Super-radiant Pre-bunched Beam FEL in the Coherent-Spontaneous and Tapered-Wiggler Regimes

Avraham Gover (Tel-Aviv Univ.),
Reuven Ianconescu (Tel Aviv Univ. and Shenkar College),

Moore Symposium on compact X-Ray FEL, UCLA Jan 2019
eSASE AND HIGH HARMONIC TES OPTION


Tapering Enhanced Superradiance
(a) Spontaneous emission
(b) Superradiant emission (coherent spontaneous emission)
Formulation of Radiation mode Expansion

\[ \tilde{E}(\mathbf{r}, \omega) = \sum_{\pm q} \tilde{C}_q(z, \omega) \tilde{E}_q(\mathbf{r}) \]

\[ \tilde{H}(\mathbf{r}, \omega) = \sum_{\pm q} \tilde{C}_q(z, \omega) \tilde{H}_q(\mathbf{r}) \]

\[ \frac{d\tilde{C}_q(z, \omega)}{dz} = -\frac{1}{4\mathcal{P}_q} \int \tilde{J}(\mathbf{r}, \omega) \cdot \tilde{E}^*_q(\mathbf{r}) dA \]

\[ \tilde{C}^{\text{out}}_q(\omega) - \tilde{C}^{\text{in}}_q(\omega) = -\frac{1}{4\mathcal{P}_q} \int \tilde{J}(\mathbf{r}, \omega) \cdot \tilde{E}^*_q(\mathbf{r}) dV \]

\[ \hat{E}(\mathbf{r}, \omega) = \sum_q C_q(z, \omega) \hat{E}_q(\mathbf{r}) \]
Particulate current:

\[ J(r, t) = \sum_{j=1}^{N} -e v_j(t) \delta(r - r_j(t)) \]

Radiation wavepackets

\[ C_{q}^{\text{out}}(\omega) = C_{q}^{\text{in}}(\omega) - \frac{1}{4\pi \mathcal{P}_q} \sum_{j=1}^{N} \Delta \tilde{W}_{qj} \]

\[ \Delta \tilde{W}_{qj} = -e \int_{-\infty}^{\infty} \tilde{v}_j \cdot \tilde{E}_q^* (r_j(t)) e^{i\omega t} dt = \Delta \tilde{W}_{qj}^{(0)} + \Delta \tilde{W}_{qj}^{st} \]

Spontaneous:

\[ \Delta \tilde{W}_{qj}^{(0)} = \Delta \tilde{W}_{qe}^{(0)} e^{i\omega t_j 0} \]

\[ \text{Where:} \quad \Delta \tilde{W}_{qe}^{(0)} = -e \int_{-\infty}^{\infty} v_{e}^{(0)}(t) \cdot \tilde{E}_q^* (r_e^{(0)}(t)) e^{i\omega t} dt \]

\[ C_{q}^{\text{out}}(\omega) = C_{q}^{\text{in}}(\omega) + \Delta C_{qe}^{(0)}(\omega) \sum_{j=1}^{N} e^{i\omega t_j 0} + \sum_{j=1}^{N} \Delta C_{qj}^{st} \]

Spectral radiant energy:

\[ \frac{dW_q}{d\omega} = \frac{2}{\pi} P_q \left| C_{q}^{\text{out}}(\omega) \right|^2 = \left( \frac{dW_q}{d\omega} \right)_{\text{in}} + \left( \frac{dW_q}{d\omega} \right)_{\text{sp/SR}} + \left( \frac{dW_q}{d\omega} \right)_{\text{st-SR}} + \left( \frac{dW_q}{d\omega} \right)_{\text{st}} \]
A. Gover, Superradiant and stimulated-superradiant emission in prebunched electron-beam radiators
PRST-AB 8, (030701) 2005
A Pulse Composed of a Periodic Train of $N_M$ Micro-Bunches
Coherent SP-SR of Undulator Radiation

For a finite train of periodic bunches:

\[
\left( \frac{dW_q}{d\omega} \right)_{SR} = \frac{N^2 e^2 Z_q (\bar{a}_w)}{16\pi} \left( \frac{\bar{a}_w}{\beta_z \gamma} \right)^2 \frac{L^2}{A_m} |M_b(\omega)|^2 |M_M(\omega)|^2 \text{sinc}^2(\theta L/2)
\]

