quickly developed in parallel to ELF studies, included waveguide geometry, multiple transverse modes, full 3D particle motion & KMR “self-design” tapering algorithm

# ELF: 35 GHz Tapered Wiggler Results

Orzechowski et al., PRL 57, 2172 (1986)

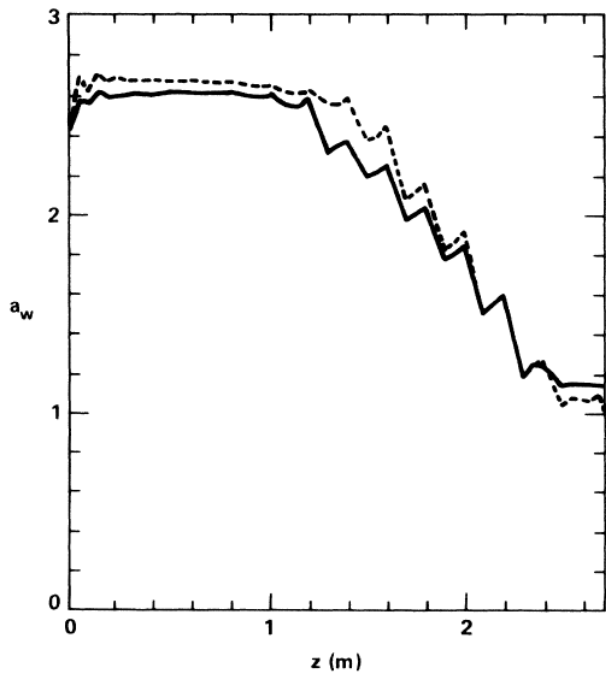


FIG. 2. Optimum wiggler field profile for tapered wiggler. The dashed line corresponds to empirical evaluation and the solid line is the numerical prediction.

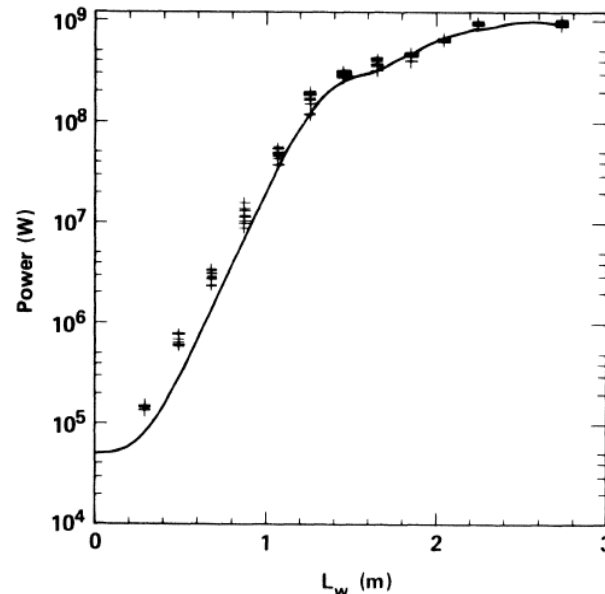
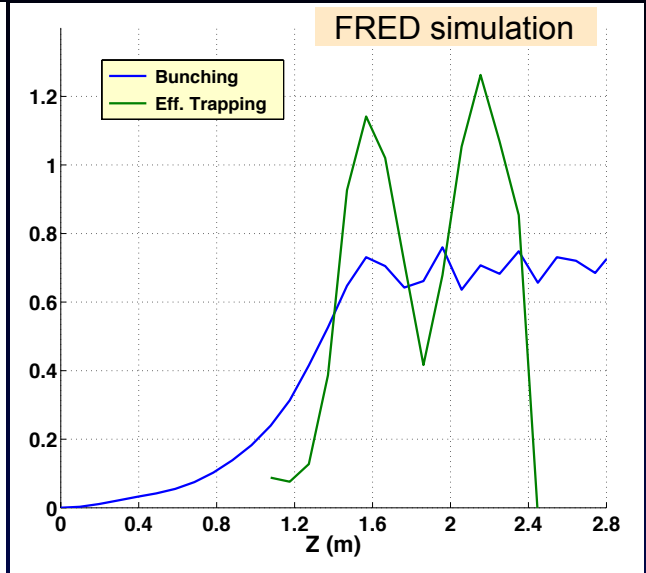


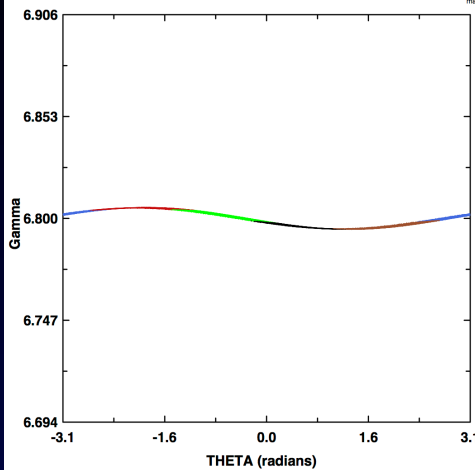
FIG. 3. Amplified signal output as a function of wiggler length for tapered wiggler field. Crosses indicate experimental values and the solid line is the results of the numerical evaluation.



