

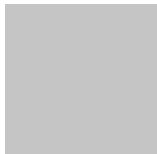



Marco Calvi:: ID group :: Paul Scherrer Institut

Intra undulator module linear taper with tilted TGU modules

Physics & Applications Of High Efficiency Free-Electron Lasers Workshop

April 11-13, 2018 at the UCLA California NanoSystem Institute

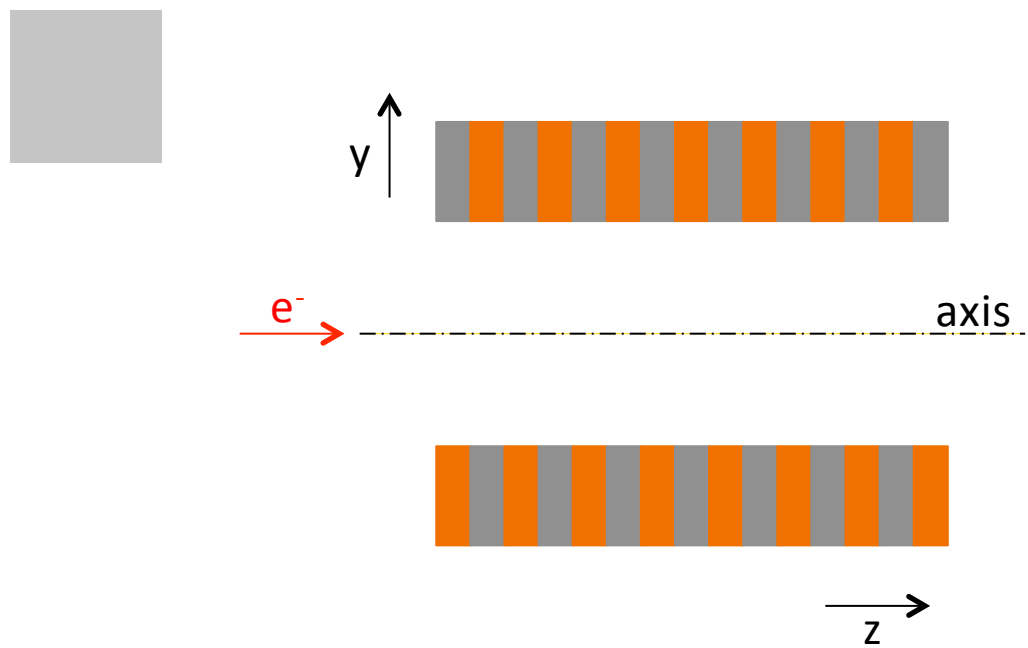


- 
- A solid grey square is positioned to the left of the main list of bullet points.
- Why linear tape undulators?
 - Transverse gradient undulator (TGU)
 - How to introduce a linear taper?
 - In-vacuum planar undulator, U15 @ SwissFEL
 - Off axis operation
 - Mover system
 - In-situ alignment
 - Apple undulators, U38 @ SwissFEL
 - Introduction of complex formalism to define K & its gradient
 - Example of Apple X operation
 - Open issues
 - Orbit distortion
 - Magnetic errors ($\Delta K/K$)
 - Conclusions

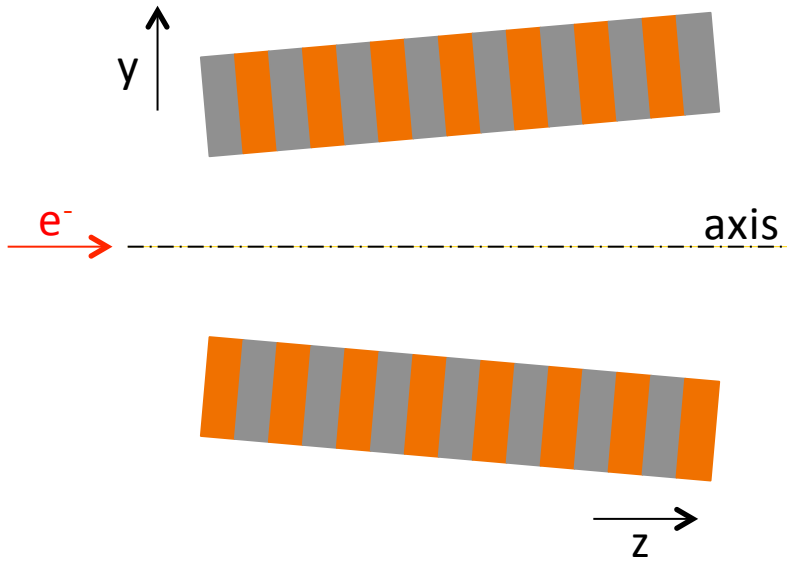
Why linear tape undulators?

- The strategy of approximating linear taper with stepwise taper with short undulator module is quickly ineffective especially when going smaller than a gain length:
 - Filling factor (effective undulator length over full length)
 - Alignment
 - Phase matching
- There are applications where the required taper is strong:
 - example of slicing of an energy modulated pulse that within one or two gain lengths the beam should be shifted out of resonance unless the slippage compensate it by a change in local mean energy of the slice over which the field (spike) slips.
- Application in high power FELs: a continuous taper will give higher power than a step-wise taper.

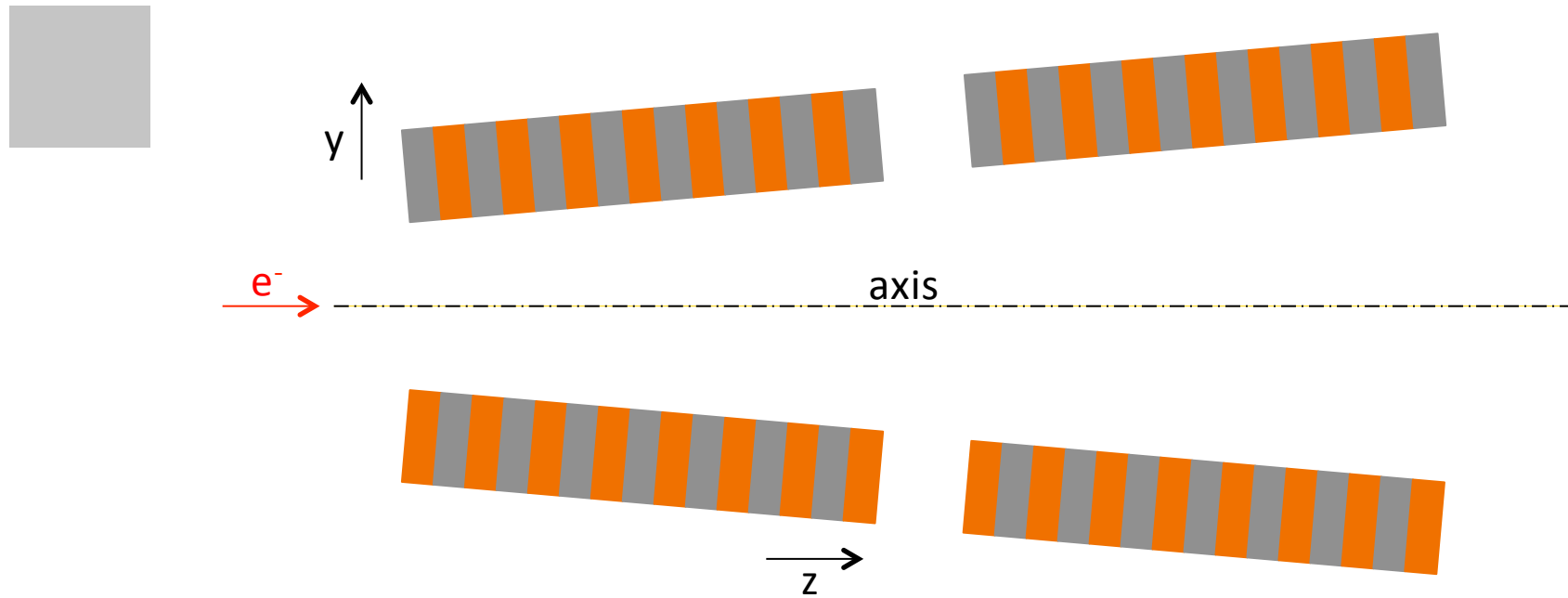
Linear taper undulators



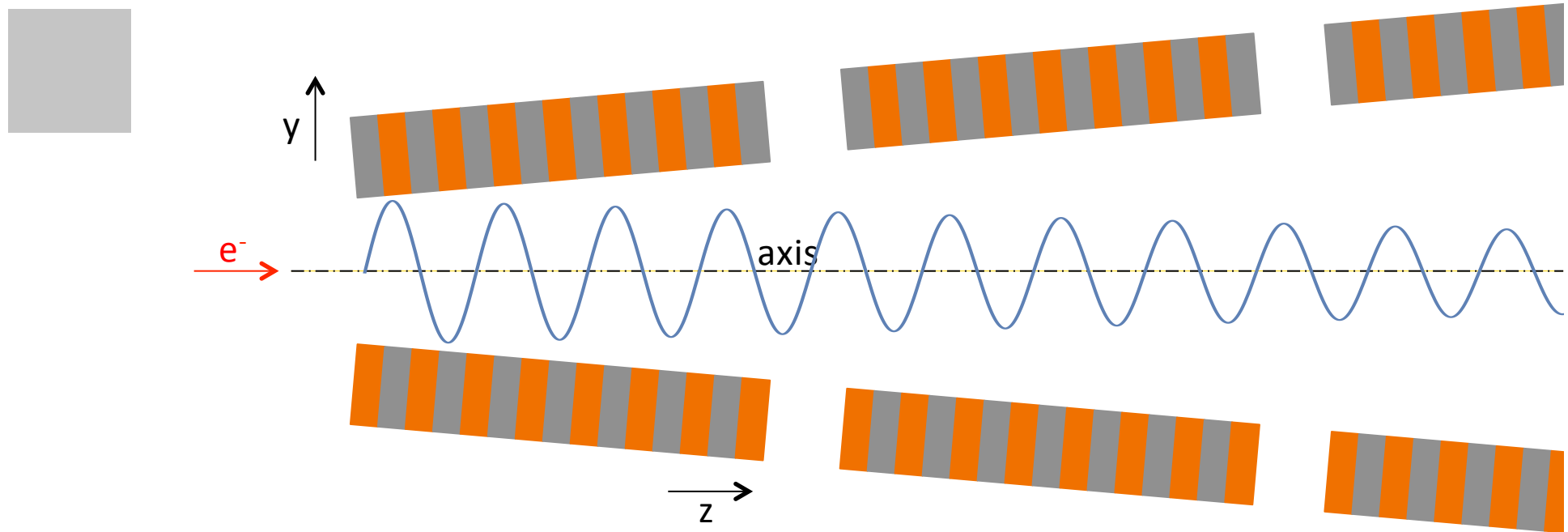
Linear taper undulators



Linear taper undulators



Linear taper undulators



If you have such an undulator, good for you and use it....
 But if you do not have this feature, like many of us, then you
 can try with a TGU

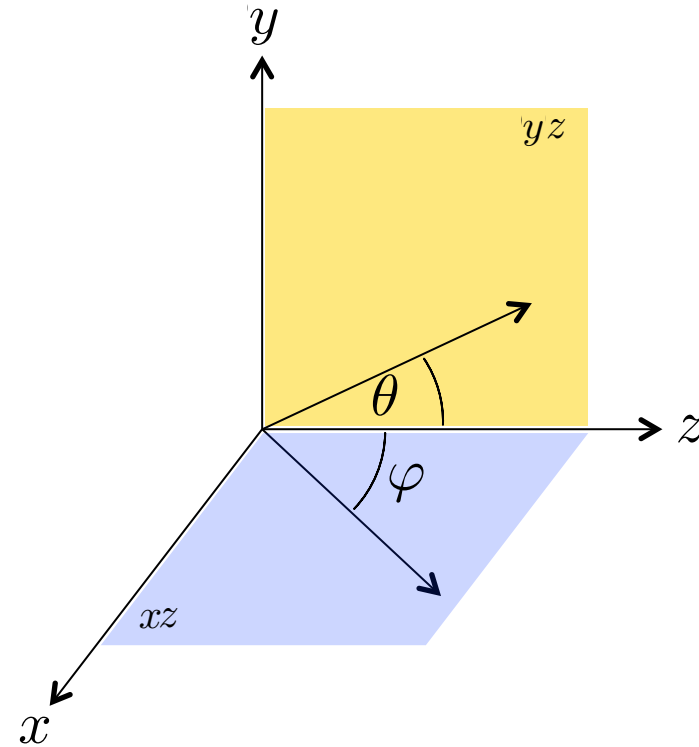
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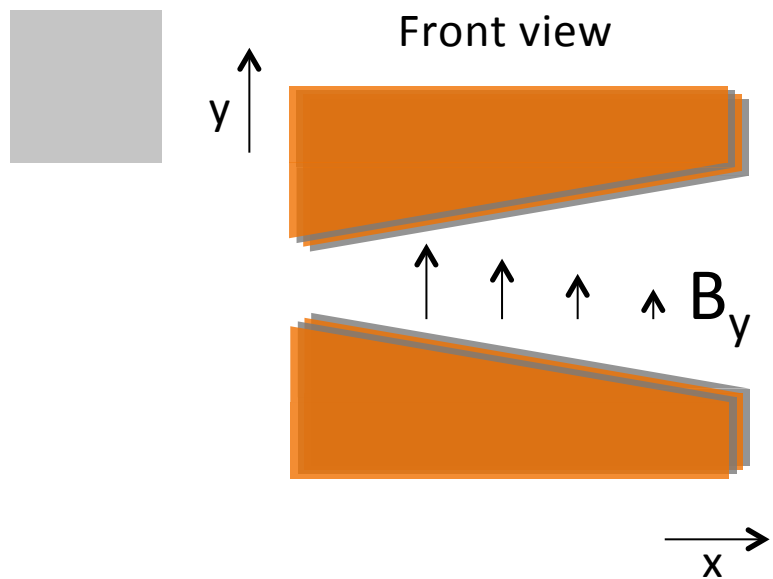
The definition of **taper (t)** used all along this presentation:

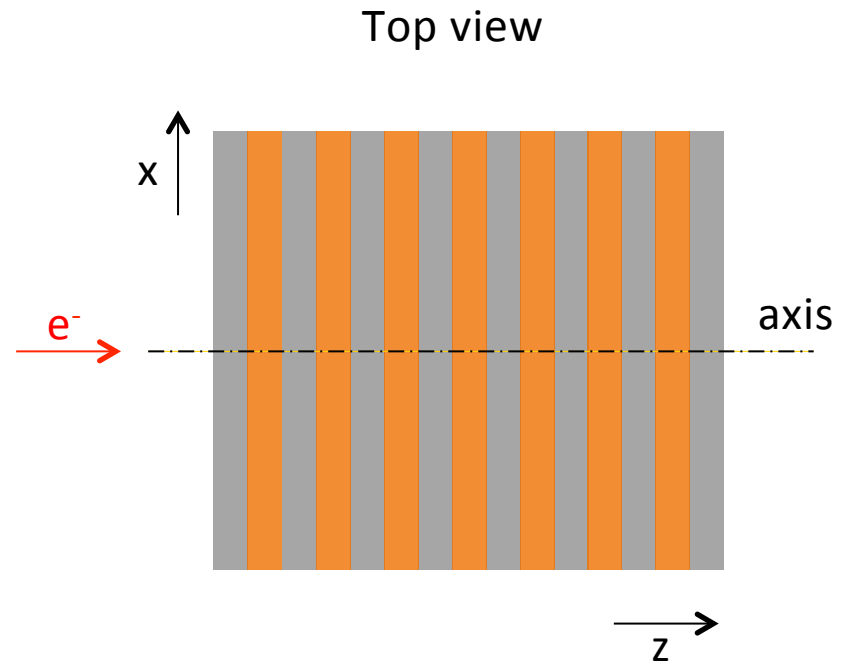
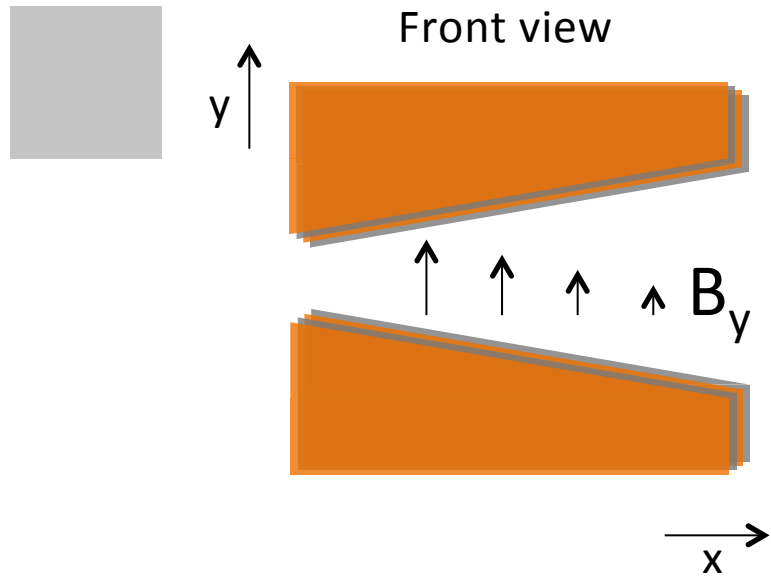
$$t = \frac{1}{K} \frac{\partial K}{\partial z} [m^{-1}]$$