Detuning parameter:

\[ \theta(\omega)L = \left( \frac{\omega}{v_z} - k_{zq}(\omega) - k_\omega \right)L \approx 2\pi \frac{\omega - \omega_0}{\Delta\omega} \]
\[ \omega_0 \approx 2\gamma^2_z c k_w \quad \Delta\omega = \omega_0/N_w \]

Pulse train form-factor:

\[ M_M(\omega) = \frac{\sin(N_M \pi \omega/\omega_b)}{N_M \sin(\pi \omega/\omega_b)} \quad \frac{\Delta\omega}{n\omega_b} = \frac{1}{nN_M} \]

Bunching coefficient:

\[ M_b(\omega) = \frac{1}{N_b} \sum_{j=1}^{N_b} e^{i\omega \Delta t_j} = e^{-\omega^2 \sigma_{tb}^2} \]

A. Gover, PRST-AB 8, (030701) 2005
The Macropulse Form-Factor Function Drawn for $N_M = 8$

$$M_M(\omega) = \frac{\sin \left( N_M \pi \frac{\omega}{\omega_b} \right)}{N_M \sin \left( \pi \frac{\omega}{\omega_b} \right)}$$

$$\frac{\Delta \omega}{n \omega_b} = \frac{1}{n N_M}$$
FIG. 7. (Color) The bunching form factor $|M_b(\omega)|^2 = \exp(-\omega^2 t_b^2/2)$ for a Gaussian e-beam bunch distribution $f(t_0) = \exp(-t_0^2/t_b^2)/\sqrt{\pi t_b}$.

BUNCHING BY LASER MODULATION AND DISPERSIVE SECTION

\[ M_{bn} = \frac{0.67}{n^{1/3}} e^{-n^2/2A^2} \]

\[ A = \frac{\delta\gamma_{\text{mod}}}{\sigma_{\gamma_0}} \]

\[ \Rightarrow n_{\text{max}} = A \]

\[ M_{b,n_{\text{max}}} = \frac{0.67}{n_{\text{max}}^{1/3}} e^{-1/2} \]

Phase-merging enhanced harmonic generation

Figure 3. Comparison of the bunching factor of PEHG and standard HGHG with different energy modulation amplitudes. The black line is the theoretical prediction of the maximal bunching factor of PEHG.

SP-SR HARMONIC POWER

For: \( N_M \gg N_w \)

\[
\omega_n = n \omega_b \approx \omega_0 = 2 \gamma^2 z_0 \lambda_w
\]

\[
\frac{dW_q(\omega)}{d\omega} = \sum_n \frac{dW_{qn}(\omega)}{d\omega}
\]

\[
\frac{dW_{qn}(\omega)}{d\omega} = \frac{N^2 e^2}{16\pi} \sqrt{\mu_0} \left( \frac{\bar{a}_w}{\beta_z \gamma} \right)^2 \frac{L_w^2}{A_{em,q}(\omega_n)} \left| M_{b,n} \right|^2 \left| M_M(\omega) \right|^2
\]

Power of harmonic \( n \):

\[
P_{q,n} = \left. \frac{dW_{qn}(\omega)}{d\omega} \right|_{\omega = \omega_n} \left( \frac{\omega_b}{N_M} \right) \frac{1}{t_p} = \frac{I_0^2}{8} \sqrt{\frac{\mu_0}{\varepsilon_0}} \left( \frac{\bar{a}_w}{\beta_z \gamma} \right)^2 \left| M_{b,n} \right|^2 \frac{L_w^2}{A_{em,q}(\omega_n)}
ST-SR IN THE NONLINEAR REGIME

\[ J(r, t) = Q_b v_e(t) f(r_\perp) \sum_{n=-\infty}^{\infty} \delta[z - z_e(t - nT_b - t_0)] \]

\[ J(z, t) J_0 + \sum_{n=1}^{\infty} 2 \text{Re} \left[ \tilde{J}_n e^{-i\omega_b t} \right] \]

\[ P_e = N_b mc^2 \left( \gamma - 1 \right) / T_b \]

\[ P_{q,n} = \left| C_{q,n} \right|^2 P_q \]