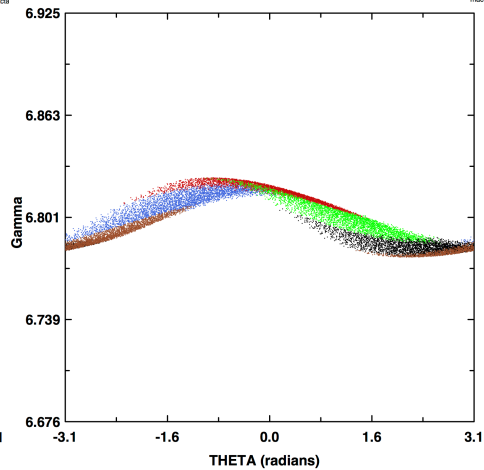
- **taper determined empirically** by optimizing power every  $2 \lambda_u$ 
  - min. allowable  $K$  reached at 2.2 m  $\rightarrow$  no additional power gain in  $z$
  - taper increases power 5.5X (7.5 dB); **50% deceleration**, **70% bunching fraction**
- empirical optimization very close to KMR-style self-design taper
- FRED code: very good agreement in taper & power

# Calculated Phase Space in ELF Taper

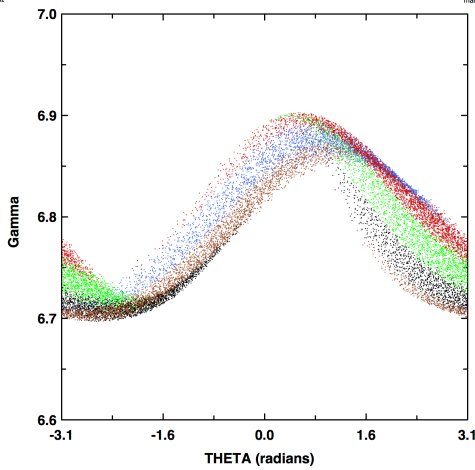
Long. Phase Space at Z=0.10 M



