Introduction to the Physics of Tapered Undulator FELs

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Workshop on the Physics and Applications of High Efficiency FELs

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Outline

• What is undulator tapering and why it is important
• Brief review of basic 1982 KMR theory and suggestions
• Summary of particular tapered FEL experiments
• The recent rebirth of high gain, amplifier taper physics
  – experimental tapering returns: both SASE and seeded
  – theoretical optimization for TW-FELs
• Some detrapping concerns
• Best strategy for tapering self-seeded FELs?
• Desire for a well-diagnosed, well-benchmarked, short wavelength deep taper FEL expt.
What is Undulator Tapering

- Undulators are characterized by their polarization (e.g., linear or circular), wavelength $\lambda_u$, and normalized undulator strength $K$ or $a_u$.
- For an electron beam particle to be in exact resonance with the radiation wave:
  \[
  \lambda_s = \frac{\lambda_u}{2\gamma^2} \times \left(1 + a_u^2 + \gamma^2 \beta_{\perp}^2\right)
  \]
- Tapering involves systematically changing either $K$ or (in principle at least) $\lambda_u$ to account for electron energy loss.
- Tapering has been studied theoretically and experimentally for nearly 4 decades.
Why Taper?

• Energy extraction efficiency --- defect the smallness of $\rho$
  - Std. result at saturation: $P_{RAD} \approx 1.6 \, \rho \, P_{BEAM}$
  - XUV & X-ray wavelength region: $\rho \leq 10^{-3}$

- For 3X or greater power increase, 50% undulator extension is *cheap!!!* (relative to *total* facility cost)

• Some other good reasons (but not covered here):
  - Reduction of spectral sideband contamination
  - Production of very intense, *ultrashort pulses* (*e.g.*, XLEAP)
  - Reverse taper to change power/bunching ratio upstream of circularly-polarized "afterburner"
  - IFEL acceleration
Free-Electron Lasers with Variable Parameter Wigglers

NORMAN M. KROLL, PHILIP L. MORTON, AND MARSHALL N. ROSENBLUTH

(Invited Paper)
KMR Theory --- Decelerating Buckets...

- Hamiltonian analysis stimulated by analogy of FEL ponderomotive wells to RF acceleration buckets in linacs (P. Morton)
- Electrons could be both trapped and then stably decelerated via reducing $K$ (ignoring effects of diffraction, betatron motion, spontaneous emission, etc.)

**Standard FEL longitudinal equations:**

$$\frac{d\psi}{dz} = k_w - \frac{k_s}{2\gamma^2} \left( 1 + a_w^2 + \gamma^2 \beta_1^2 - 2a_w a_s \cos \psi \right) + \frac{d\varphi}{dz}$$

$$\frac{d\gamma}{dz} = -\frac{a_w e_s}{\gamma} \sin \psi .$$

To keep a resonant “design” electron at constant $\psi_R$, balance gamma loss by reduction in $a_w (\equiv K_{RMS})$; eikonal phase derivative can be important.

KMR often remembered for a **constant $\psi_R$ (z)** approach

Resultant bucket area is a strong function of $\psi_R$

It is convenient for discussion and probably desirable as a design characteristic to choose $\gamma_r$ so that $\psi_r$ is constant. Then

$max (\sin \psi_R \times \text{area}) = 0.17$
@ $\psi_R = 0.43$
• “Fine-tuning” $\Psi_R$ and $K$ in saturation region can improve trapping --- this is now well-appreciated by many workers.

• FRED/GINGER “self-design” algorithm (mid-1980’s):

$$\Psi_R = \Psi_R^0 + g(z - z_0) \text{ for } z_0 \leq z \leq z_M,$$
then constant with $z$; $z_0$ typically $z_{sat} - 2L_G$.

• Many FEL codes including GENESIS, FAST, etc., allow one to pre-specify a given $K(z)$.

☞ KMR were no dummies – they knew $\Psi_R$ could be varied:

mum one. Again, in anticipation of the amplifier case, we note that an increase of $a_s$ with $z$ can eliminate the detrapping associated with an increase in $\psi_r$ so that an increase of $\psi_r$ with $z$ should have some advantages for an amplifier.
1984-6 ELF Expts. @ Livermore

- Joint LBNL – LLNL experiment to study physics of high gain, high energy extraction FELs
- Initial expt. @ 35 GHz (8 mm) in over-moded waveguide seeded with low power (~50 kW; $P_{\text{SAT}} \sim 100$ MW) magnetron