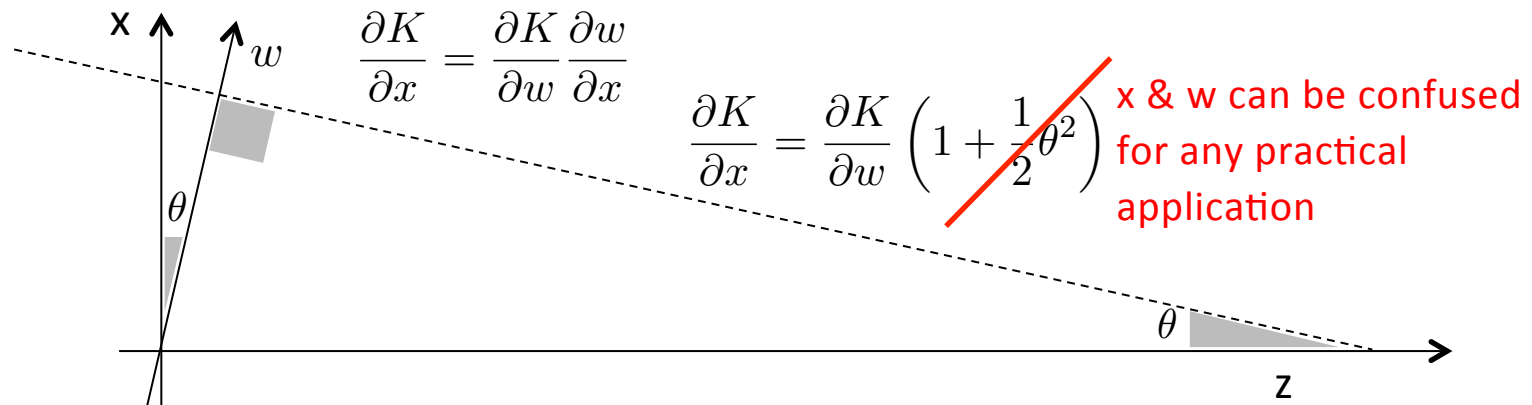
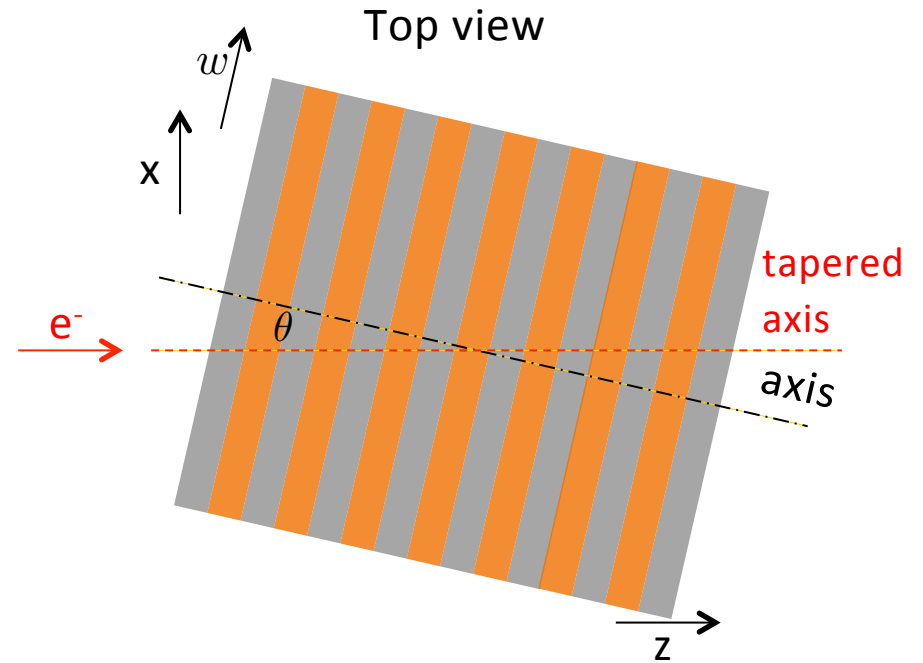
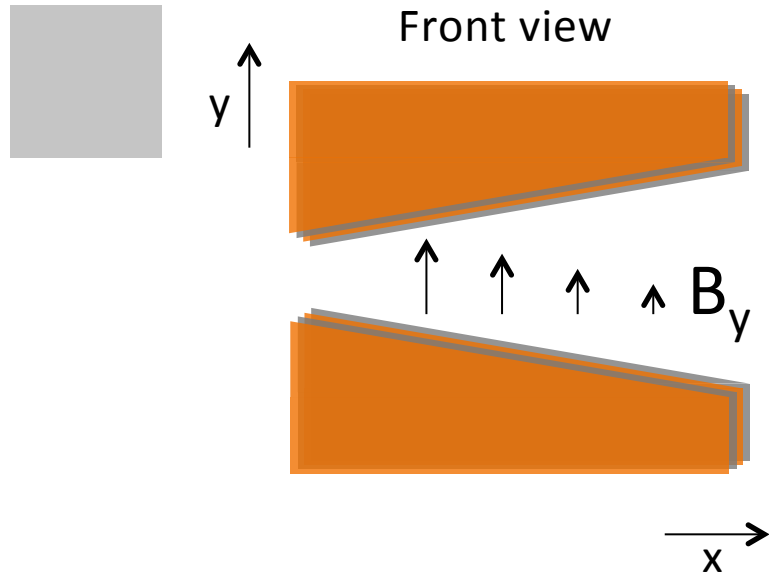
$$= \frac{1}{K} \frac{\partial K}{\partial y} \frac{\partial y}{\partial z} \text{ *Pitch angle } \theta$$


$$= \frac{1}{K} \frac{\partial K}{\partial x} \frac{\partial x}{\partial z} \text{ *Yaw angle } \varphi$$



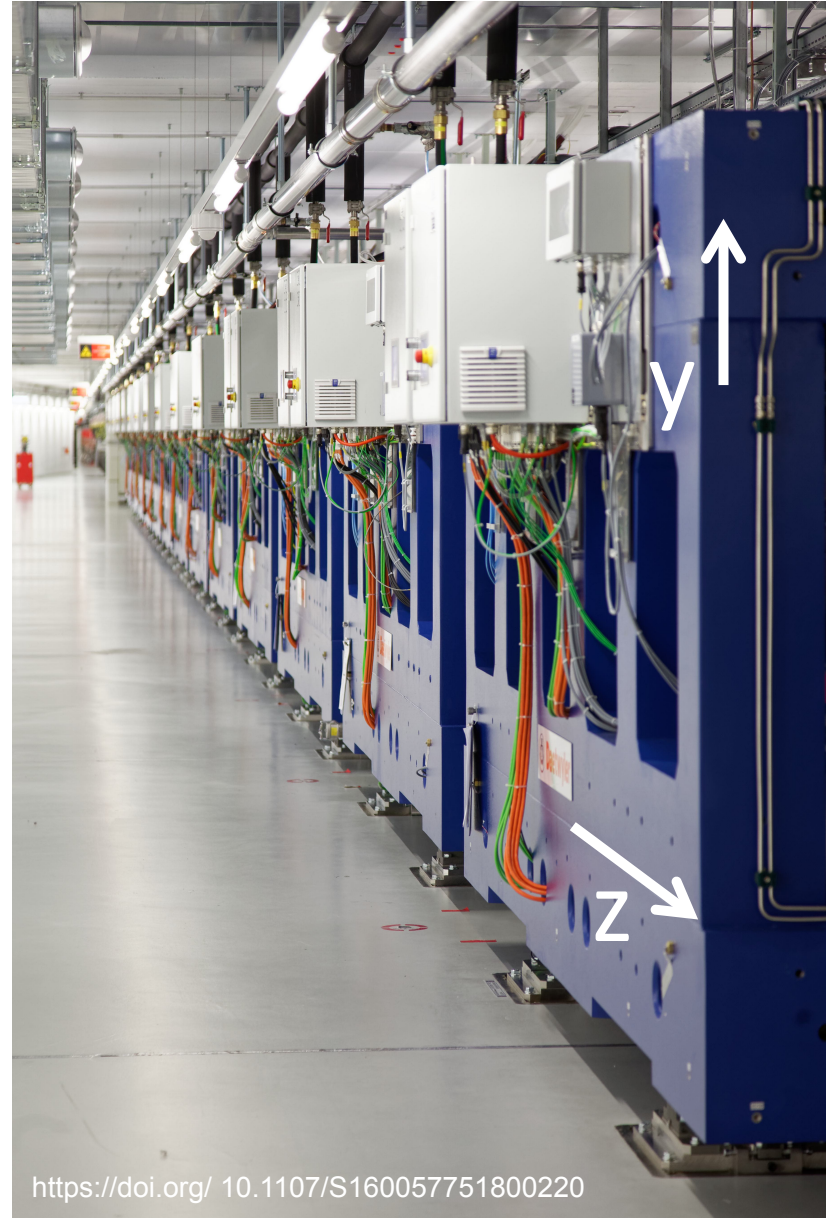
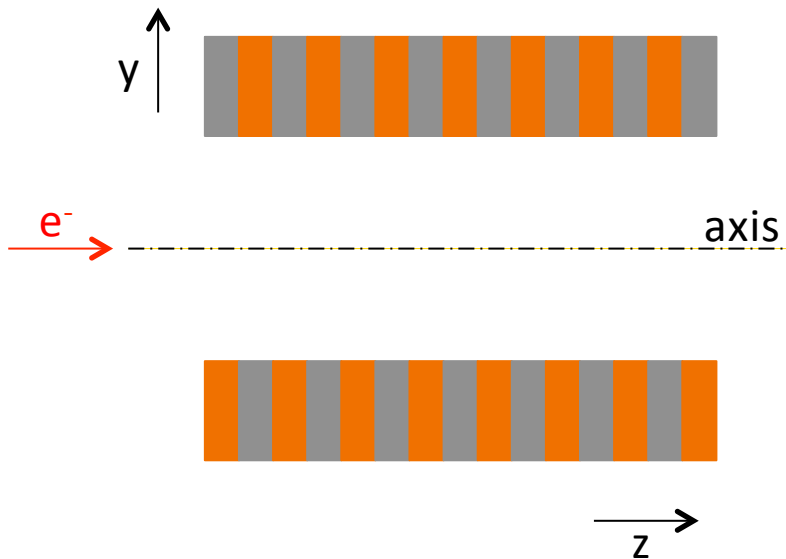
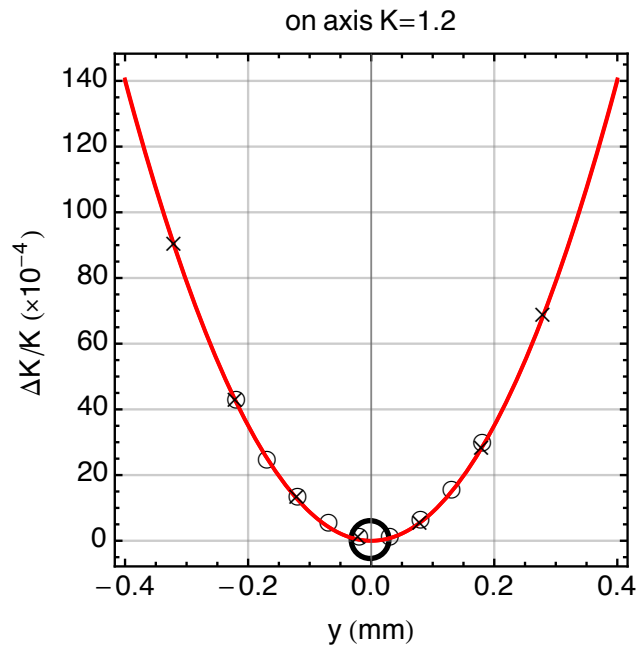






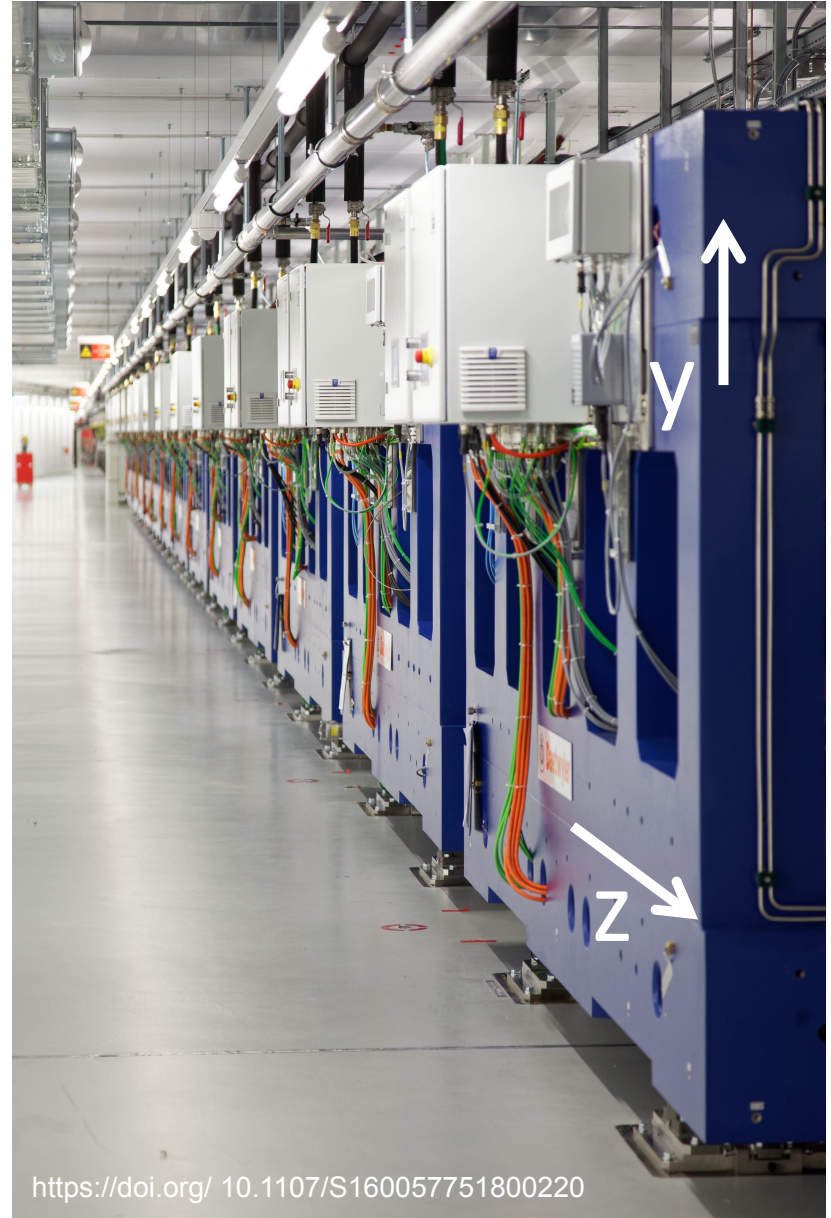
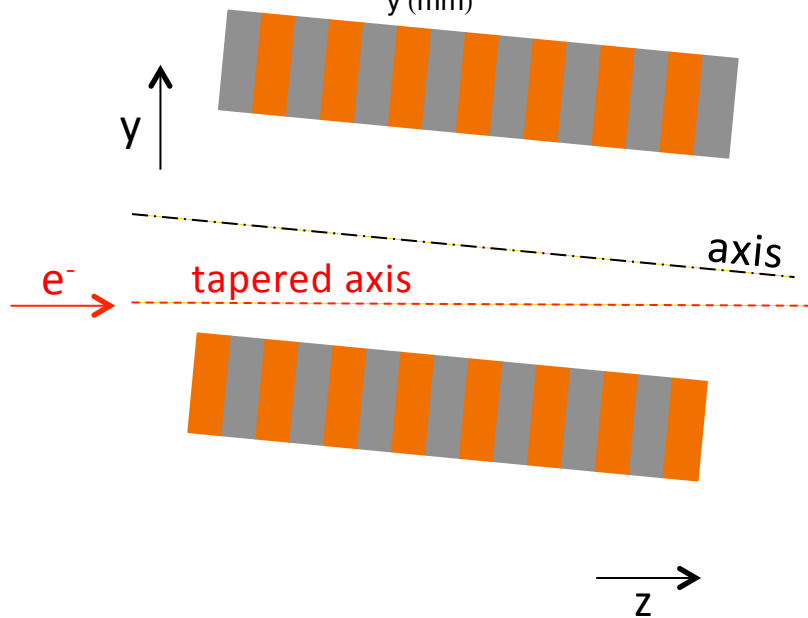
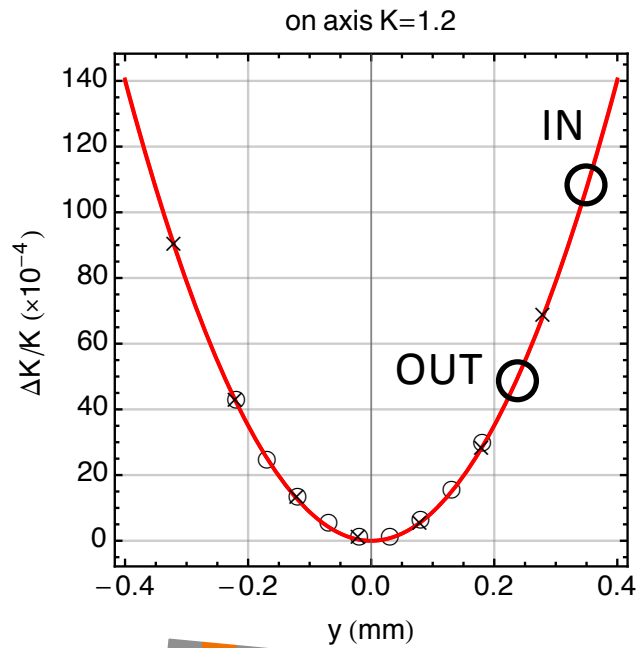
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In-vacuum undulator