Self-consistent nonlinear model formulation for an infinite pulse or finite pulse with zero-slippage:

\[ \frac{dp_{q,n}}{dz} + \frac{dp_e}{dz} = 0 \]
TILTED PENDULUM EQUATION MODEL FOR TAPERED WIGGLER

\[
\frac{d|\tilde{C}_q|}{dz} = B \sin \psi,
\]

\[
\frac{d\delta \gamma}{dz} = -\frac{\beta_{zr}^3 \gamma^2_r \gamma_r}{k_0} K_s^2(z) \left[ \sin \psi - \sin \psi_r \right],
\]

where \(0 < \psi_r < \pi/2\).

\[
\frac{d\psi}{dz} = \frac{k_0}{\beta_{zr}^3 \gamma_r^2} \delta \gamma + \frac{B}{|\tilde{C}_q|} \cos \psi
\]


ST-SR IN THE NONLINEAR REGIME

2nd ORDER PERTURBATIVE SOLUTION

For short interaction length \( u = z / L_w \):

\[
\frac{\Delta P_q}{P_{\text{REF}}} = \bar{E}^2(u) - \bar{E}^2(0) = \\
[2u\bar{E}(0)(\sin \psi(0) - \sin \psi_r) + u^2 \sin^2 \psi(0)] + 2u\bar{E}(0)\sin \psi_r.
\]

\[\psi(0) = \psi_r:\]

\[
\frac{\Delta P_{em}}{P_{\text{REF}}} = 2u\bar{E}(0)\sin \psi_r + u^2 \sin^2 \psi_r.
\]
Uniform wiggler: maximal gain
Stimulated Superradiance
Tapering Enhanced Stimulated Superradiance (TESSA): $\psi(0) = \psi_r$
Tapering Enhanced Stimulated Superradiance (TESSA): $\psi(0) = \frac{\pi}{2}$
Superradiance in the nonlinear regime (self interaction) - uniform wiggler
Tapering Enhanced Superradiance (TES)

E. C. SNIVELY, J. XIONG, P. MUSUMECI, A. GOVER “Broadband THz Amplification and Superradiant Spontaneous Emission in a Guided FEL” (SUBMITTED)
Phase – space trajectories of a realistic bunch in a tapered wiggler FEL

TESSA: EXTENTION TO DISTRIBUTED BUNCH

Bunches get trapped in the tapered undulator in ponderomotive buckets of the SP-SR generated harmonic radiation.

\[ f_t \cdot \text{fraction trapped depends on phase between beam and radiation} \]

\[ P_{rad}(z) = P_0 + E_0 \frac{\bar{a}_w(0)}{\gamma_0} f_t I z \sin \psi_r + \frac{Z_0}{4A_{emq}} \left( \frac{\bar{a}_w(0)}{\gamma_0} \right)^2 (f_t I z \sin \psi_r)^2 \]

Need to optimize \( f_t (\psi_r, \Delta \gamma_{mod}) \sin \psi_r \)

N. Sudar, P. Musumeci et al “High Efficiency Energy Extraction ...Tapered Undulator”
PRL 117, 174801 (2016)
OPTIMAL TAPERING PHASE

Assume:
\[ \Delta \gamma_{\text{mod}} \ll \delta \gamma_{\text{trap}} \]

For \( n_{\text{max}} (= A) \), \[ 2\sigma_z \approx \lambda_{n_{\text{max}}} \]

\[ f_t = f_b = \frac{\psi_2 - \psi_1}{2\pi} \]

*RIVER ROBLES - STUDENT UCLA PHYSICS DEPT.*
A Model Problem with a SAMURAI beam

1 GeV, 800 A microbunched beam

Short wiggler (10 periods, 24 cm) 
$\bar{K} = 10$

Focusing optics

Tapered wiggler (125 periods, 3 m) 
$\bar{K}(0) = 10$

320 nm radiation
Non-tapered Undulator Radiation

- In undulator, beam slips back 10 radiation wavelengths
- Choosing tenth harmonic for resonance avoids “gaps” in the resulting laser distribution
MODEL PROBLEM RESULTS

- Beam generates 2.4 GW of SP-SR radiation in the uniform wiggler which acts as seed radiation for the tapered wiggler.
- An additional 13.4 GW is produced in the tapered wiggler.
- Energy extraction efficiency: 2%
Conclusions

1. High power coherent spontaneous superradiant emission (SP-SR) may be feasible at high UV harmonic (x30) of modulating laser.