- Findings included:
  - importance of good matching to undulator transport
  - high gain and saturation for untapered undulator
  - confirmation of “launching losses” (factor of $1/9$ in power coupling)
  - SASE studies: expt. stimulated K-J Kim classic SASE paper
  - very high energy extraction efficiency (>35%) for tapered undulator (starting with $P \ll P_{\text{SAT}}$)
ELF: Undulator & Untapered 35 GHz Results

1-m section of ELF Undulator
Designed by K. Halbach LBNL
fully electromagnetic
every 2 periods individually controllable

Directed by

FRED code quickly developed in parallel to ELF studies, included
waveguide geometry, multiple transverse modes, full 3D particle
motion & KMR “self-design” tapering algorithm

Orzechowski et al., PRL 57, 2172 (1986)

FIG. 1. Amplified signal output as a function of wiggler
length for uniform (flat) wiggler. Crosses indicate experi-
mental values and the solid line is the result of numerical
evaluation.

WM Fawley --- Tapered Undulator Physics ---- UCLA Workshop on Physics and Applications of High Efficiency FELS --- 11-13 April 2018
ELF: 35 GHz **Tapered Wiggler Results**

Orzechowski et al., PRL 57, 2172 (1986)

**Taper determined empirically** by optimizing power every 2 $\lambda_u$  
- min. allowable $K$ reached at 2.2 m → no additional power gain in z  
- taper increases power 5.5X (7.5 dB); 50% deceleration, 70% bunching fraction

- **empirical optimization** very close to KMR-style self-design taper  
- **FRED code**: very good agreement in taper & power
Calculated Phase Space in ELF Taper

GINGER time-steady simulation ("FRED-mode") using expt.-determined taper
2016 UCLA 10.6μm Strong Tapering Expt.

- Sudar et al. (PRL, 117, 174801 (2016)) used 200 GW CO₂ to investigate strong trapping regime
- Key results:
  - very strong trapping fraction
  - good control of ponderomotive bucket while extracting >30% ebeam energy
  - very good agreement with GPT/GENESIS simulations including accurate mapping of detrapping with z
  - found prebunching very helpful to optimize initial trapping

- As a historical note, original LANL FEL group did a similar CO₂ tapered wiggler expt. (3.7% extraction) and IFEL in early 1980’s to confirm no-gain FEL dynamics at sub-microwave wavelengths (see Warren et al., J. Quant. Elec., QE-19, 391 (1983))
Tapering *SASE* Amplifiers

- Question arose in early 2000’s during mid-LCLS design: can a taper strongly increase SASE FEL output beyond saturation?
- Study by WMF, Huang, Kim, Vinokurov (FEL01, NIM A 483, 537–541 [2002]) showed 4X power increase over untapered case & reasonable trapping fraction (~30%, decreasing slowly over last 100 m)
  >> necessary to reduce asymptotic tapering rate ($\psi_R$) to 0.2 from ~ 0.4 optimum found for time-steady case
  – mode quality looks so-so
Real-life SASE Tapering: Hard X-ray Results @ LCLS

- After achieving saturation in spring 2009, LCLS team quickly explored tapering to increase SASE power at 8-keV:

Results from Ratner et al., Paper TUOA03, FEL09:

- Tapering: ~3X output pulse energy gain relative to saturation
- Reasonable agreement with simulation modeling
  - uncertainty if wakefields increased current spike intensity & emission
- New XTCAV: successful deployment in 2013 now gives shot-by-shot time-resolved indications of FEL energy extraction, trapping phenomena
LCLS 10.2keV SASE tapering example: 15.2 GeV, 150 pC
XTCAV diagnostic results

~ quadratic tapering
0.5% deceleration
evidence for trapped
particle group at U33

Data analysis and slide courtesy Y. Ding
≥ 2011: Renewed Interest in Strong Tapering

• New desire to reach **TW-power level at hard x-rays** stimulates new work in **self-seeded** tapering optimization
  
  – Jiao et al. (SLAC, see PRST-AB, **15**, 050704 (2012))
    bottom line: optimizing power for fixed undulator length ➔ ~20% power increase relative to constant $\psi_R$ (self-seeded, 8-keV LCLS-1)
  
  – Mak-Curbis-Werin (Lund; see PRST-AB **18**, 040702 (2015))
    variable $\psi_R$
  
  – Schneidmiller&Yurkov (DESY, see PRST-AB **18**, 040702 (2015))
    taper study with scaled equations – universal solution as function of diffraction/FEL gain ratio
  