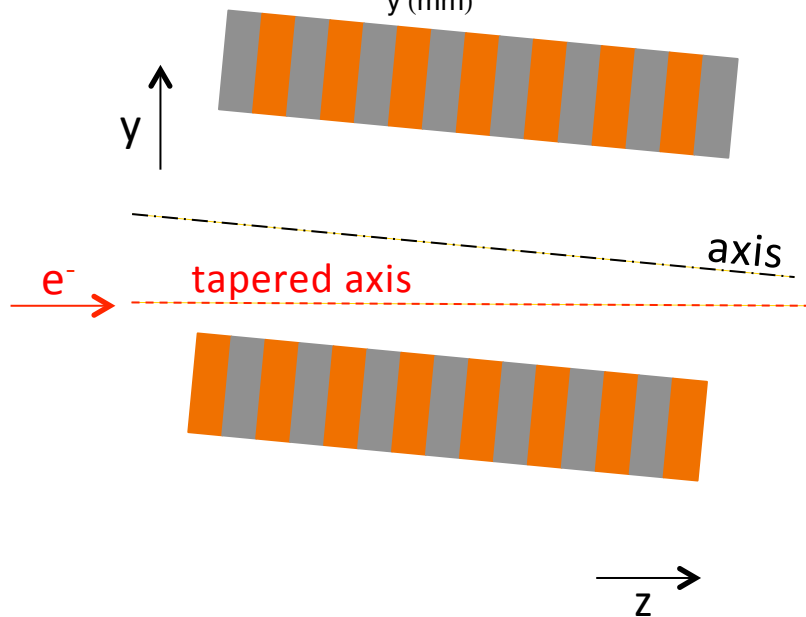
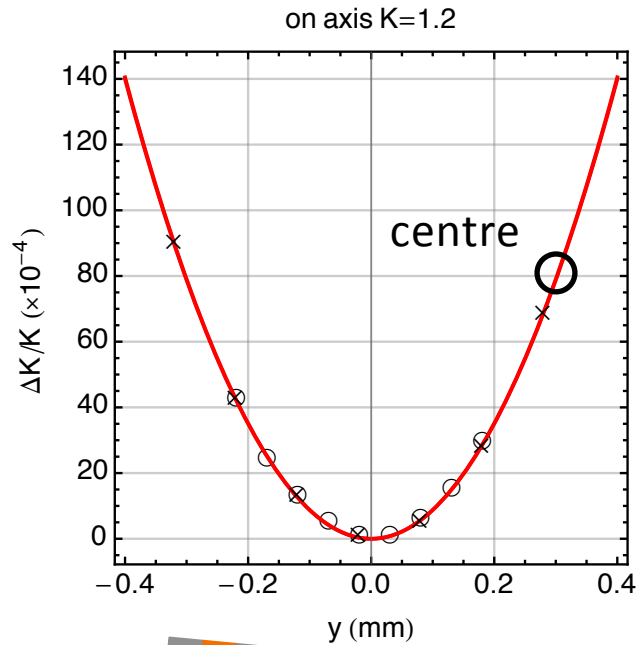


[https://doi.org/ 10.1107/S160057751800220](https://doi.org/10.1107/S160057751800220)

In-vacuum undulator



In-vacuum undulator: *operation example*



$$\frac{\Delta K}{K} = \alpha y^2$$

$$\alpha = 8.83 \cdot 10^{-2} \text{mm}^{-2}$$

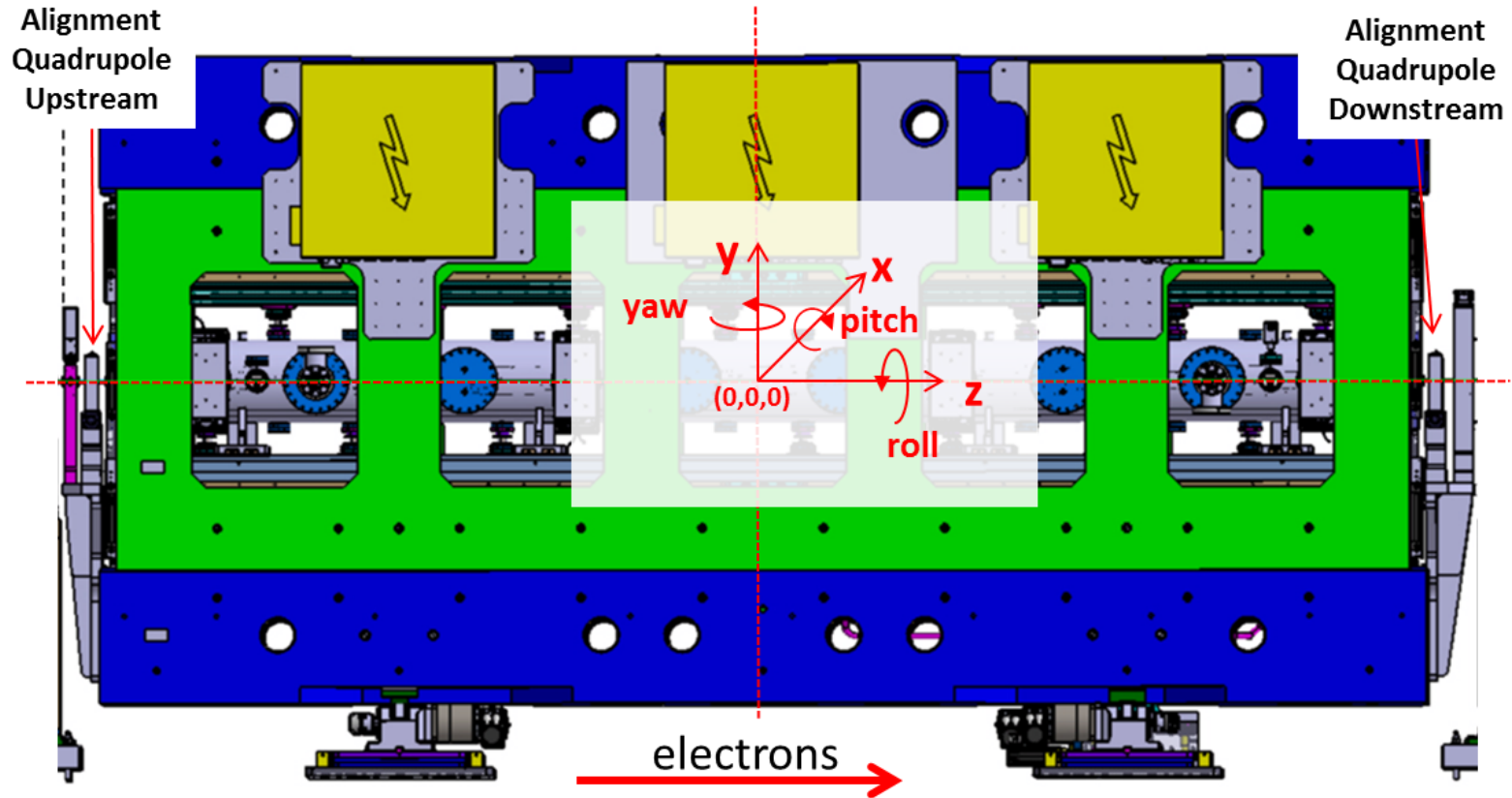
$$y = 0.3 \text{mm}$$

$$\theta = 100 \mu\text{rad}$$

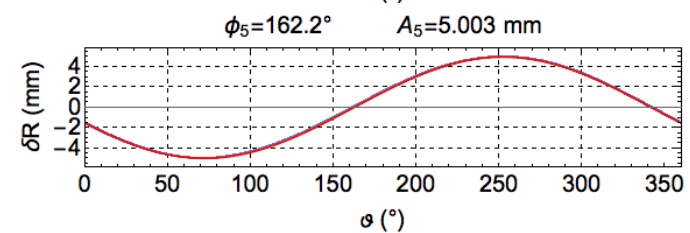
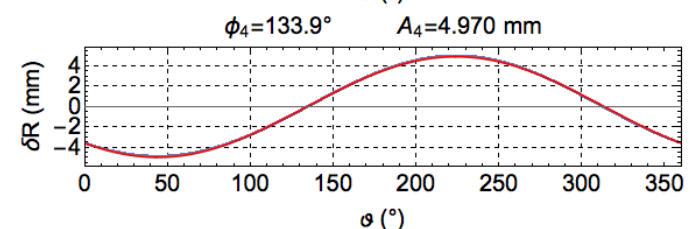
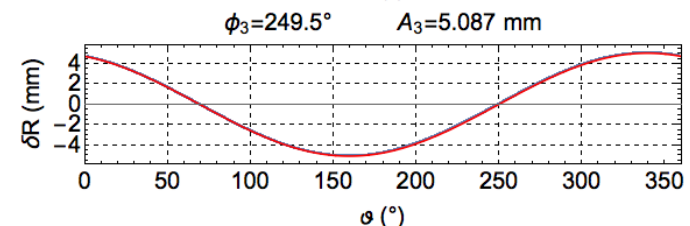
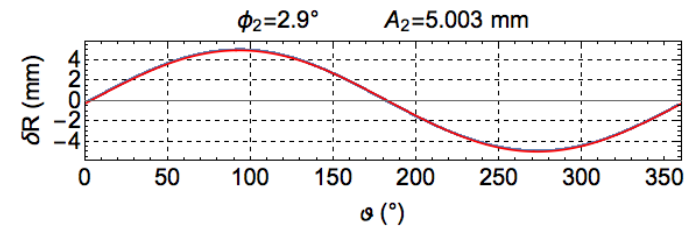
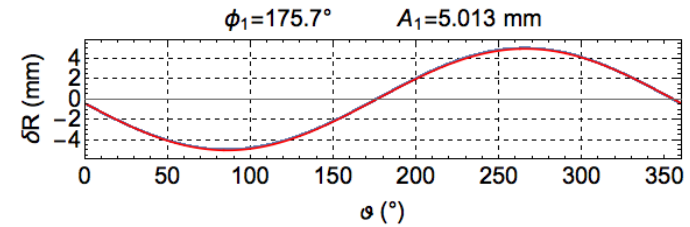
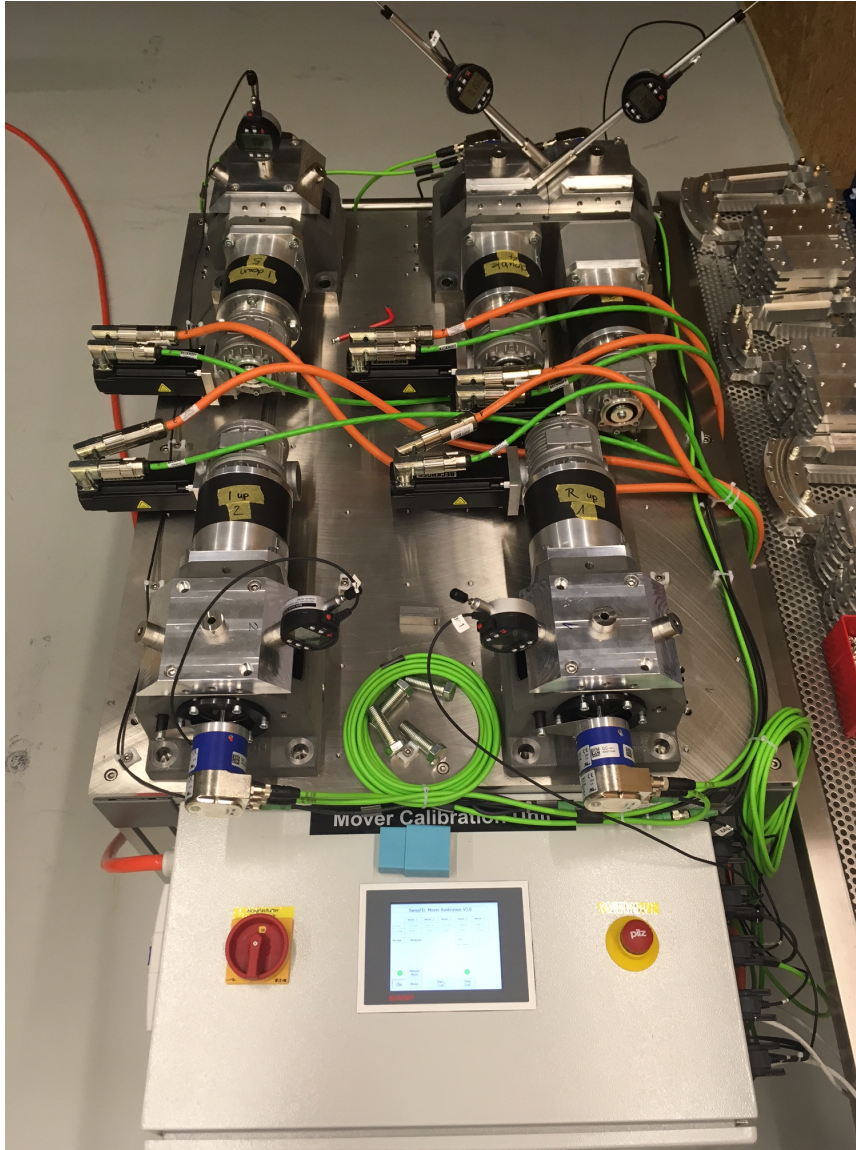
$$\frac{1}{K} \frac{\partial K}{\partial y} = 2\alpha y = 5.3 \cdot 10^{-2} \text{mm}^{-1}$$