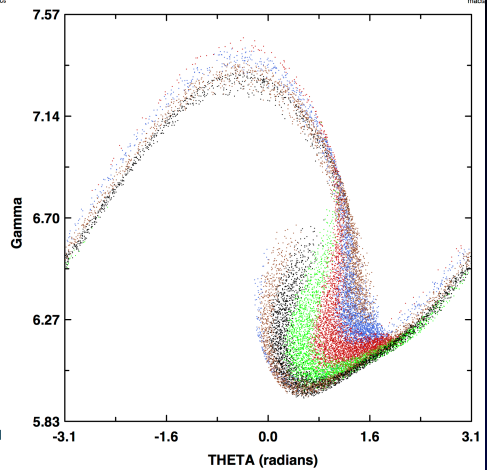
Long. Phase Space at Z=0.59 M



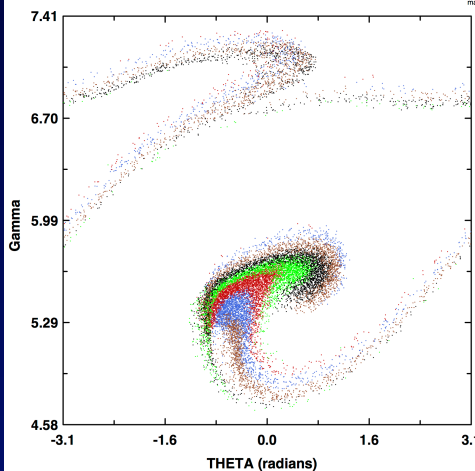
Long. Phase Space at Z=1.08 M



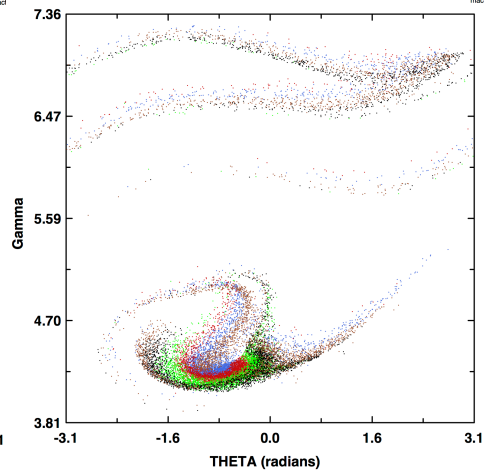
Long. Phase Space at Z=1.57 M



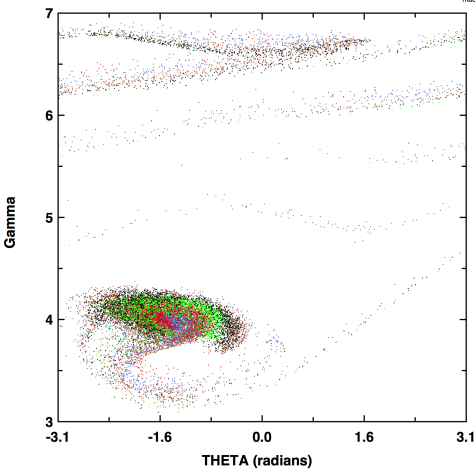
Long. Phase Space at Z=1.96 M



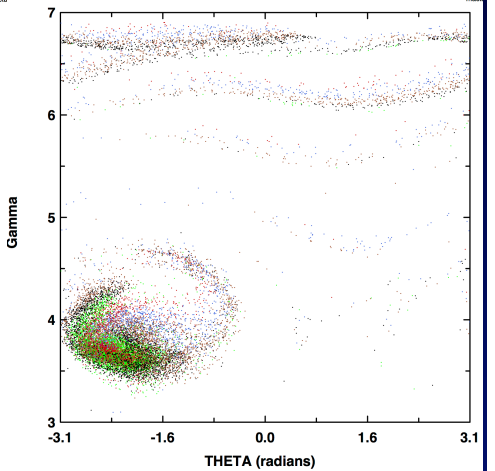
Long. Phase Space at Z=2.25 M



Long. Phase Space at Z=2.55 M



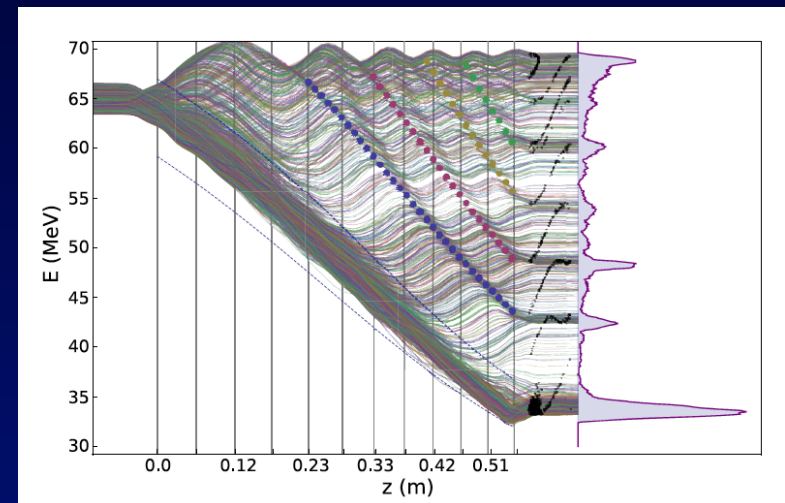
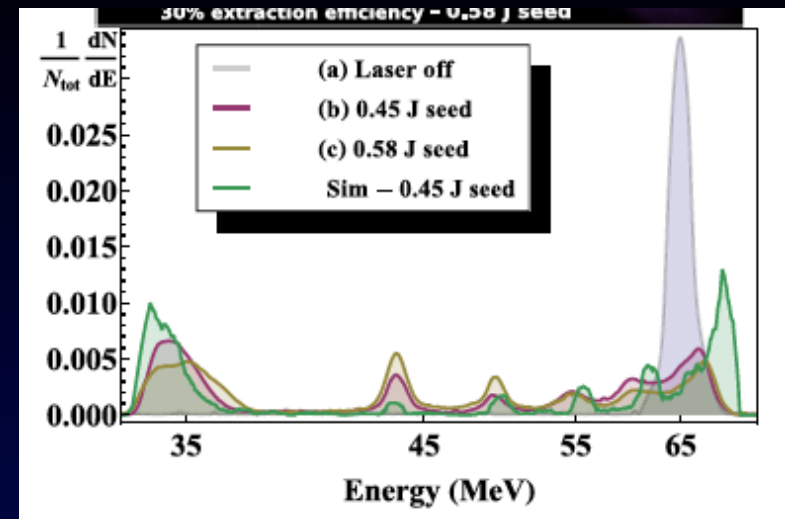
Long. Phase Space at Z=2.84 M