2. High power extraction may be attainable in a Tapering Enhanced Superradiant (TES) scheme where the bunched beam is trapped by its own generated SP-SR.

3. The advantage of TES is self phase matching to the bunch train.

4. Can be companion to e-SASE facility.

5. Consider option of oscillator (TESO). No need for seed.
RESERVE
Analytic Output Predictions

- Spontaneous power in normal wiggler: 2.4 GW
- Power gained in tapered wiggler: 13.4 GW
- Radiation wavelength: 320 nm
- Radiation pulses: 250
- Pulse spacing: 320 nm
Experimental Setup Numbers

- Microbunching period: 3.2 microns
- Input beam energy: 1 GeV
- Average bunch current: 800 A
- Bunch charge: 200 pC
- Normal wiggler period: 24 mm
- Wiggler periods: 10
- Resonant wavelength: 320 nm
- Normalized vector potential: 10

- Tapered wiggler period: 24 mm
- Wiggler periods: 125
- Norm. vector potential (entrance): 10
- Resonant wavelength: 320 nm
EXTENTION TO DISTRIBUTED BUNCH

\[ \Delta P_q = \left(8P_q(0)\right)^{1/2} \left(P_{SR}\right)^{1/2} + P_{SR} \]

\[ P_{SR} = \frac{1}{4} \sqrt{\frac{\mu_0 I_0^2}{\varepsilon_0} \frac{L_w^2}{A_{em}(\omega_n)} \left(\frac{\bar{a}_w(0)}{\gamma_0}\right)^2 f(\psi_r, A/n)} \]

\[ A = \delta \gamma_{mod} / \sigma \gamma_0 \]

\[ \Delta z = \lambda_0 / 2A \]

\[ 2\Delta z = \lambda_0 / n_{max} \Rightarrow n_{max} = A \]


eSASE AND HIGH HARMONIC TES OPTION

Phase-merging enhanced harmonic generation


Figure 2. The longitudinal phase space evolution in scheme 2: (a) the initial phase space after passing through the dogleg; (b) the phase space at the exit of the conventional modulator; (c) the phase space at the exit of the TGU; (d) the phase space at the exit of the DS.
Mesured multi-bunch coherent Smith-Purcell linewidth (MIT - S.E. Korbly et al PRL 2005)

\[ f_{or} = mf_b = 240\,GHz \]
\[ \Delta f = f_b / N_M = 28\,MHz \]
\[ \Delta f / f_{or} = 1 / mN_M = 1.3 \cdot 10^{-4} \]
SR and ST-SR of Undulator Radiation

For a finite train of periodic bunches:

\[
\left( \frac{dW_q}{d\omega} \right)_{SR} = \frac{N^2 e^2 Z_q}{16\pi} \left( \frac{\bar{a}_w}{\beta z \gamma} \right)^2 \frac{L^2}{A_m} |M_b(\omega)|^2 |M_M(\omega)|^2 \text{sinc}^2 (\theta L / 2)
\]

\[
\left( \frac{dW_q}{d\omega} \right)_{ST-SR} = \left| \tilde{c}_q^{\text{in}}(\omega) \right| \frac{Ne}{2\pi} \left( \frac{\bar{a}_w}{\beta z \gamma} \right) \sqrt{\frac{2Z_q P_q}{A_{emq}}} L |M_b(\omega)||M_M(\omega)| \text{sinc}(\theta L / 2) \cos(\varphi - \theta L / 2)
\]

Detuning parameter:
\[
\theta(\omega)L = \left( \frac{\omega}{v_z} - k_{zq}(\omega) - k_\omega \right) L \approx 2\pi \frac{\omega - \omega_0}{\Delta \omega} \quad \omega_0 \approx 2\gamma_z^2 c k_w \quad \Delta \omega = \omega_0 / N_w
\]

\(N_w=\text{# of undulator periods}\)

Pulse train form-factor:
\(N_M=\text{# of bunches in macropulse}\)