$$t = \theta \frac{1}{K} \frac{\partial K}{\partial y} = 0.53\%/\text{m}$$

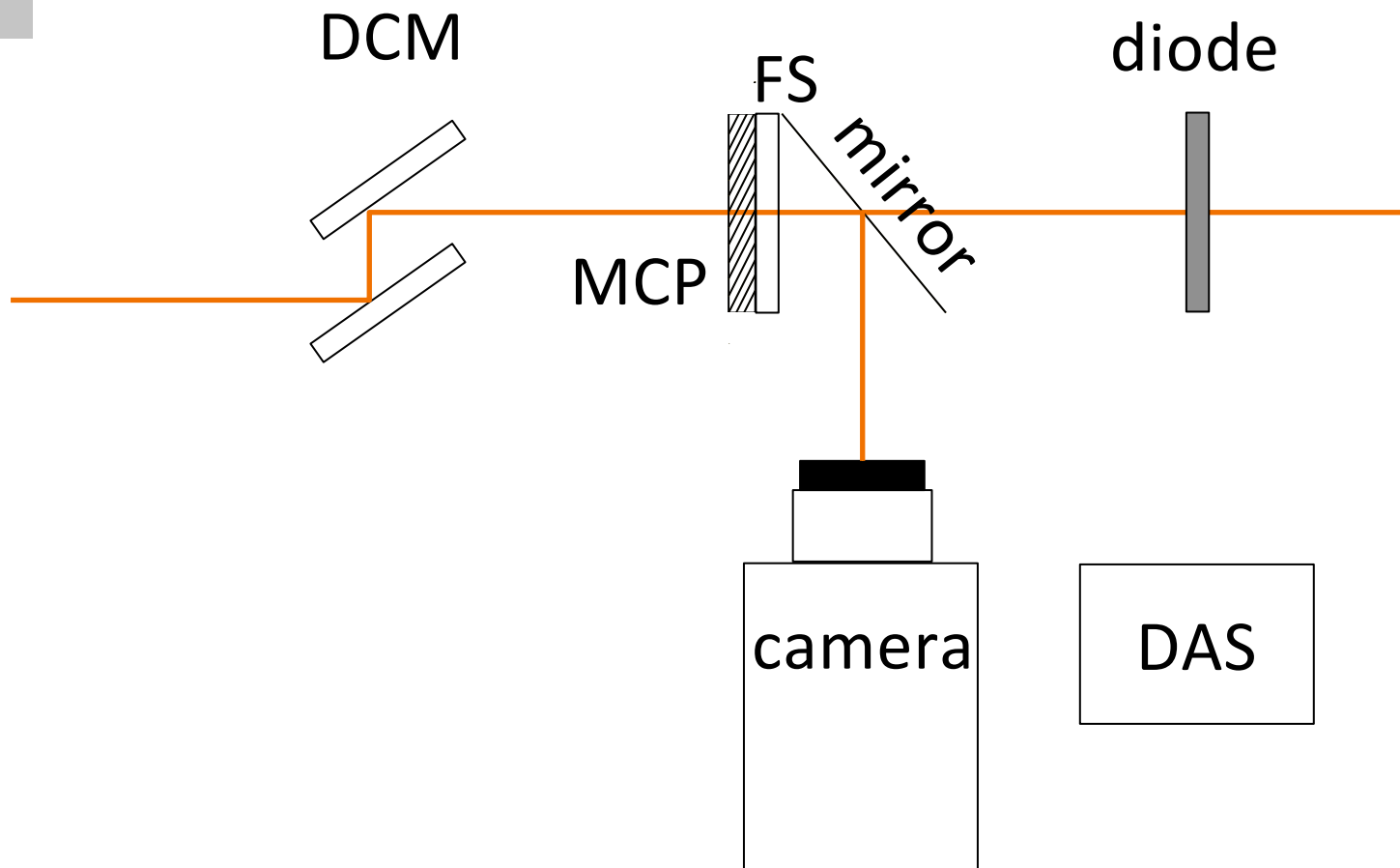
Camshaft Mover system

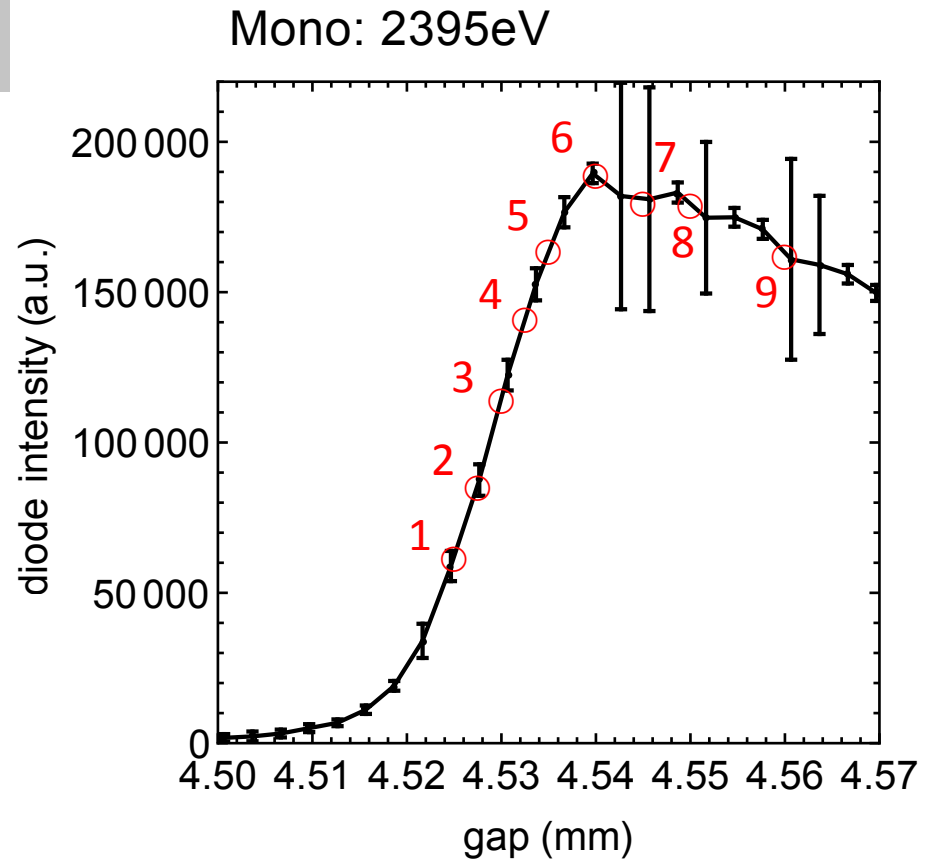
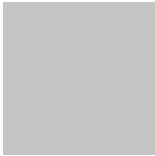


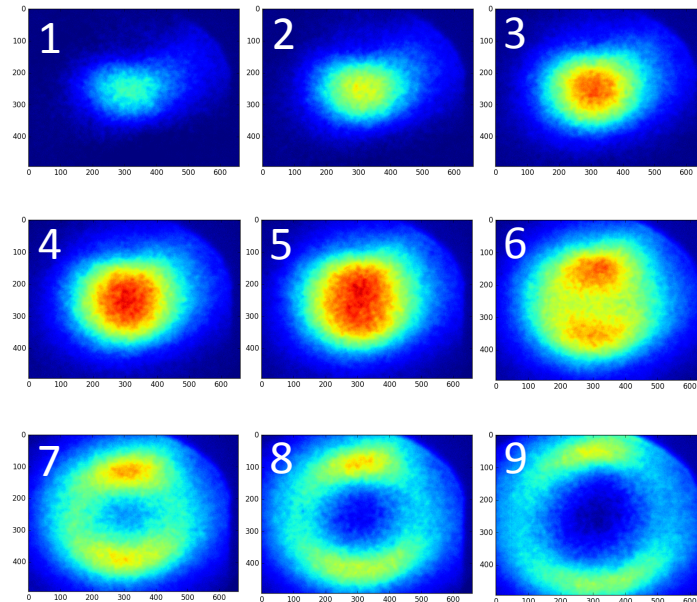
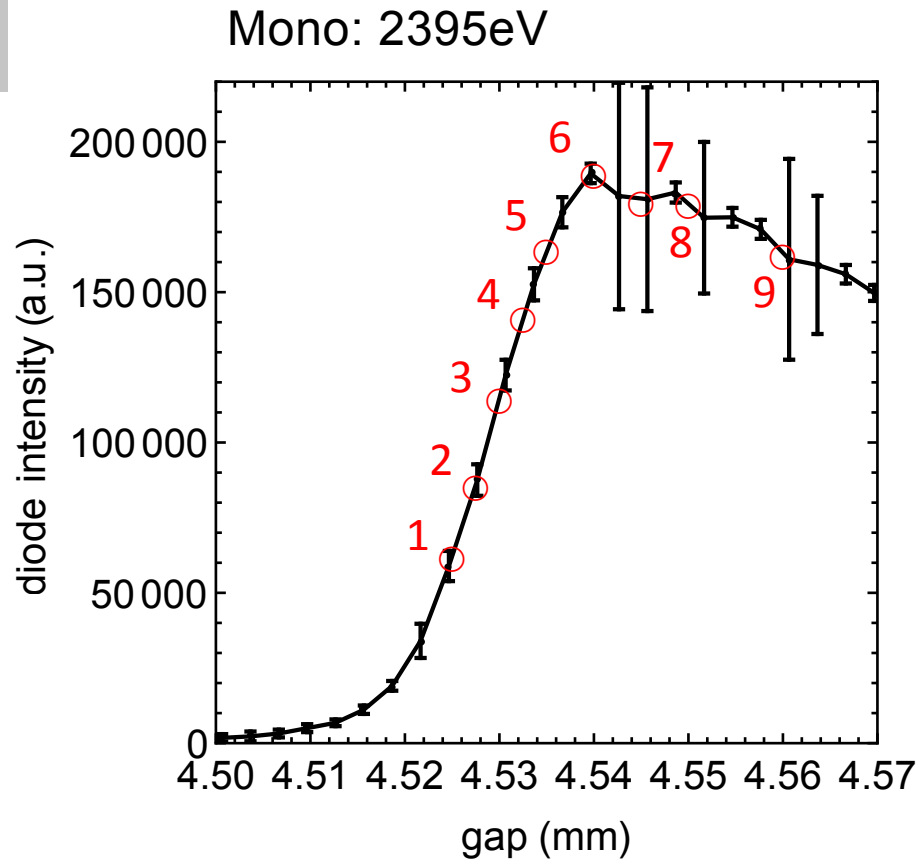
Camshaft Mover system



In-situ alignment

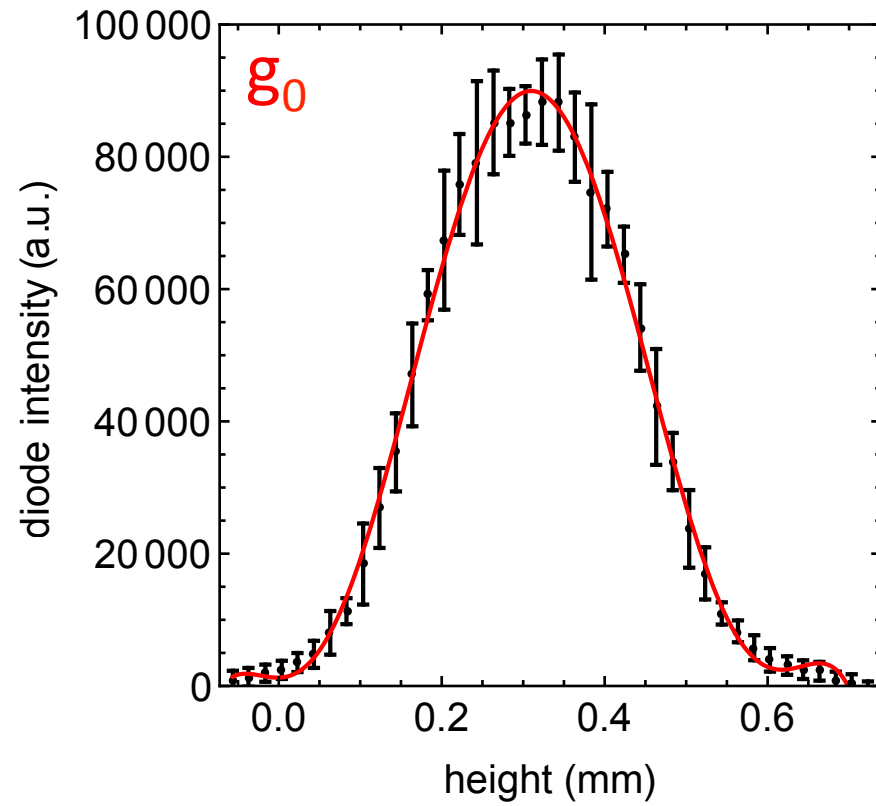
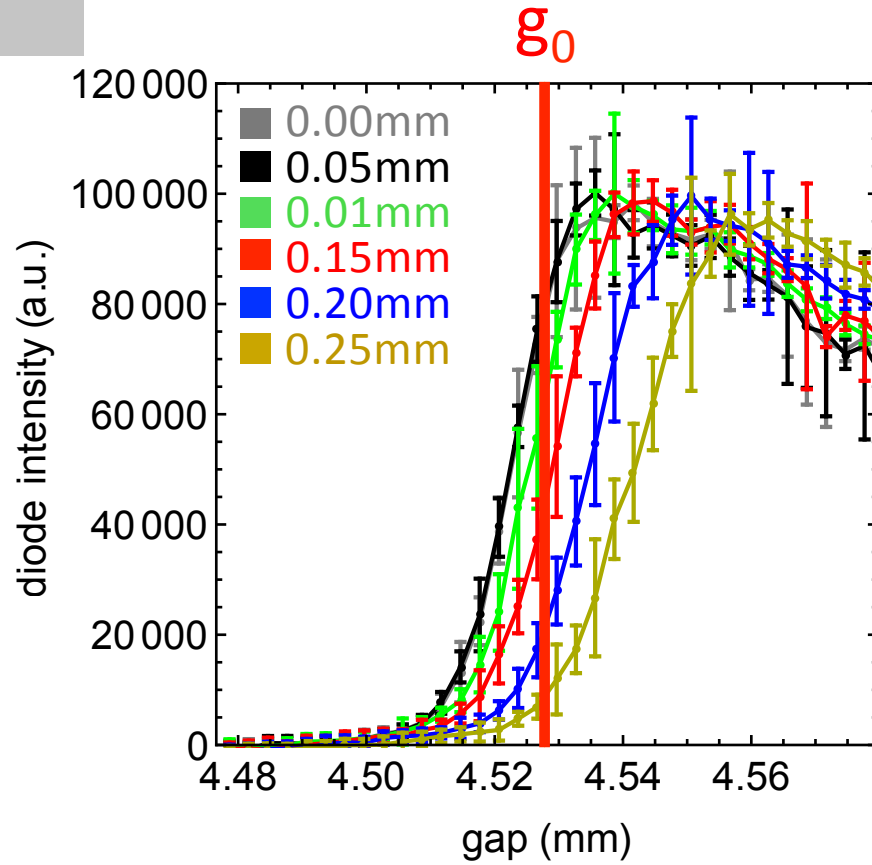






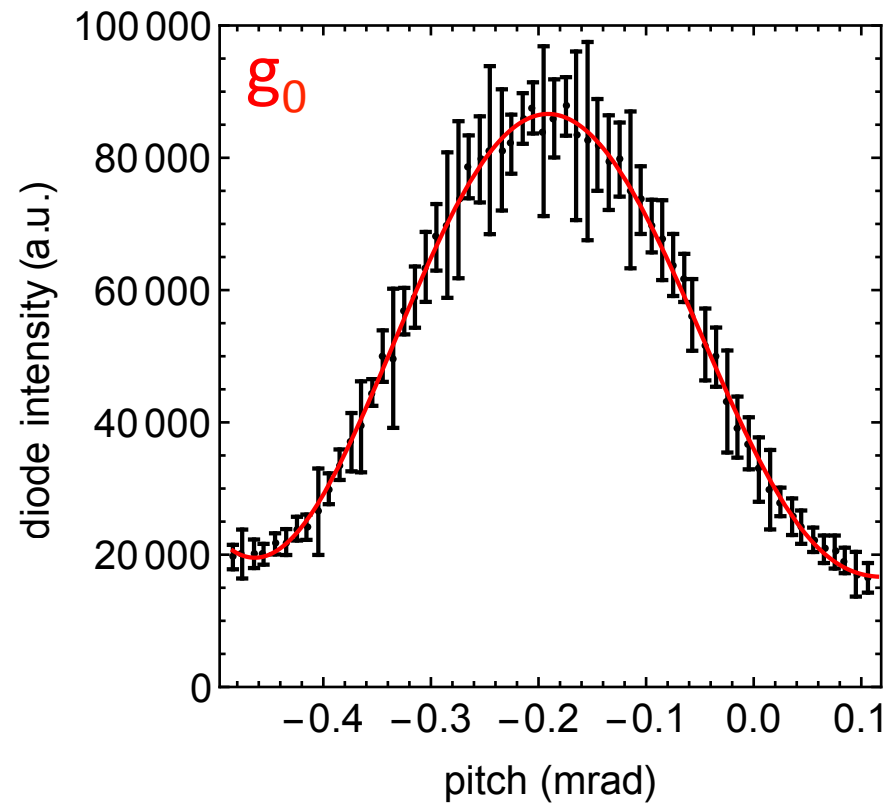
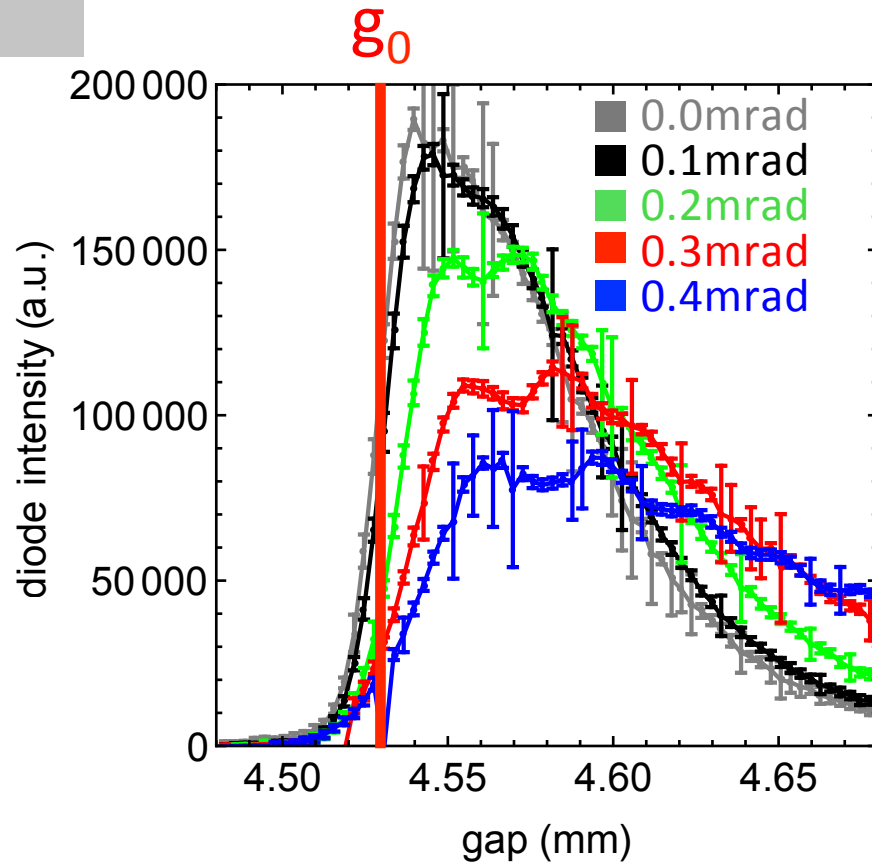
In-situ alignment: height tuning


Mono: 2395eV
1st harmonic



In-situ alignment: pitch tuning

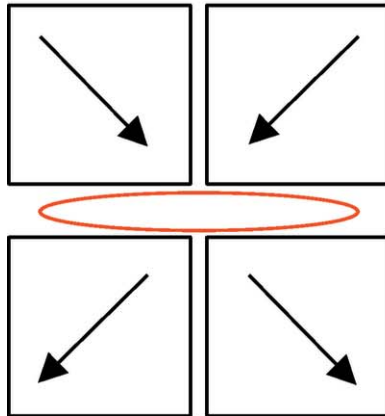
Mono: 2395eV
1st harmonic



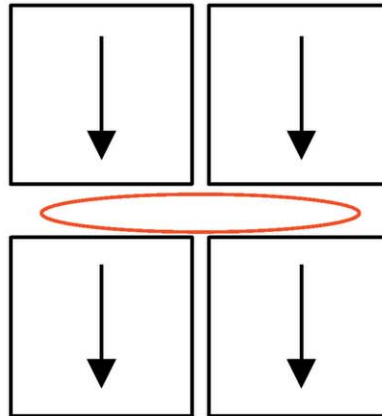
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Apple Undulators