GINGER time-steady simulation ("FRED-mode") using **expt.-determined** taper

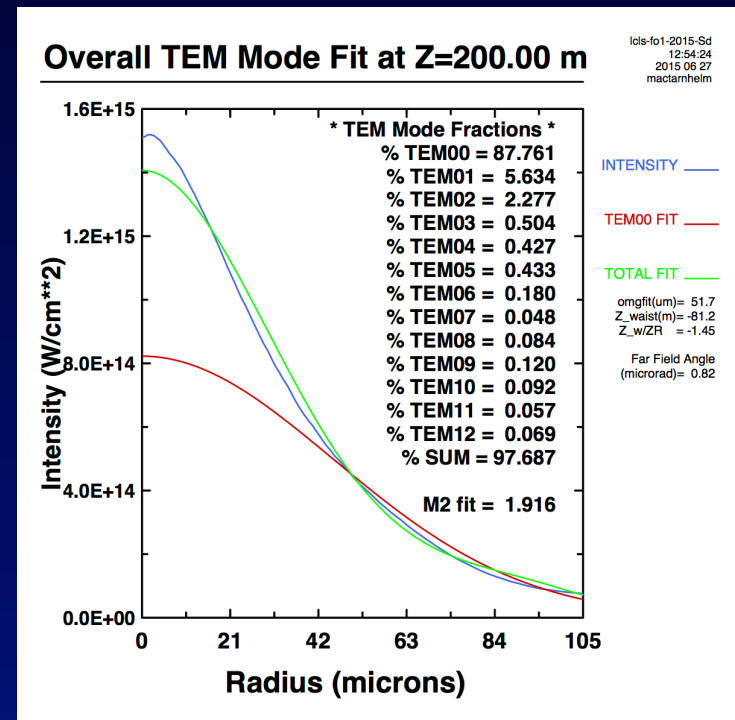
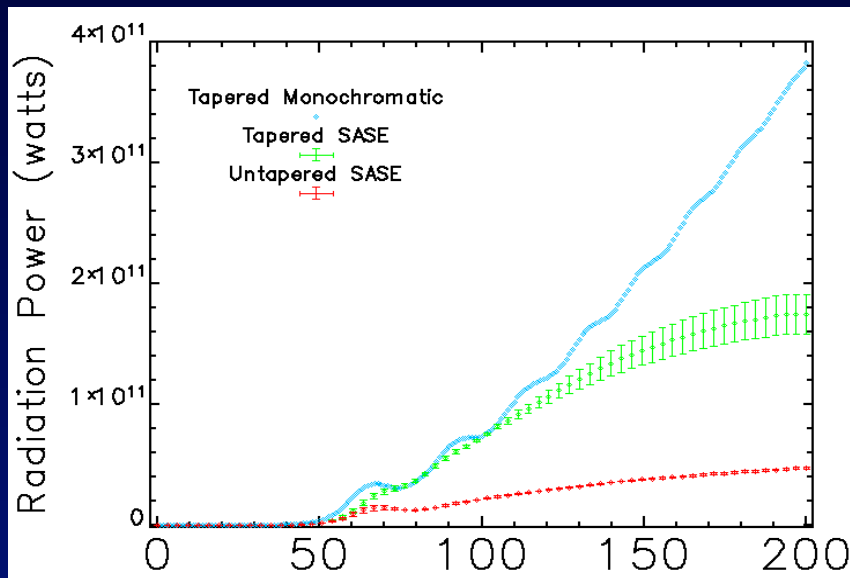
# 2016 UCLA 10.6 $\mu$ m Strong Tapering Expt.

- Sudar *et al.* (PRL, 117, 174801 (2016) ) used 200 GW CO<sub>2</sub> to investigate strong trapping regime
- Key results:
  - very strong trapping fraction
  - good control of ponderomotive bucket while **extracting >30%** ebeam energy
  - very good agreement with GPT/GENESIS simulations including accurate mapping of detrapping with  $z$
  - found **prebunching** very helpful to optimize initial trapping
- As a historical note, original LANL FEL group did a similar CO<sub>2</sub> tapered wiggler expt. (**3.7% extraction**) **and IFEL** in **early 1980's** to confirm no-gain FEL dynamics at sub-microwave wavelengths (see Warren *et al.*, *J. Quant. Elec.*, QE-19, 391 (1983) )



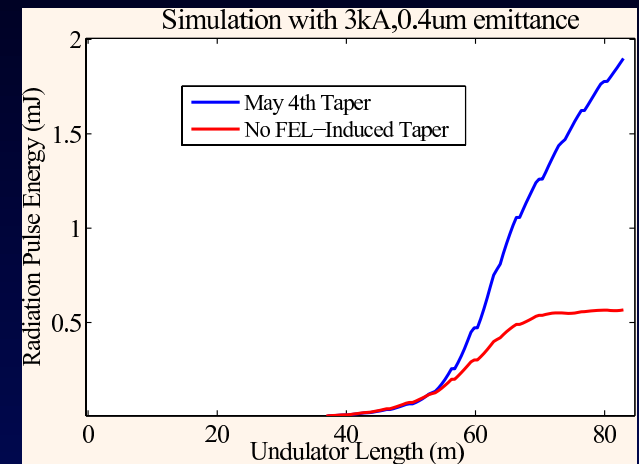
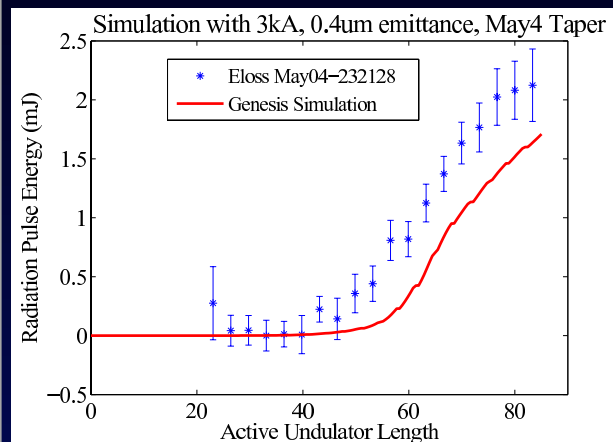
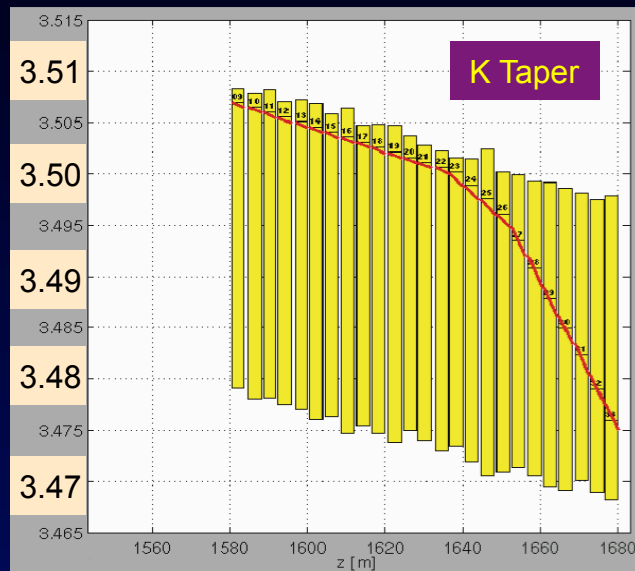
# Tapering \*SASE\* Amplifiers