  – “I due Claudi”: C. Emma & C. Pellegrini (UCLA/SLAC, see PRAB **19**, 020705 (2016), PRAB **20**, 110701 (2017)) optimizing tapering in presence of time-dependent effects such as sidebands (2D study), tapering with strong pre-bunching (1D study)
Schneidmiller-Yurkov analysis of optimized taper

- Details presented at FEL14, PRST-AB 18, 030705 (2015);

- **Fundamental** work for understanding FEL physics behind optimizing high gain amplifier tapers (low $\varepsilon_N$, $\sigma_E$ limit)
  - Critical parameters are normalized diffraction parameter $B \approx Z_R/L_G$ and Fresnel number $N \equiv Z_R/z$
  - Details of the formation of trapping/bunching modulation in the region $2L_G$ before $z_{SAT}$ relatively insensitive to $B$ ⇒ start taper there
  - Many normalized quantities (power, bunching fraction, optimum deceleration, mode characteristics) follow self-similar solution

- Optimal deceleration shows FEL power initially follows quadratic $z^2$ dependence followed by eventual asymptotic linear $z$ dependence (linear dependence due to limits of “optical guiding”)

Optimized S&Y Taper Results

**FAST** simulations with initial $\sigma_E = 0$; $L_G$ is field gain length

Left: Solid line is radiation power “W” normalized to nominal saturation power
Dashed line is deceleration “C” normalized to gain parameter

➤ Note B=1 (diffraction effects important ) case quickly enters linear gain regime, while B=40 (quasi-1D) remains ~quadratic for P vs. z

Right: high diffraction case shows better bunching!

Note bunching squared plotted!
Detrapping Concerns

- Multi-TW output power levels require 20X or greater increase from nominal saturation power
- This requires good initial trapping and deceleration equivalent \( \geq 10 \) synchrotron rotations
- Detrapping \( \geq 2.5\% / \) oscillation (e.g., 50\% \( \rightarrow \) 25\% trapping) would seriously decreases net power extraction
- \( \exists \) many *conceivable* detrapping mechanisms:
  - “phase shake” due effects such as classic sideband growth, synchrotron-betatron resonances, drift space phase corrector errors
  - \( K(z) \) taper near end equivalent to strong increase in \( \psi_R \) (C. Emma); this leads to rapid detrapping
  - Nominal \( K(z) \) may be too rapid for those self-seeding shots with significantly lower post-monochromator powers
  - at very high energies, quantum effects such as temporal\&spatial coarseness of radiation field, incoherent spontaneous emission \( \rightarrow \) non-adiabatic e- energy or ponderomotive phase change
- Remember synchrotron oscillation period \( \rightarrow \infty \) near bucket edge
What is Best Tapering Strategy for Self-Seeding?

- Self-seeding starts from stochastic, multi-mode SASE ...
- If possible, is it best to operate with a single longitudinal mode (temporally coherent but large shot-to-shot variation)?
  - Figs. to right from Paper TUOA4, Fawley et. al., FEL11; Genesis sims. by J. Wu for 13.6 GeV \( E_B \), 4 kA, 8.3-keV, 0.4 mm-mrad
- Or better to allow a moderate mode # (e.g., \( \sim M=5 \) (\( \Rightarrow \) less post-monochromator pulse energy shot-to-shot variability but strong temporal variations)?
- How deep into nominal saturation before starting tapering?
- Depending on user application, should one strive for highest average pulse energy (i.e., semi-conservative taper) OR go for a “1 in a million” super shot (with very aggressive taper)?
- Active work on AI guiding of tapering optimization at SLAC and elsewhere (J. Wu talk this meeting)
What can a well-designed taper experiment show?

• Before big $$ are spent on a TW-class x-ray FEL undulator, we need systematic **experimental** studies of best tapering strategies:
  – best $K(z)$ for max. power in a given undulator length
  – best $K(z)$ for min. spectral bandwidth & **sideband** control
  – best $K(z)$ for minimizing shot-to-shot fluctuations

• One would like a good diagnostic suite, *e.g.*,:
  – parasitic $f(y,t)$ diagnostic like the SLAC XTCAV for single shot estimates of time-resolved trapping/detrappping vs. $z$
  – near- and far-field **radiation pattern**, single shot measurements for mode quality; it may be possible to determine local bunching via sensitivity of power&mode content to controlled, local (in $z$) change of $K$ and/or phase shifter

  FERMI: FEL-1@20 nm – interference of undulator segments #1, 4, & 6

  – **accurate** power measurements likely essential (improved gas ionization monitors?) for single shot comparisons with other diagnostics