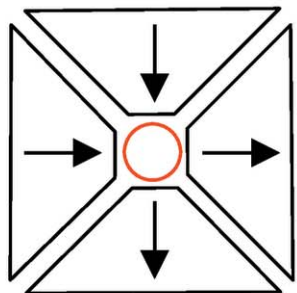
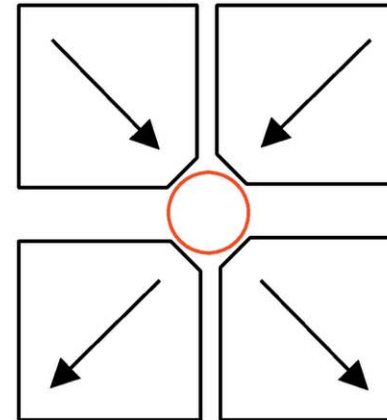
Apple I



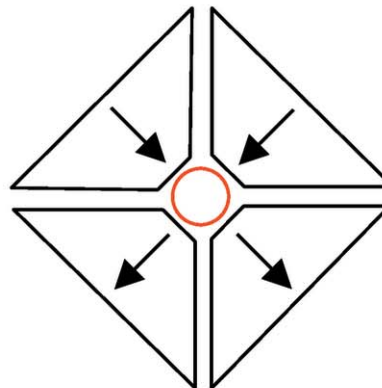
Apple II



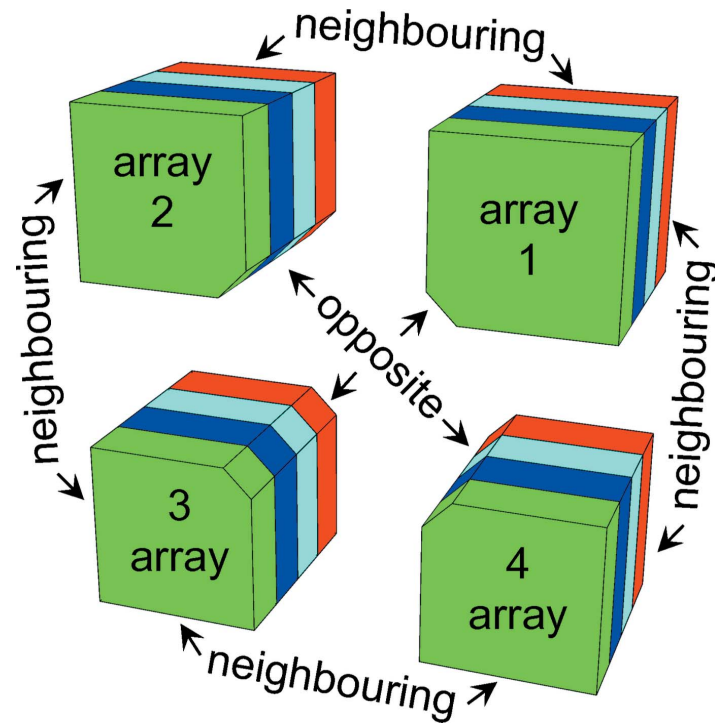
Apple III



Delta
LEPP-CHESS

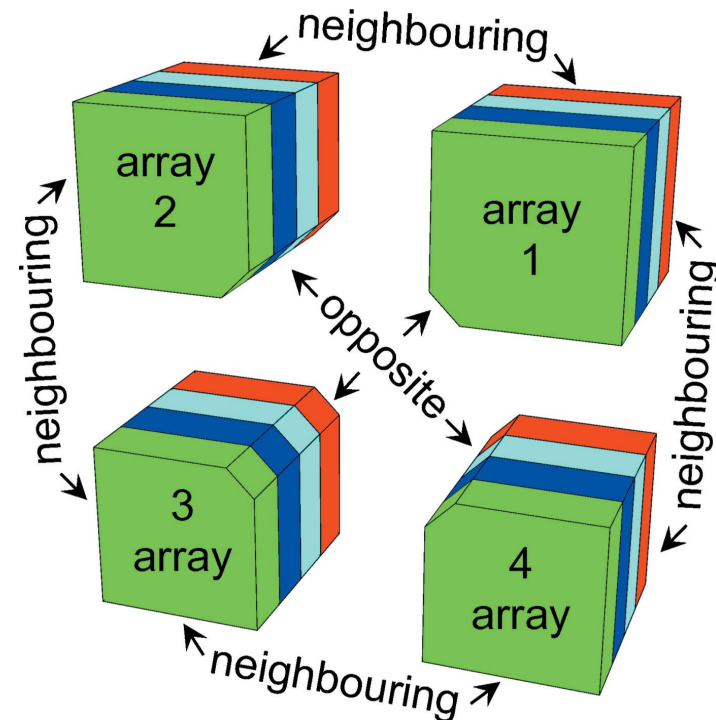


Delta



- Parallel mode (p):
elliptical polarisation
- Anti parallel mode (\bar{p}):
linear polarisation with arbitrary angle
- Energy mode (e):
to change the photon energy
@ fix gap

This last mode introduces gradients on axis for elliptical polarisation!!



$$\mathbf{B}(\mathbf{X}, z) = \sum_{n=1}^4 \mathbf{B}_n(\mathbf{X}, z - z_n)$$

$$\mathbf{B}_n(\mathbf{X}, z) = \mathbf{R}_n \cdot \mathbf{B}_1(\mathbf{R}_n^{-1} \cdot \mathbf{X}, z)$$

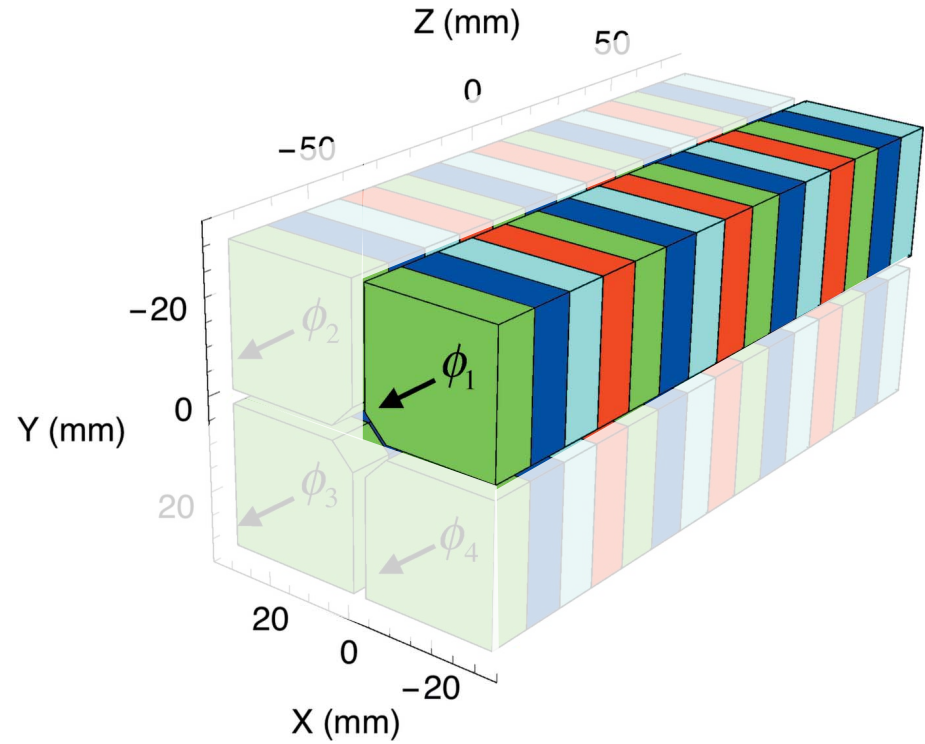
$$\mathbf{R}_1 = \begin{bmatrix} +1 & 0 \\ 0 & +1 \end{bmatrix} \quad \mathbf{R}_2 = \begin{bmatrix} -1 & 0 \\ 0 & +1 \end{bmatrix}$$

$$\mathbf{R}_3 = \begin{bmatrix} -1 & 0 \\ 0 & -1 \end{bmatrix} \quad \mathbf{R}_4 = \begin{bmatrix} +1 & 0 \\ 0 & -1 \end{bmatrix}$$

$$\hat{\mathbf{B}}_n(\mathbf{X}, \omega) = \int_{-\infty}^{+\infty} \mathbf{B}_n(\mathbf{X}, z) \exp(-i\omega z) dz$$

$$\hat{\mathbf{B}}(\mathbf{X}) = \sum_{n=1}^4 \exp(i\phi_n) \mathbf{R}_n \cdot \hat{\mathbf{B}}_1(\mathbf{R}_n^{-1} \cdot \mathbf{X})$$

$$\hat{\mathbf{J}}(\mathbf{X}) = \sum_{n=1}^4 \exp(i\phi_n) \mathbf{R}_n \cdot \hat{\mathbf{J}}_1(\mathbf{R}_n^{-1} \cdot \mathbf{X}) \cdot \mathbf{R}_n^{-1}$$



$$\mathbf{B}(\mathbf{X}, z) = \sum_{n=1}^4 \mathbf{B}_n(\mathbf{X}, z - z_n)$$

$$\mathbf{B}_n(\mathbf{X}, z) = \mathbf{R}_n \cdot \mathbf{B}_1(\mathbf{R}_n^{-1} \cdot \mathbf{X}, z)$$

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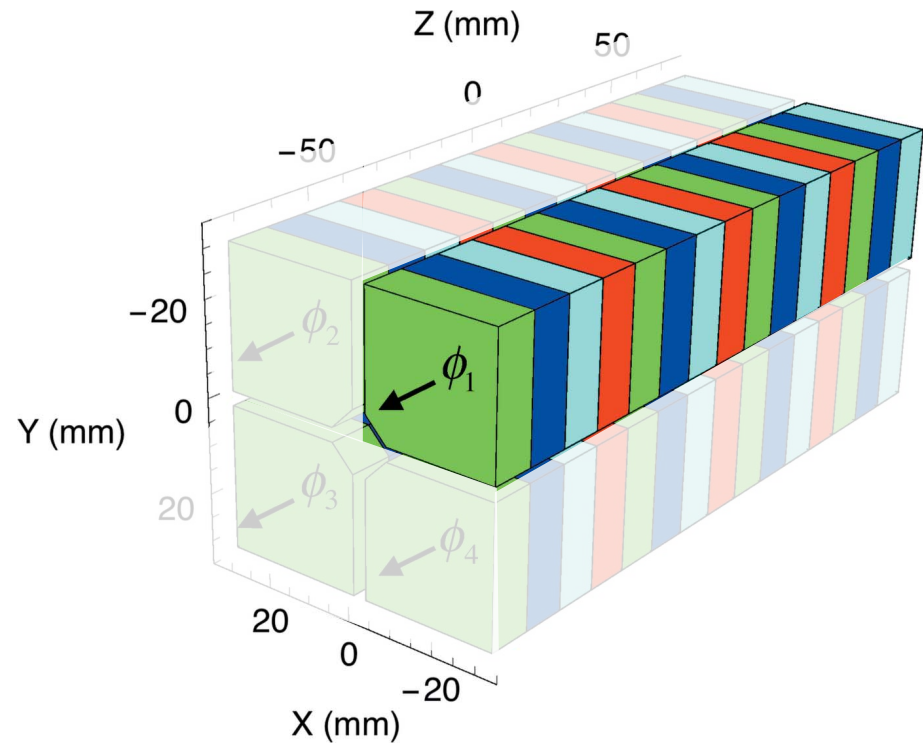
$$\mathbf{R}_3 = \begin{bmatrix} -1 & 0 \\ 0 & -1 \end{bmatrix} \quad \mathbf{R}_4 = \begin{bmatrix} +1 & 0 \\ 0 & -1 \end{bmatrix}$$

$$\hat{\mathbf{B}}_n(\mathbf{X}, \omega) = \int_{-\infty}^{+\infty} \mathbf{B}_n(\mathbf{X}, z) \exp(-i\omega z) dz$$

$$\hat{\mathbf{B}} = \left[\sum_{n=1}^4 \exp(i\phi_n) \mathbf{R}_n \right] \cdot \hat{\mathbf{B}}_1$$

$$\hat{\mathbf{J}} = \sum_{n=1}^4 \exp(i\phi_n) \mathbf{R}_n \cdot \hat{\mathbf{J}}_1 \cdot \mathbf{R}_n^{-1}$$

Reducing our investigation to the on axis fields



B-domain

$$\hat{\mathbf{B}} = \left[\sum_{n=1}^4 \exp(i\phi_n) \mathbf{R}_n \right] \cdot \hat{\mathbf{B}}_1$$

$$\hat{\mathbf{J}} = \sum_{n=1}^4 \exp(i\phi_n) \mathbf{R}_n \cdot \hat{\mathbf{J}}_1 \cdot \mathbf{R}_n^{-1}$$

K-domain

$$K^2 \equiv \mathbf{K} \cdot \mathbf{K}^*$$

$$\mathbf{K} = \kappa \hat{\mathbf{B}} \quad \kappa = \frac{e\lambda_U}{2\pi mc}$$

$$\nabla K = \kappa \cdot \Re(\hat{\mathbf{J}} \cdot \mathbf{\Gamma}^*)$$

$$\mathbf{\Gamma} = \mathbf{K}/K$$

The K and its gradient for a generic Apple undulator in elliptical polarisation configuration:

$$K = 2\sqrt{2}\kappa\hat{B}_1 \cos\frac{1}{2}\phi_e \quad \hat{B}_1 = [\hat{B}_{1x}^2(1 - \cos\phi_p) + \hat{B}_{1y}^2(1 + \cos\phi_p)]^{1/2}$$

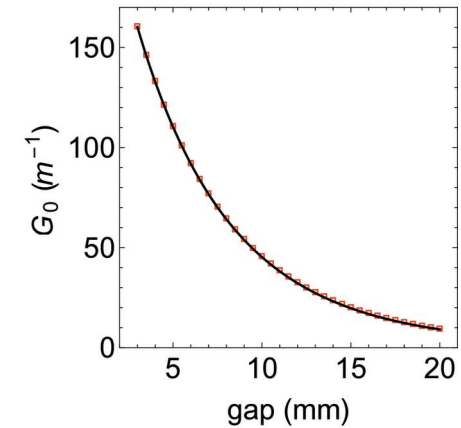
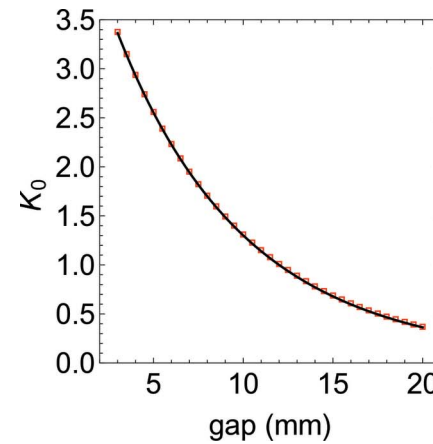
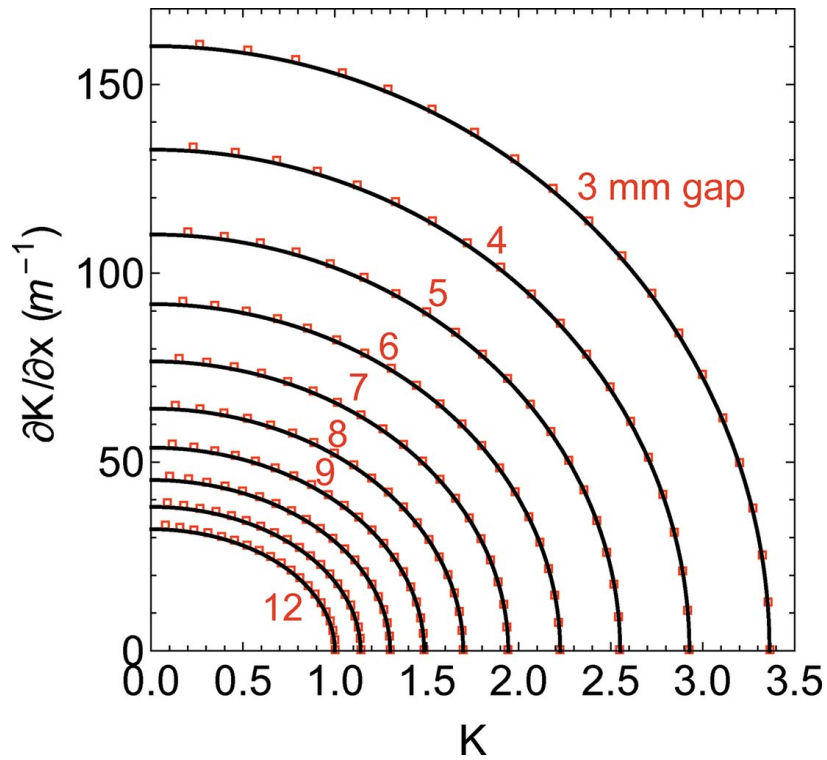
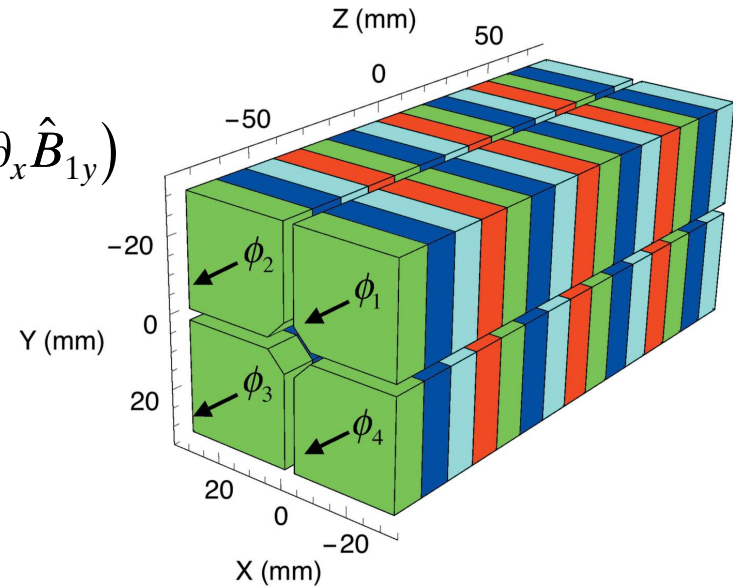
$$\partial_x K = G_0 \sin\frac{1}{2}\phi_e \sin\phi_p, \quad G_0 = 2\sqrt{2}\kappa(\hat{B}_{1x}\partial_x\hat{B}_{1x} - \hat{B}_{1y}\partial_x\hat{B}_{1y})/\hat{B}_1$$