- Question arose in early 2000's during mid-LCLS design: can a taper **strongly** increase SASE FEL output beyond saturation?
- Study by WMF, Huang, Kim, Vinokurov (FEL01, NIM A 483, 537–541 [2002]) showed **4X power increase** over untapered case & reasonable trapping fraction ( $\sim 30\%$  , decreasing slowly over last 100 m)
  - >> necessary to **reduce** asymptotic tapering rate ( $\psi_R$ ) to **0.2** from  $\sim 0.4$  optimum found for time-steady case
  - mode quality looks so-so



# Real-life SASE Tapering: Hard X-ray Results @ LCLS

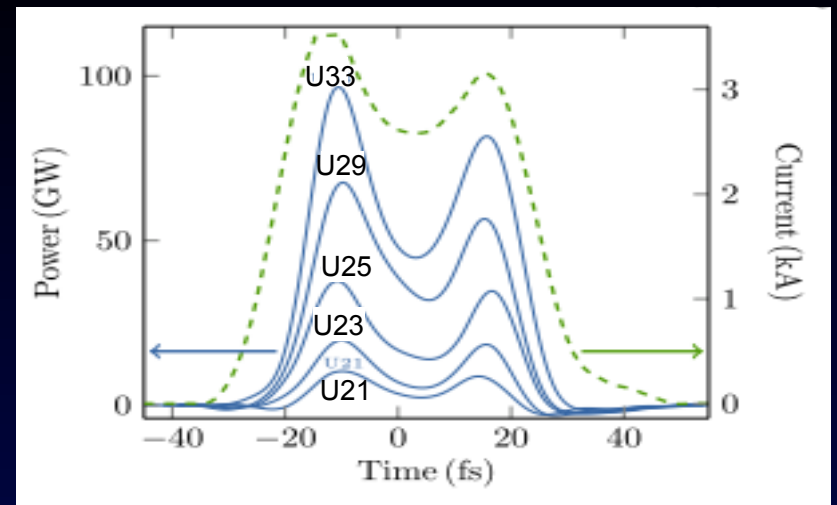
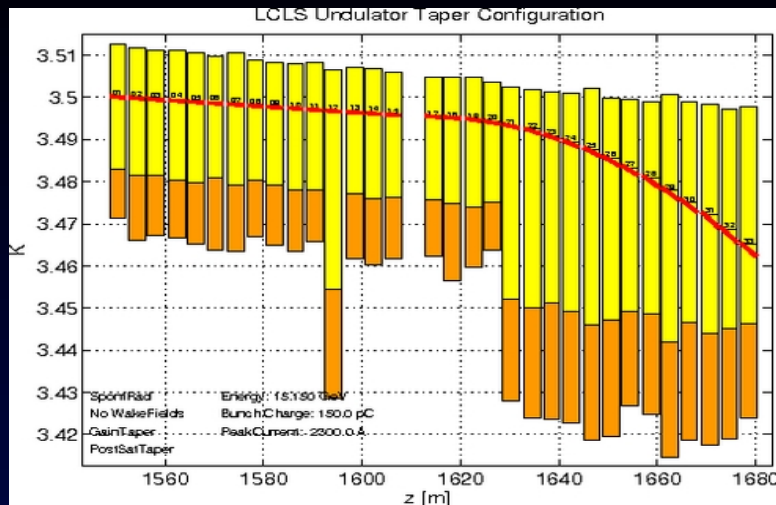
- After achieving saturation in spring 2009, LCLS team quickly explored tapering to increase SASE power at 8-keV:



Results from Ratner et al., Paper TUOA03, FEL09:

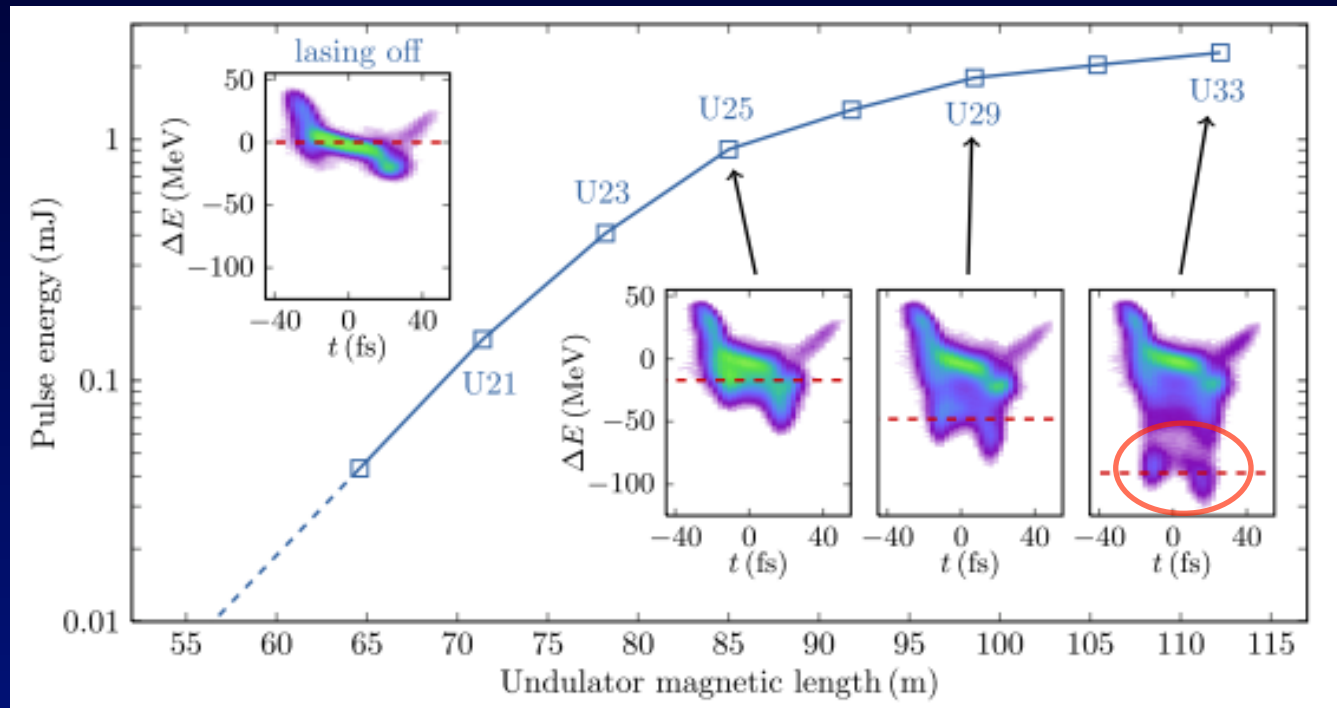
- Tapering: **~3X output pulse energy gain** relative to saturation
- Reasonable agreement with simulation modeling
  - uncertainty if wakefields increased current spike intensity & emission
- New **XTCAV**: successful deployment in 2013 now gives **shot-by-shot time-resolved** indications of FEL energy extraction, trapping phenomena

# LCLS 10.2keV SASE tapering example: 15.2 GeV, 150 pC XTCAV diagnostic results



~ quadratic tapering  
0.5% deceleration  
evidence for trapped  
particle group at U33

Data analysis and  
slide courtesy Y. Ding



## ≥ 2011: Renewed Interest in Strong Tapering

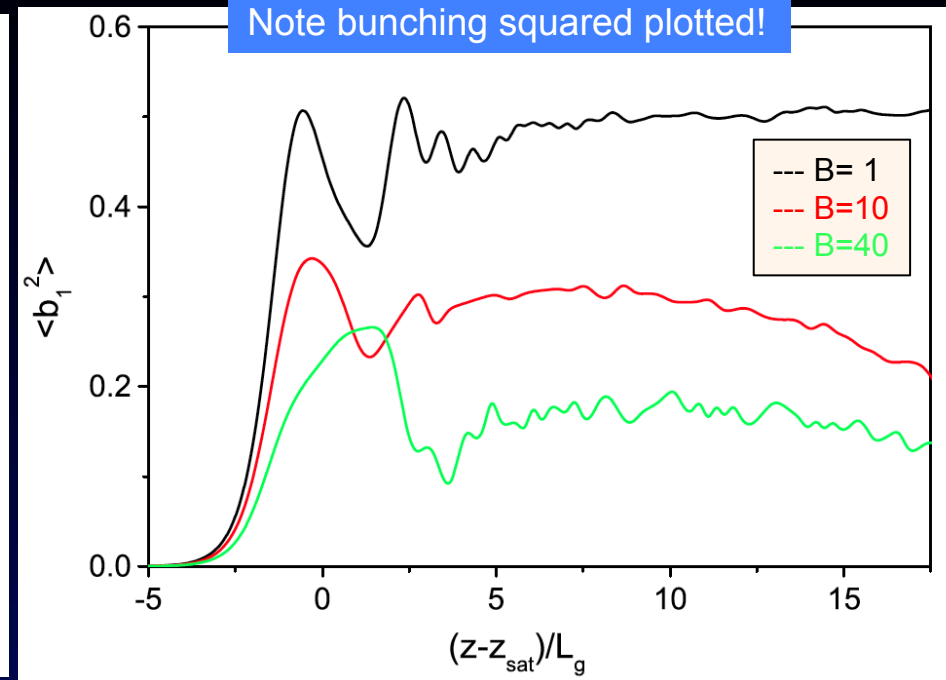
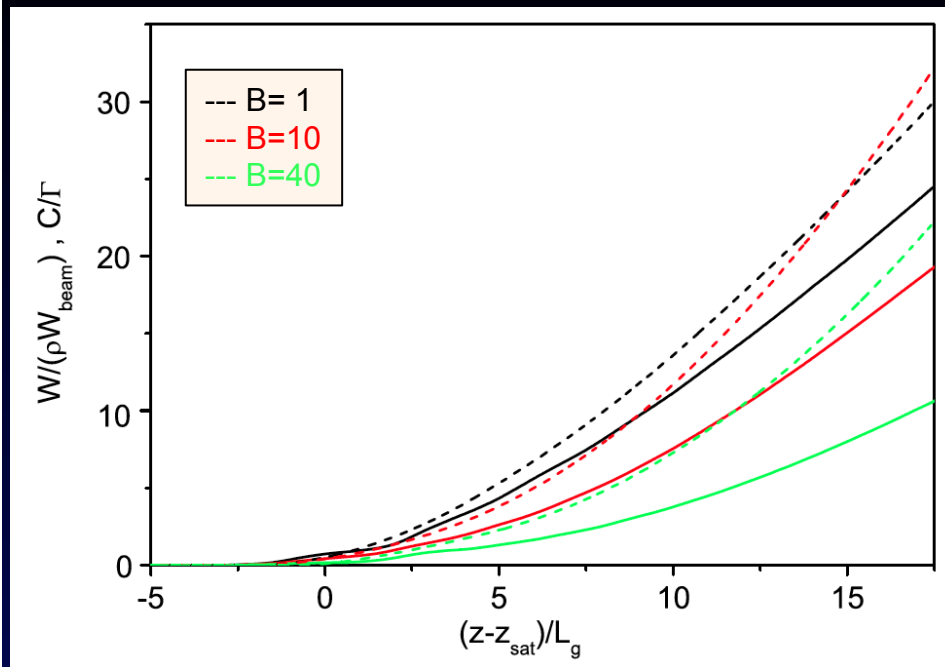
- New desire to reach TW-power level at hard x-rays stimulates new work in self-seeded tapering optimization
  - Jiao *et al.* (SLAC, see [PRST-AB, 15, 050704 \(2012\)](#) )  
bottom line: optimizing power for fixed undulator length  $\rightsquigarrow$  ~20% power increase relative to constant  $\psi_R$  (self-seeded, 8-keV LCLS-1)
  - Mak-Curbis-Werin (Lund; see [PRST-AB 18, 040702 \(2015\)](#) )  
variable  $\psi_R$
  - Schneidmiller&Yurkov (DESY, see [PRST-AB 18, 040702 \(2015\)](#) )  
taper study with scaled equations – universal solution as function of diffraction/FEL gain ratio
  - “I due Claudi”: C. Emma & C. Pellegrini (UCLA/SLAC, see [PRAB 19, 020705 \(2016\)](#), [PRAB 20, 110701 \(2017\)](#) ) optimizing tapering in presence of time-dependent effects such as sidebands (2D study), tapering with strong pre-bunching (1D study)



# Schneidmiller-Yurkov analysis of optimized taper

- Details presented at FEL14, PRST-AB 18, 030705 (2015) ;
- Fundamental work for understanding FEL physics behind optimizing high gain amplifier tapers (low  $\epsilon_N$  ,  $\sigma_E$  limit)
  - Critical parameters are normalized **diffraction parameter**  $B \sim Z_R/L_G$  and **Fresnel number**  $N \equiv Z_R/z$
  - Details of the formation of trapping&/bunching modulation in the region  $2 L_G$  before  $z_{SAT}$  relatively insensitive to  $B$  → start taper there
  - Many normalized quantities (power, bunching fraction, optimum deceleration, mode characteristics) follow self-similar solution
- Optimal deceleration shows FEL power initially follows quadratic  $z^2$  dependence followed by eventual asymptotic linear  $z$  dependence (linear dependence due to limits of “optical guiding”)

# Optimized S&Y Taper Results



**FAST** simulations with initial  $\sigma_E = 0$ ;  $L_G$  is field gain length

Left: Solid line is radiation power “W” normalized to nominal saturation power  
Dashed line is deceleration “C” normalized to gain parameter

- Note B=1 (diffraction effects important) case quickly enters linear gain regime, while B=40 (quasi-1D) remains ~quadratic for P vs. z