Apple X (or Delta II): *all get simpler*

$$K = K_0 \cos \frac{1}{2} \phi_e \quad K_0 = 4\kappa \hat{B}_{x1}$$

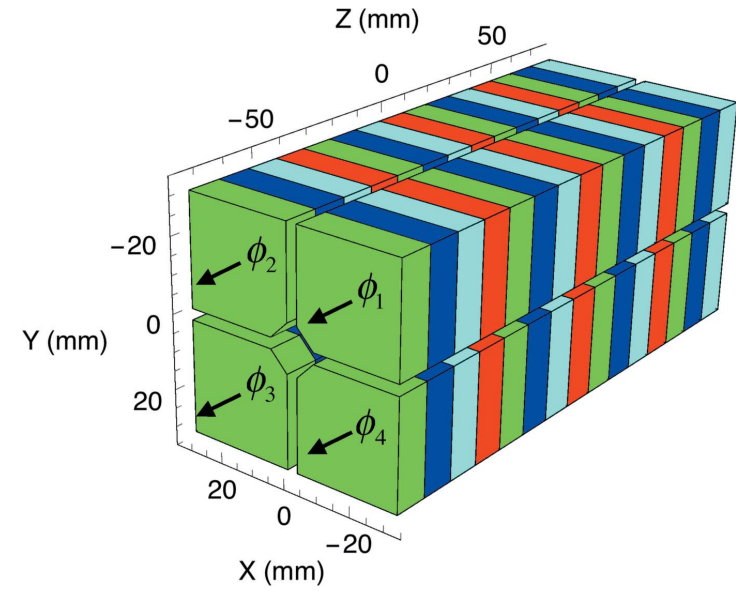
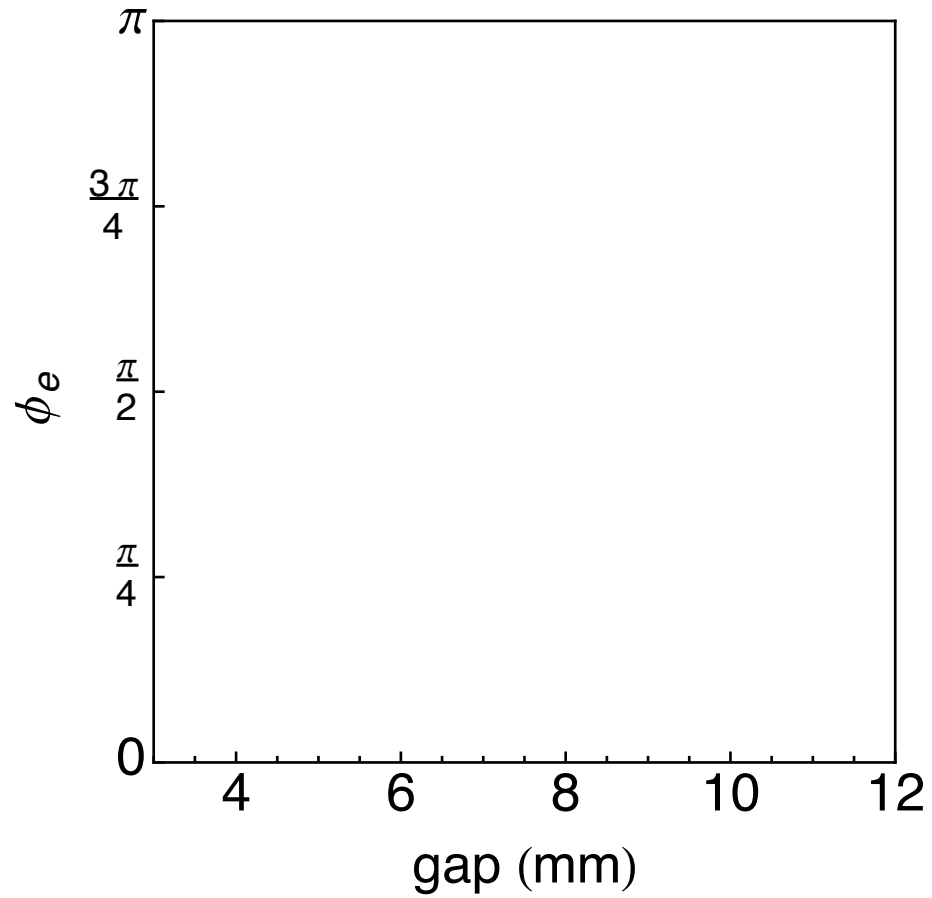
$$\partial_x K = G_0 (1 - \xi^2)^{1/2} \quad G_0 = 2\kappa (\partial_x \hat{B}_{1x} - \partial_x \hat{B}_{1y})$$

$$\xi = K/K_0$$

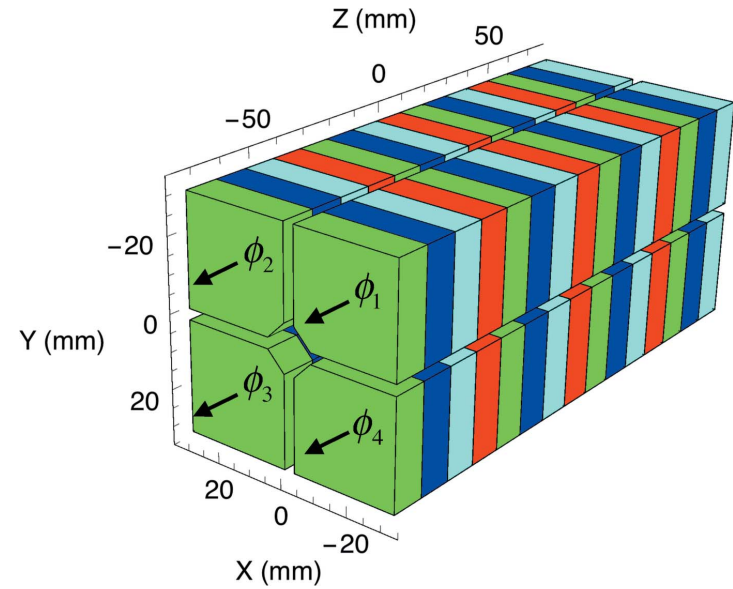
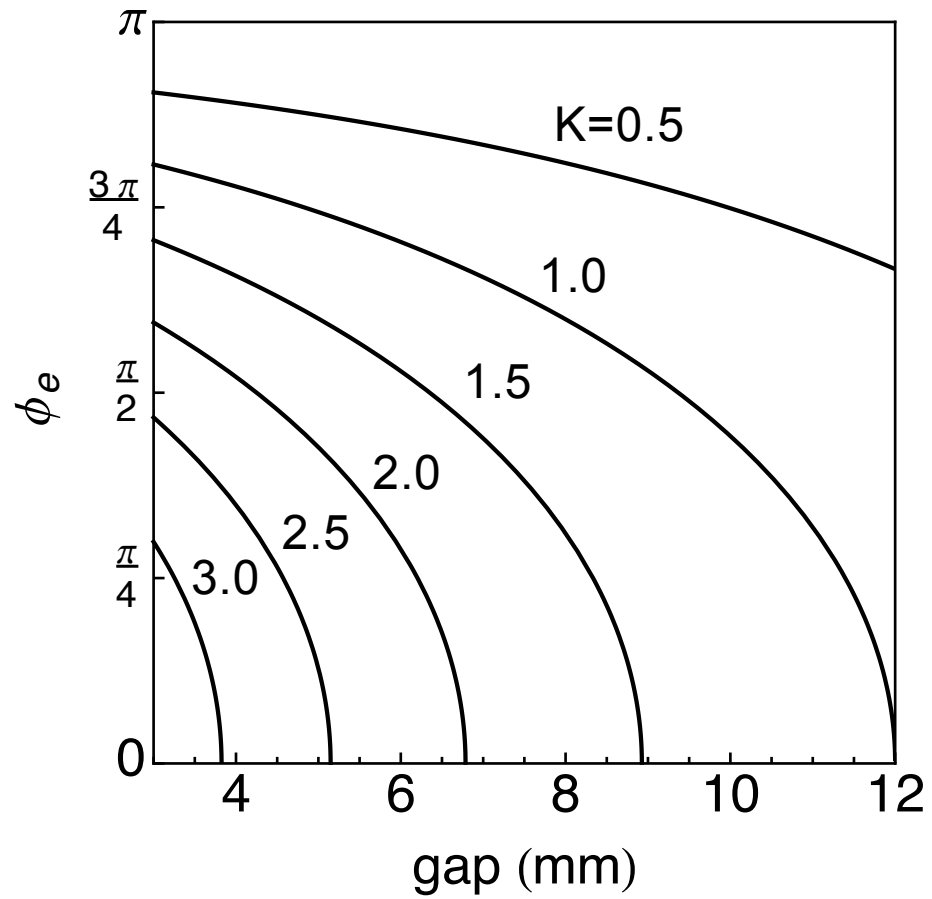


Example of U40 for Athos with Br=1.08T

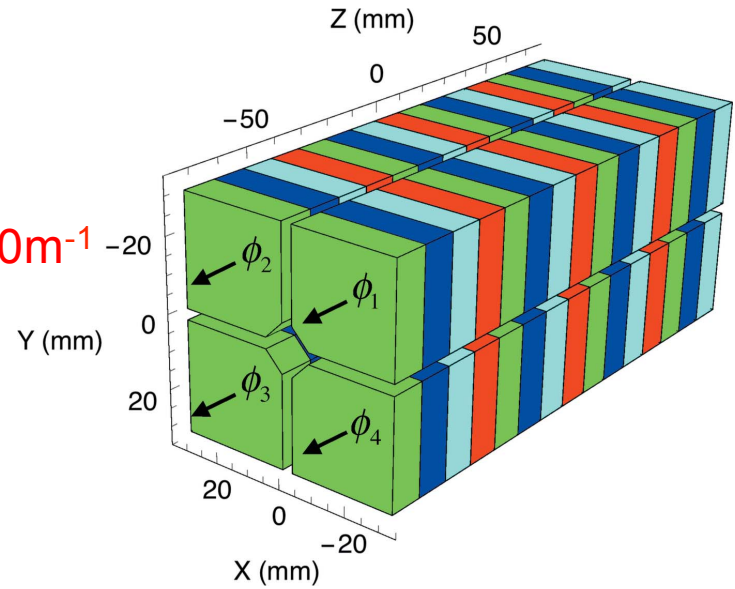
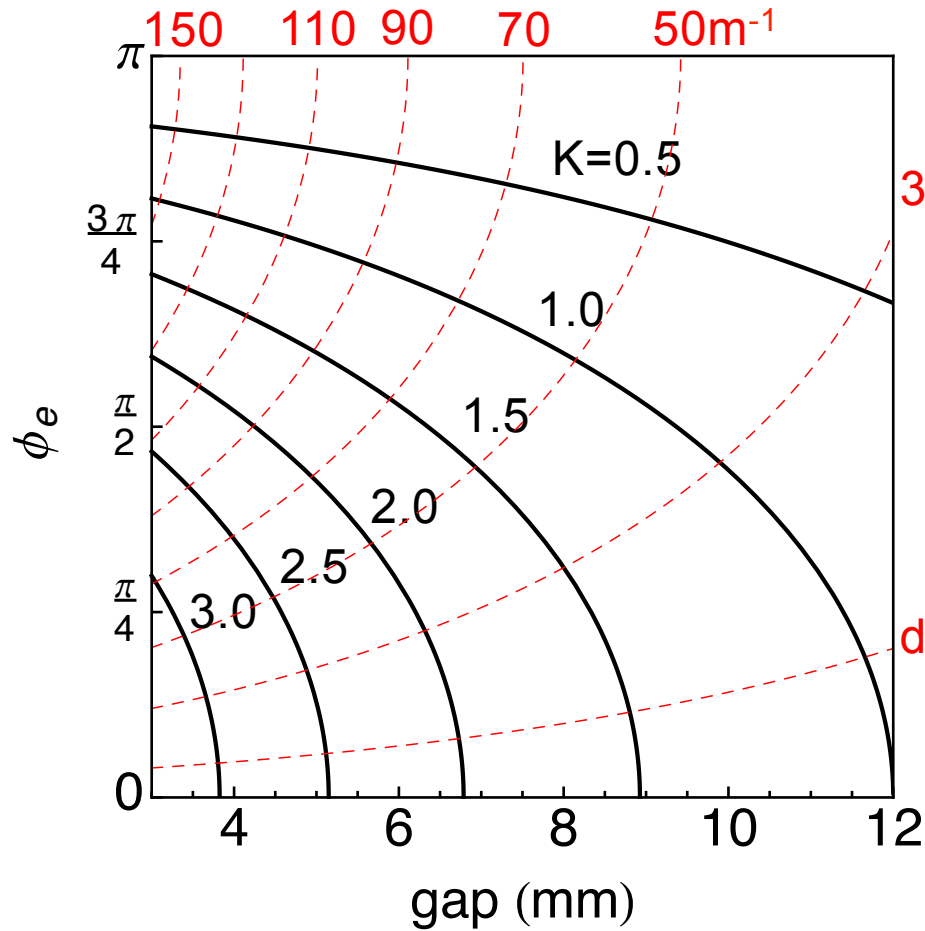
Apple X (or Delta II): *Operational example*



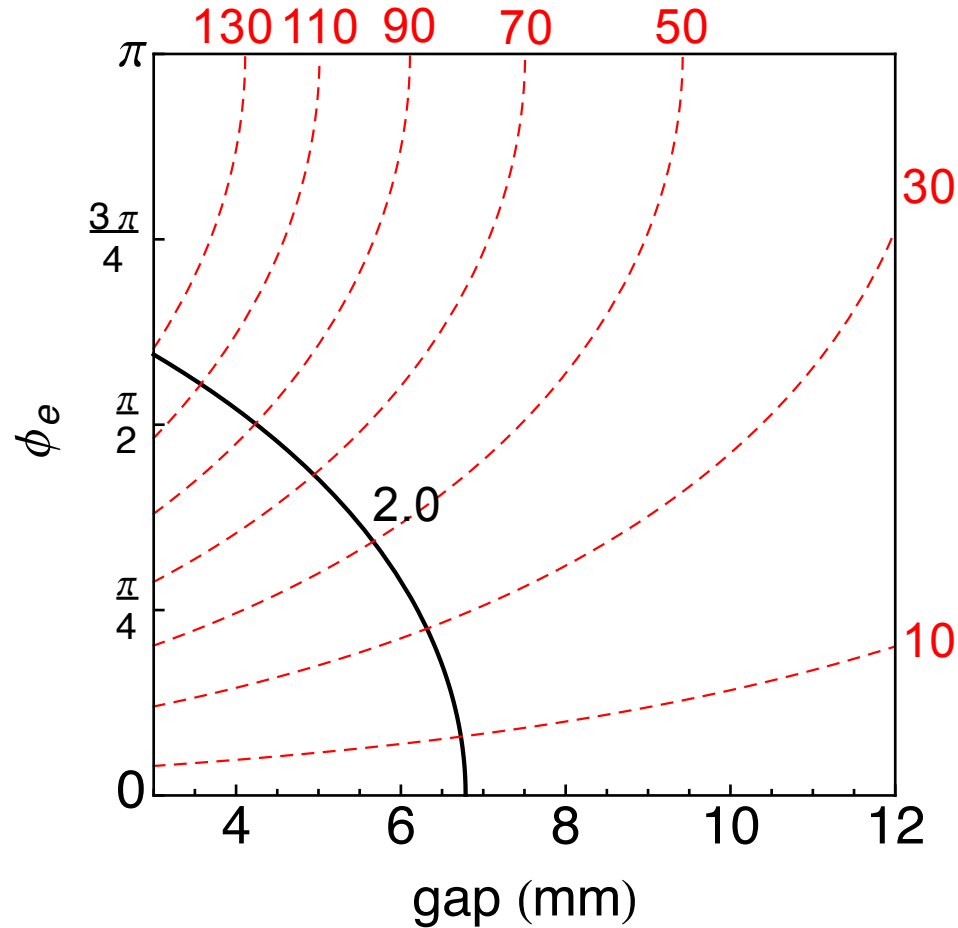
Apple X (or Delta II): *Operational example*