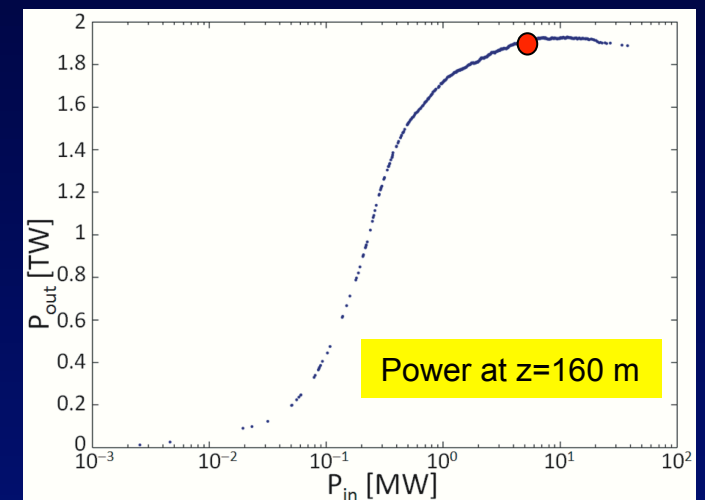
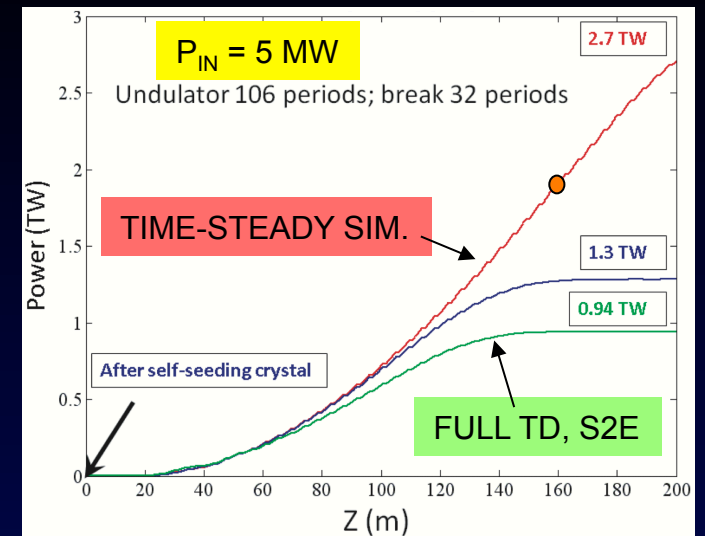
**Right: high diffraction case shows better bunching!**

# Detrapping Concerns

- Multi-TW output power levels require 20X or greater increase from nominal saturation power
- This requires good initial trapping and deceleration equivalent  $\geq 10$  synchrotron rotations
- Detrapping  $\geq 2.5\%$  / oscillation (e.g., 50%  $\rightarrow$  25% trapping) would seriously decrease net power extraction
- $\exists$  many *conceivable* detrapping mechanisms:
  - “phase shake” due effects such as classic sideband growth, synchrotron-betatron resonances, drift space phase corrector errors
  - $K(z)$  taper near end equivalent to strong increase in  $\psi_R$  (C. Emma); this leads to rapid detrapping
  - Nominal  $K(z)$  may be too rapid for those self-seeding shots with significantly lower post-monochromator powers
  - at very high energies, quantum effects such as temporal&spatial coarseness of radiation field, incoherent spontaneous emission  $\rightarrow$  non-adiabatic e- energy or ponderomotive phase change
- Remember synchrotron oscillation period  $\rightarrow \infty$  near bucket edge

# What is Best Tapering Strategy for Self-Seeding?

- Self-seeding starts from stochastic, multi-mode SASE ...
- If possible, is it best to operate with a **single longitudinal mode** (temporally coherent but large shot-to-shot variation)?
  - Figs. to right from Paper TUOA4, Fawley et al. , FEL11; Genesis sims. by J. Wu for 13.6 GeV  $E_B$ , 4 kA, 8.3-keV, 0.4 mm-mrad
- Or better to allow a **moderate mode #** (e.g.,  $\sim M=5$  ( $\rightarrow$  less post-monochromator pulse energy shot-to-shot variability but strong temporal variations)?
- How deep into nominal saturation before starting tapering?
- Depending on user application, should one strive for highest average pulse energy (i.e., semi-conservative taper) OR go for a **“1 in a million”** super shot (with very aggressive taper)?
- Active work on AI guiding of tapering optimization at SLAC and elsewhere (J. Wu talk this meeting)



# What can a well-designed taper experiment show?

- Before big \$\$\$ are spent on a TW-class x-ray FEL undulator, we need systematic **experimental** studies of best tapering strategies:
  - best  $K(z)$  for max. power in a given undulator length
  - best  $K(z)$  for min. spectral bandwidth & **sideband** control
  - best  $K(z)$  for minimizing shot-to-shot fluctuations
- One would like a good diagnostic suite, *e.g.*, :
  - parasitic  $f(\gamma, t)$  diagnostic like the SLAC XTCAV for single shot estimates of time-resolved trapping/detrapping vs.  $z$
  - near- and far-field **radiation pattern**, single shot measurements for mode quality; it may be possible to determine local bunching via sensitivity of power&mode content to controlled, local (in  $z$ ) change of  $K$  and/or phase shifter

FERMI: FEL-1@20 nm – interference of undulator segments #1, 4, & 6