Apple X (or Delta II): *Operational example*



Apple X (or Delta II): *Operational example*




$$K = 2.0$$

$$\varphi = 100 \mu rad$$

$$t = \varphi \frac{1}{K} \frac{\partial K}{\partial x}$$

$$0 < t < 0.65\%/m$$

$$-0.65 < t < 0.65\%/m$$

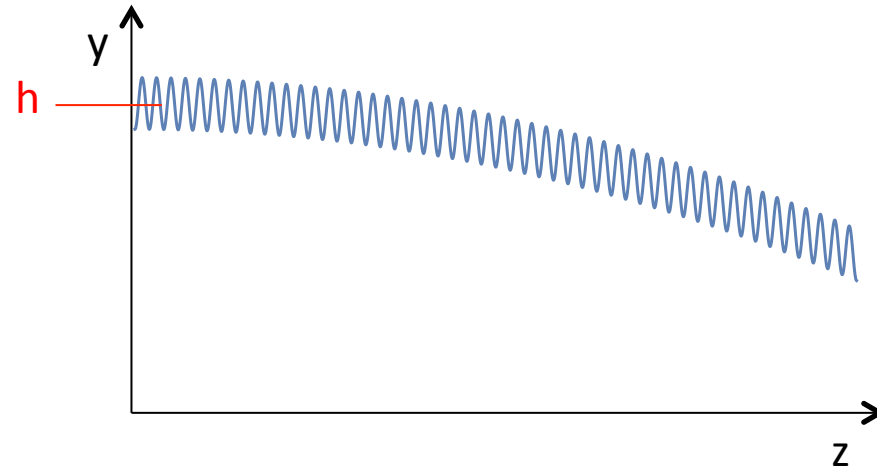
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Natural focusing

$$\frac{d^2 y}{dz^2} + \hat{k}y = 0$$

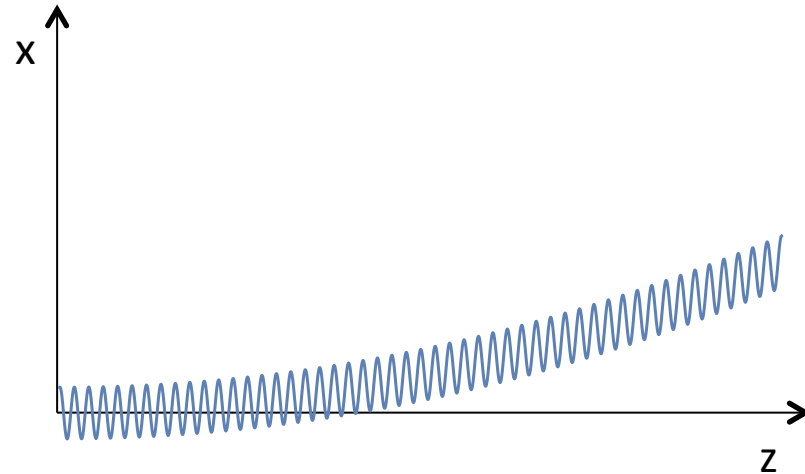
$$\hat{k} = \left(\frac{e}{\gamma mc} \right)^2 \frac{b^2}{2}$$

$$\varphi = \sqrt{\hat{k}L}$$

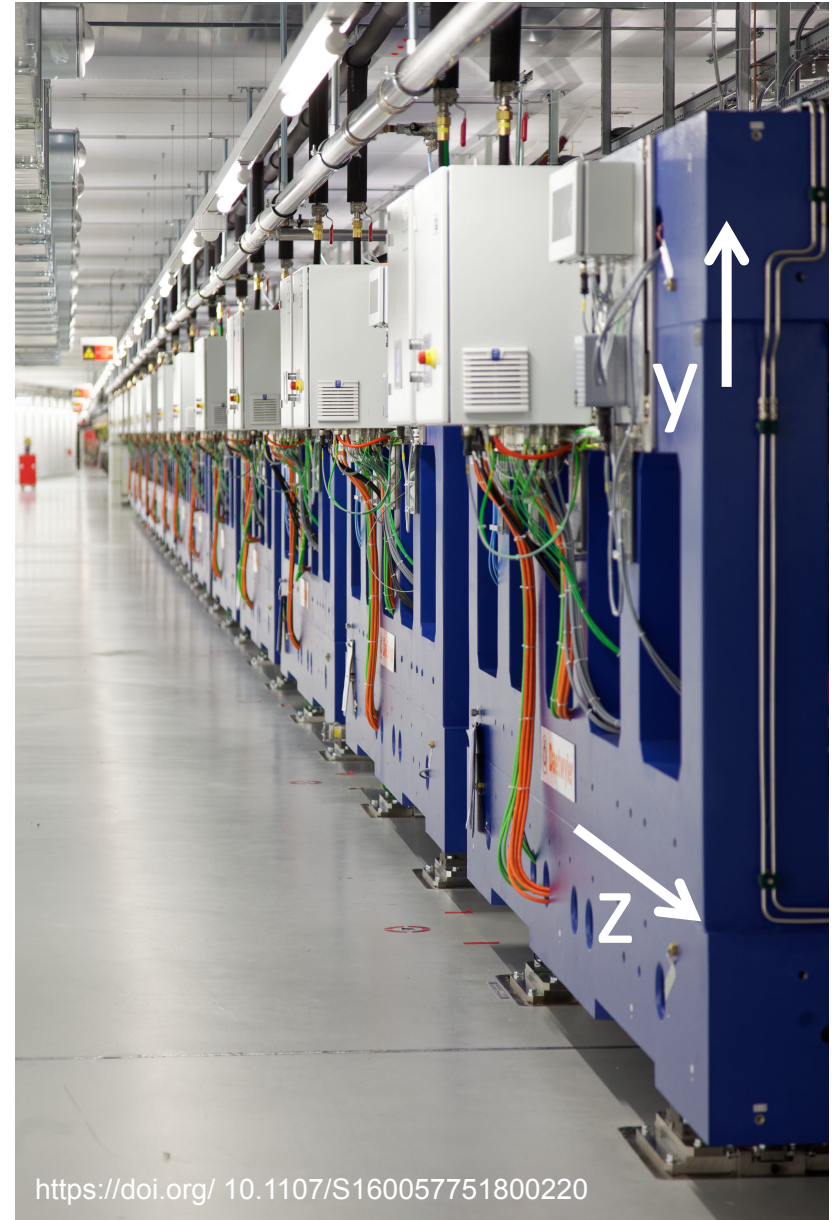
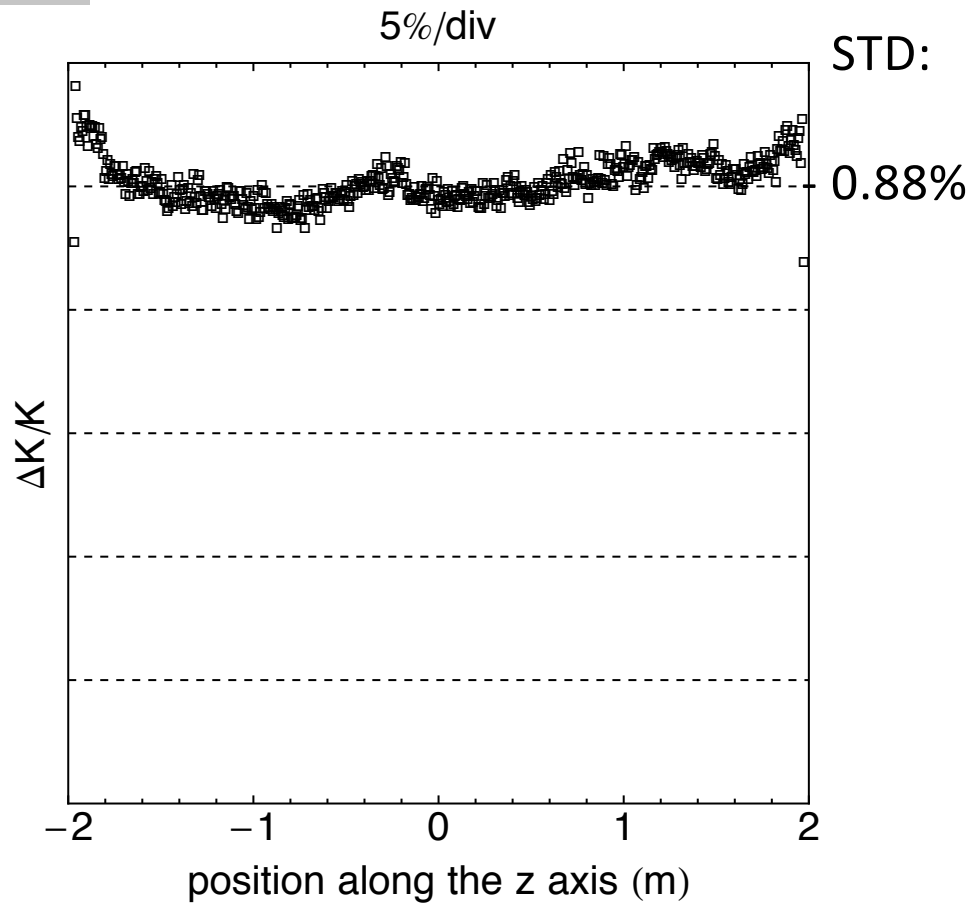


Gradient bending

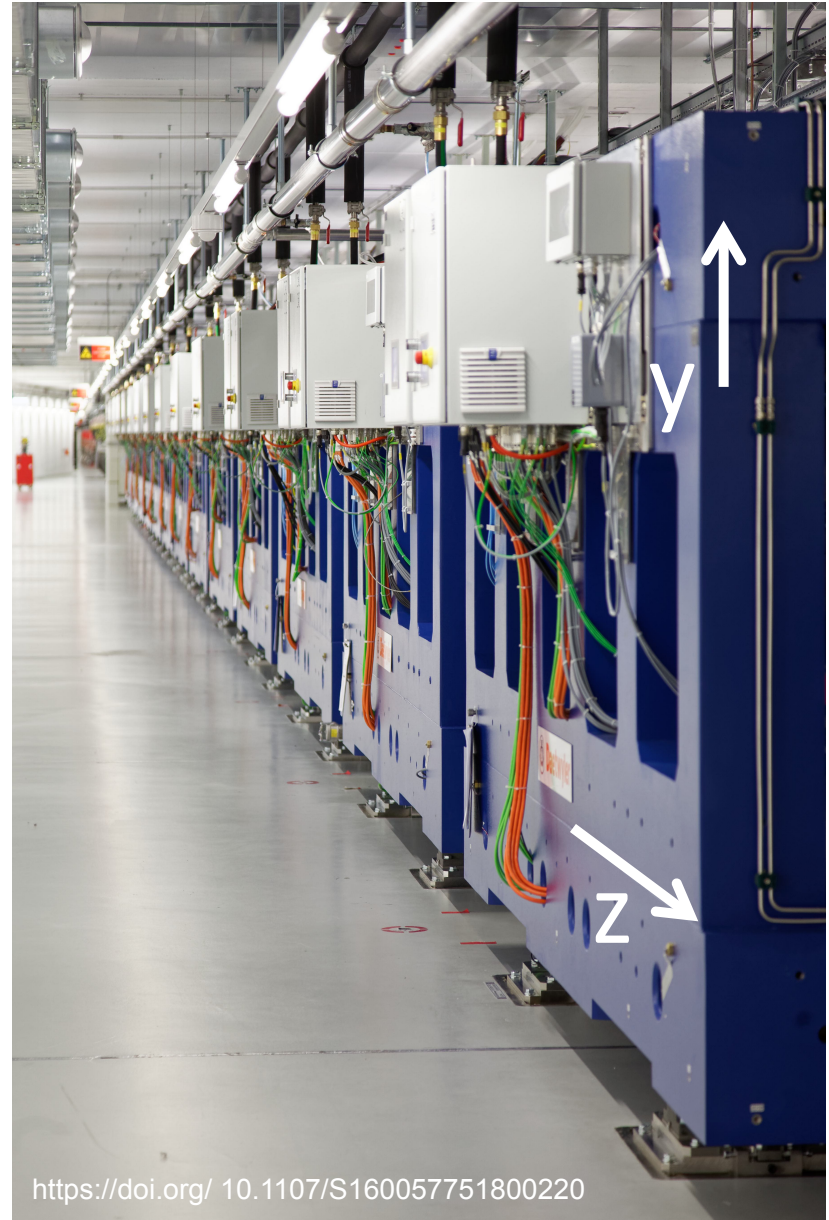
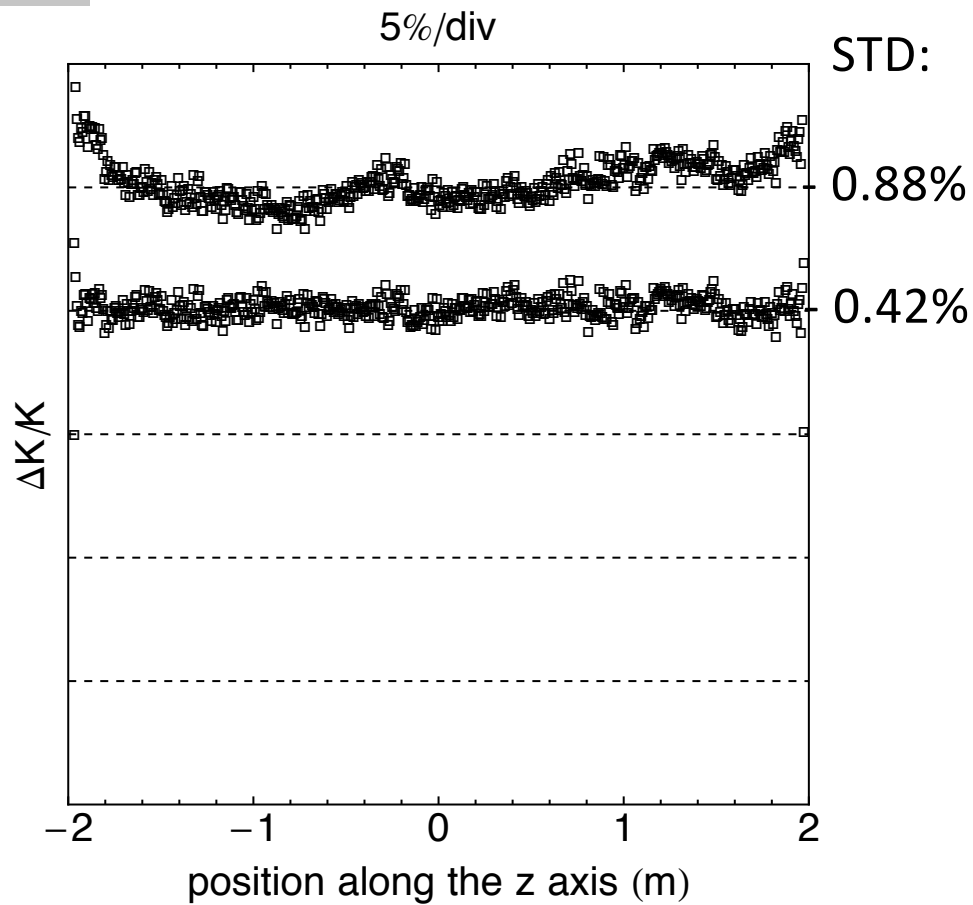
$$\frac{d^2 x}{dz^2} = \frac{e}{\gamma mc} (b + gx) \sin(2\pi z / \lambda_u)$$



Measurements of the local K profiles

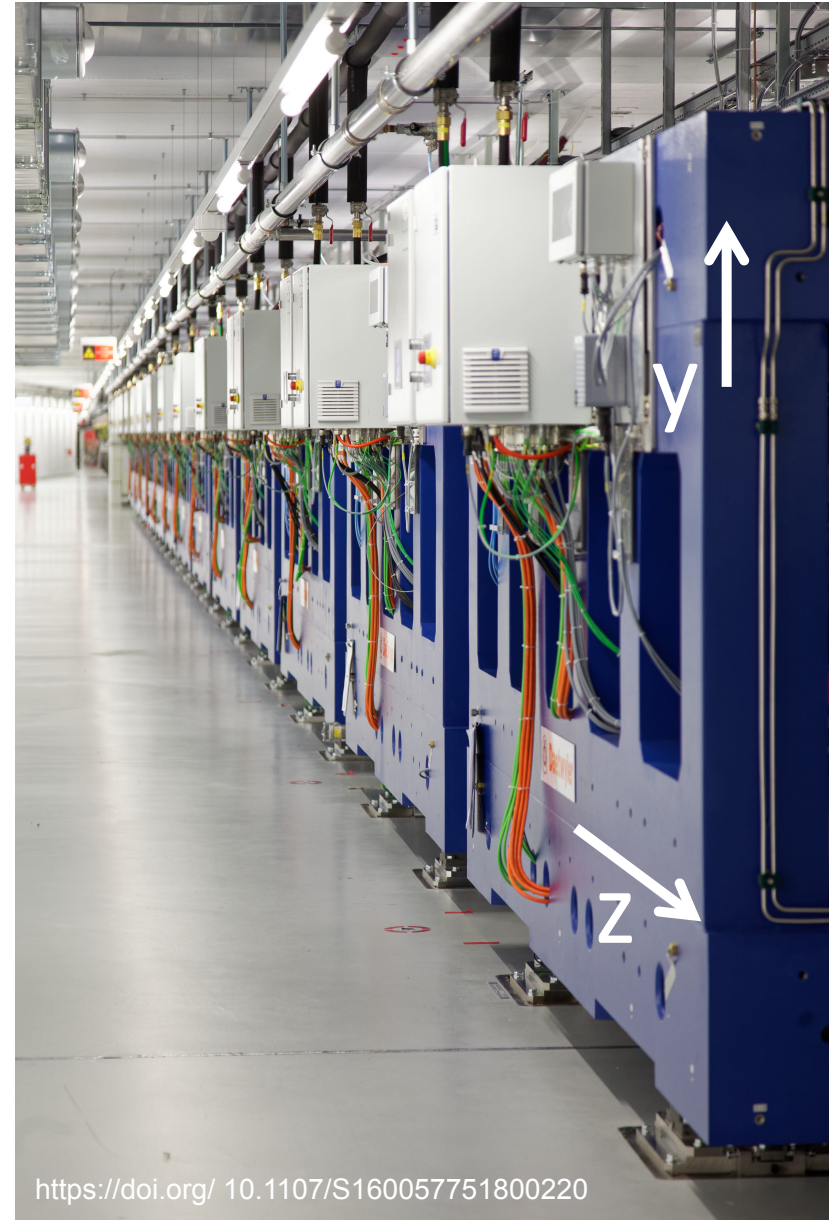
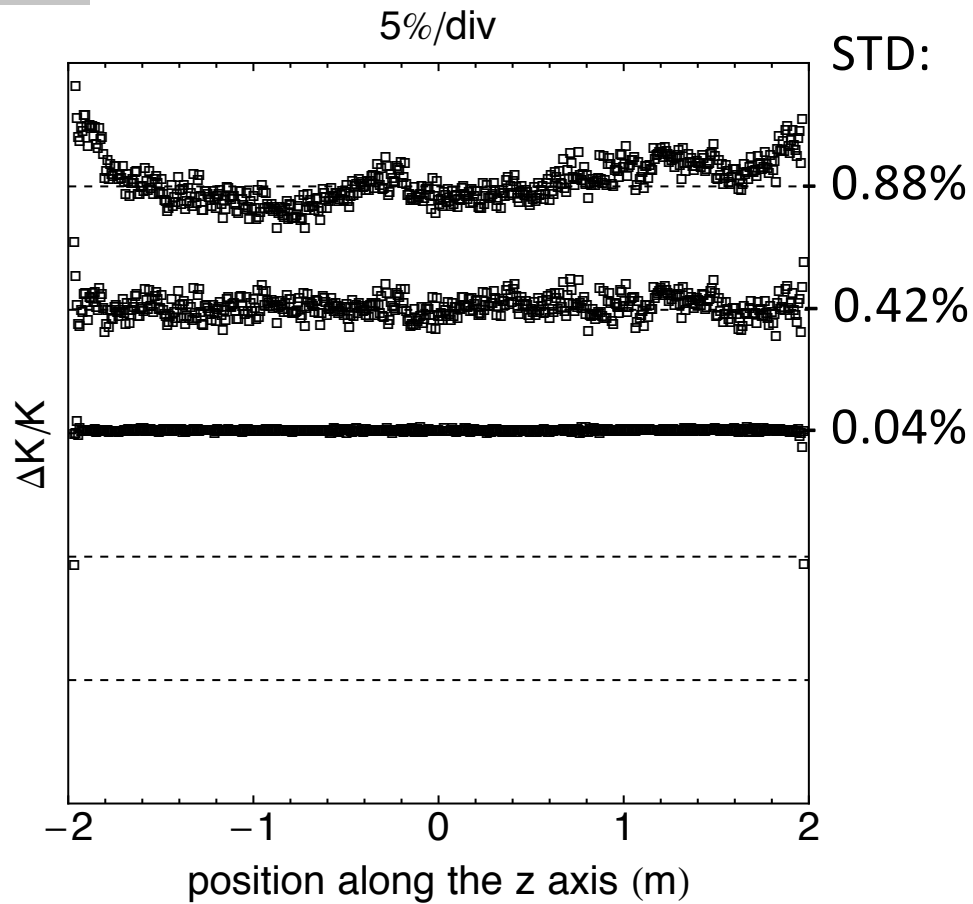


Measurements of the local K profiles

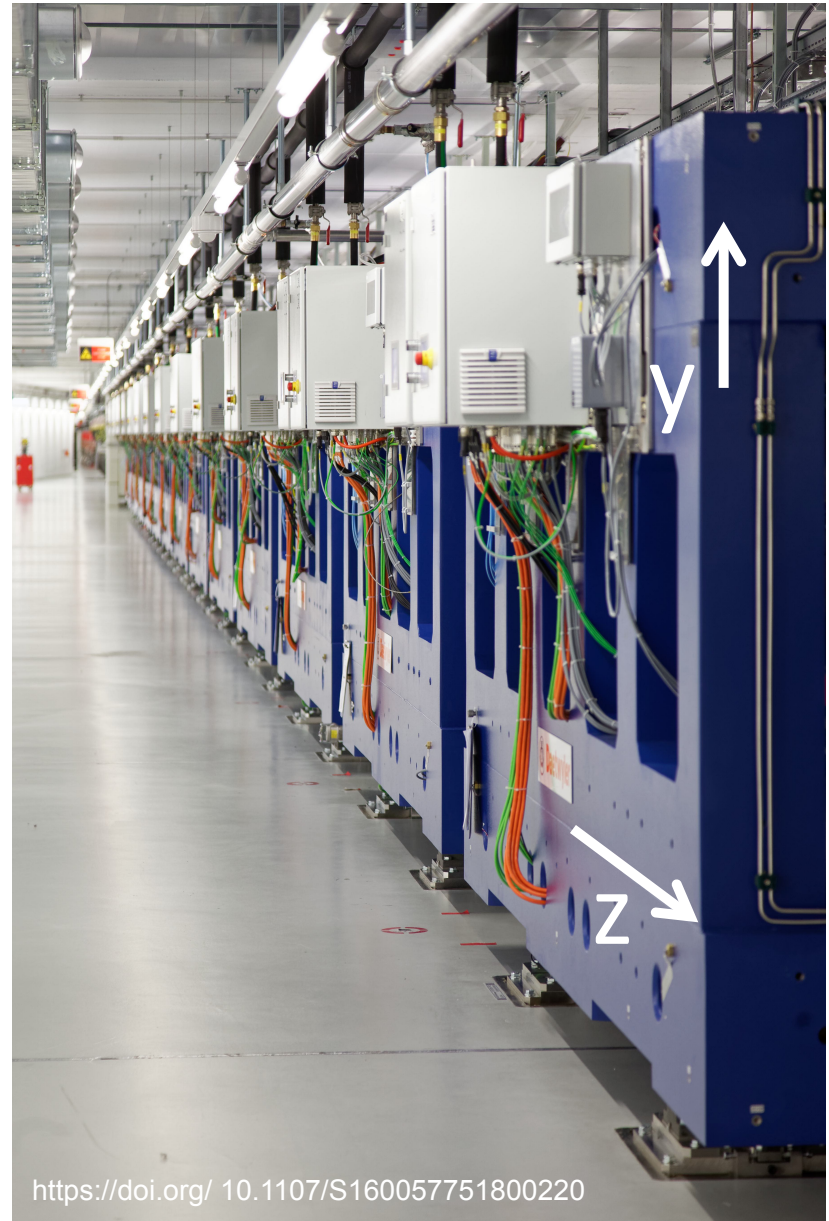
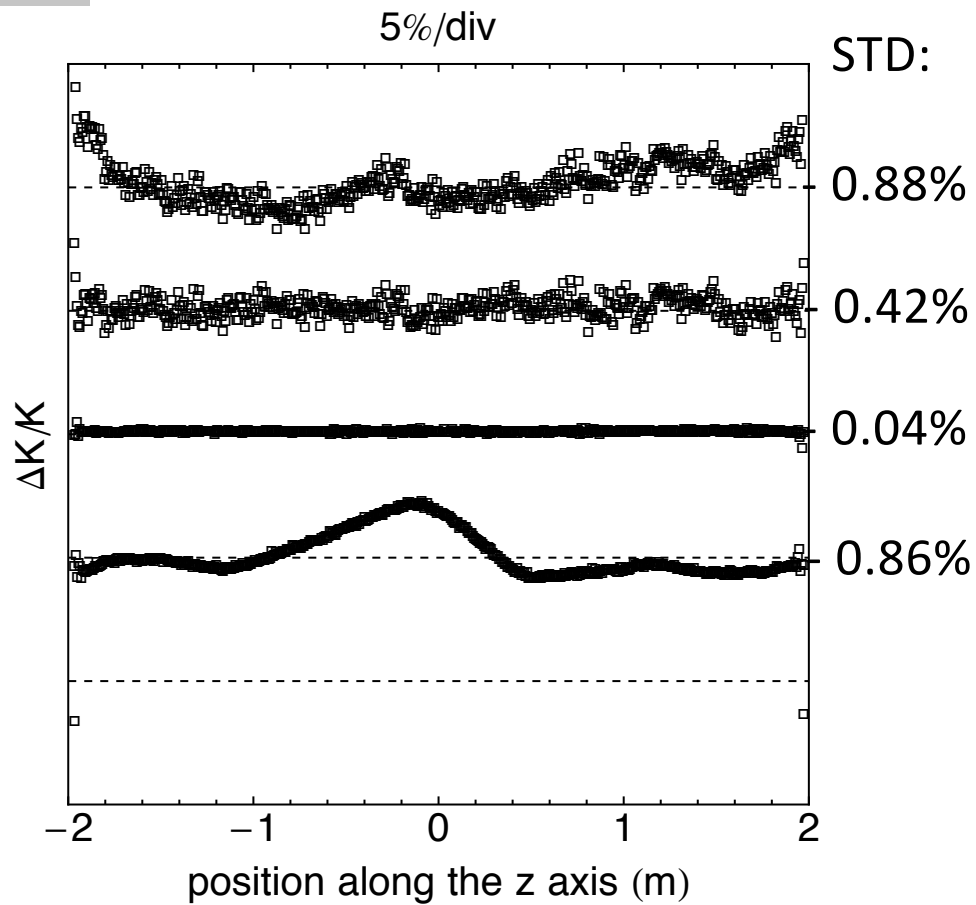
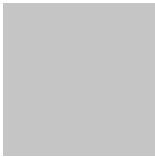


[https://doi.org/ 10.1107/S160057751800220](https://doi.org/10.1107/S160057751800220)

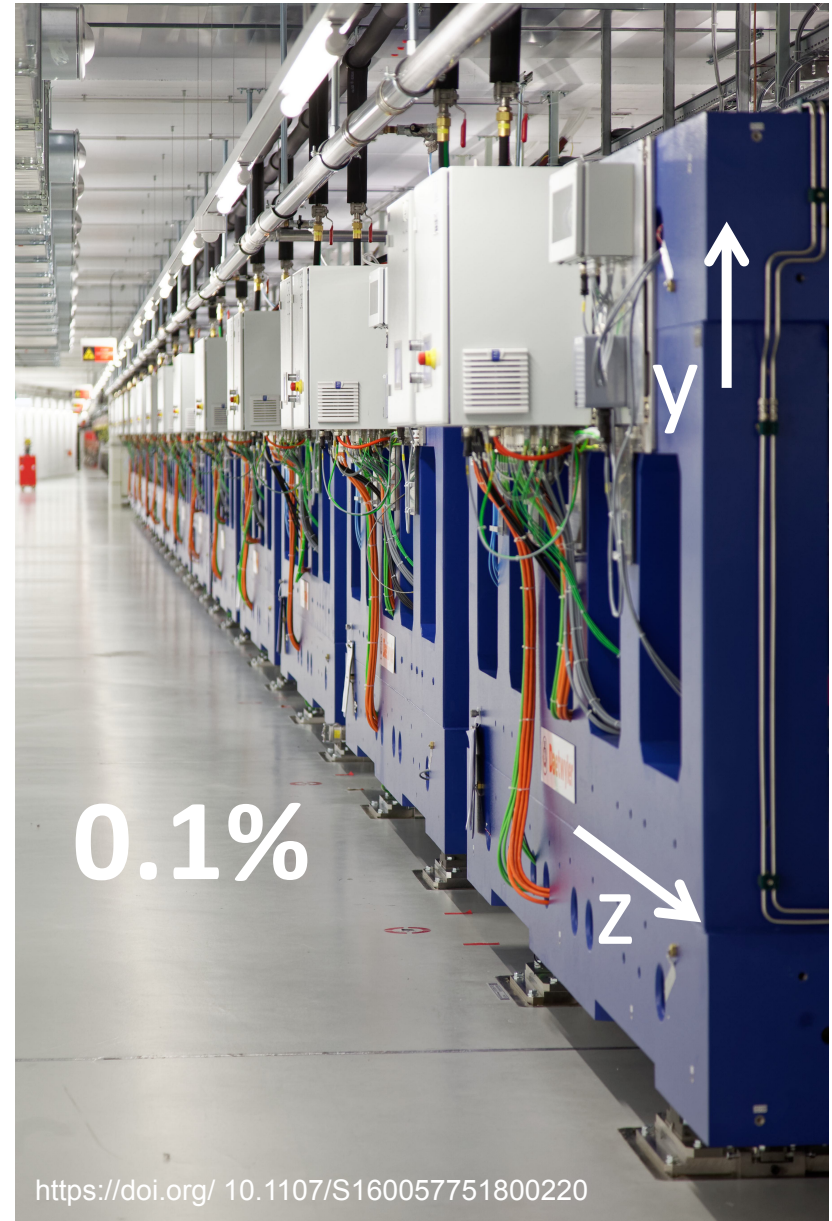
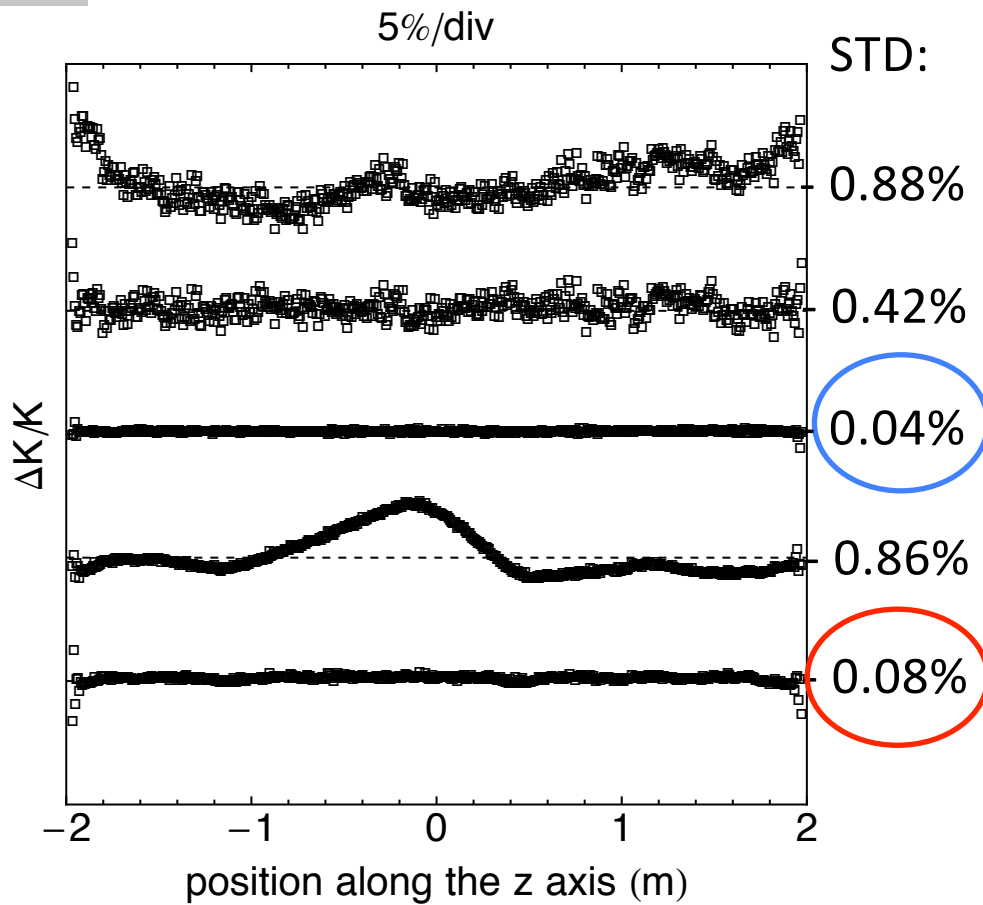
Measurements of the local K profiles



Measurements of the local K profiles



Measurements of the local K profiles



- This approach can be used with any undulator type:
 - A calibrated mover system is required to tune the gradient in a TGU
 - A photon diagnostic system (mono + diode) would simplify the operation of a TGU very much
- In Apple undulators (in p mode) it is possible to introduce a gradient on axis and changing it continuously to get the optimum taper
 - In Apple X (Delta) it is simpler than in regular Apple I,II,III
 - In Apple X (Delta) it is possible to introduce gradient on axis also in linear polarisation if operated in asymmetric mode
- Long dipole corrector coil should be designed to compensate the bending introduced by natural focusing and gradients, **if required**
- The actual pole to pole scattering DK/K should be considered during simulation:
 - 0.1% is the status of the art
 - 0.01% could be achieved with additional efforts **if required**

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