Bunching coefficient:
\(N_b=\text{# of particles in bunch}\)

\[
M_M(\omega) = \frac{\sin(N_M \pi \omega / \omega_b)}{N_M \sin(\pi \omega / \omega_b)} \quad \Delta \omega = \frac{1}{n \omega_b} = \frac{1}{nN_M}
\]

\[
M_b(\omega) = \frac{1}{N_b} \sum_{j=1}^{N_b} e^{i \omega \Delta t_j} = e^{-\omega^2 \sigma_{tb}^2}
\]

A. Gover, PRST-AB 8, (030701) 2005
Uniform wiggler: maximal extraction
RUBICON/NOCIBUR EXPERIMENT

Fig. 11 Conceptual description of Tapering Enhanced Stimulated Superradiance Amplifier configuration.

Fig. 12 Tesa emission process in a tapered wiggler. (a) Lab frame phase space, (b) Radiation and beam energy power increments, (c) Multi-particle trajectories in the multi-particle trap frame.

Single Electron Emission
Frequency Domain

\[
C_{qj}^{\text{out}}(\omega) = e^{-\frac{v_w \cdot E_q^*(r_{\perp 0})}{8v_z}} L \text{sinc} \left( \frac{\theta L}{2} \right) e^{i\frac{\theta L}{2}} e^{i\omega t_{0j}}
\]

\[
\theta(\omega) \equiv \left( \frac{\omega_0}{v_z} - k_z(\omega_0) - k_w \right) = (\omega - \omega_0) t_{sl} \equiv 2\pi \frac{\omega - \omega_0}{\Delta \omega}
\]

Where:

\[
\theta(\omega_0) = \frac{\omega_0}{v_z} - k_z(\omega_0) - k_w = 0 \quad \text{ - Synchronism}
\]

\[
t_{sl} = \frac{2\pi}{\Delta \omega} = \frac{L}{v_z} - \frac{L}{v_g} \quad \text{ - Slippage Time}
\]

(v_g=c)
Spectral energy flow of super radiant emission for rectangular 5×5 mm waveguide

\[ E_b = 2.8 \text{ MeV} \]

\[ Q_b = 10 \text{ pCl} \]
PB — FEM SUPERRADIANCE MEASUREMENT*

PB-FEL STIMULATED SUPERRADIANCE MEASUREMENT
the general expression for spontaneous SR and ST-SR spectral energy of a finite train of periodic bunches:

\[
\left( \frac{dW_q}{d\omega} \right)_{SR} = \frac{N^2 e^2 Z_q}{16\pi} \left( \frac{\bar{a}_w}{\beta_z \gamma} \right)^2 \frac{L^2}{A_m} |M_b(\omega)|^2 |M_M(\omega)|^2 \text{sinc}^2(\theta L/2) \tag{48}
\]

and the stimulated superradiant term is

\[
\left( \frac{dW_q}{d\omega} \right)_{ST-SR} = |\tilde{C}_q^{\text{in}}(\omega)| \frac{Ne}{2\pi} \left( \frac{\bar{a}_w}{\beta_z \gamma} \right) \sqrt{\frac{2Z_q P_q}{A_{em,q}}} L |M_b(\omega)| |M_M(\omega)| \text{sinc}(\theta L/2) \cos(\varphi - \theta L/2) \tag{49}
\]

\[
\theta(\omega) = \frac{\omega}{v_z} - k_{zq}(\omega) - k_w.
\]

\[
\theta(\omega)L \simeq (\omega - \omega_0)t_s = 2\pi \frac{\omega - \omega_0}{\Delta \omega}.
\]

\[
M_M(\omega) = \frac{\sin(N_M \pi \omega/\omega_b)}{N_M \sin(\pi \omega/\omega_b)}
\]

\[
\frac{\Delta \omega}{n \omega_b} = \frac{1}{nN_M}
\]
Formulation of Radiation mode Expansion

\[
\tilde{E}(\mathbf{r}, \omega) = \sum_{\pm q} \tilde{c}_q(z, \omega) \tilde{E}_q(\mathbf{r})
\]

\[
\tilde{H}(\mathbf{r}, \omega) = \sum_{\pm q} \tilde{c}_q(z, \omega) \tilde{H}_q(\mathbf{r})
\]

\[
\frac{d\tilde{c}_q(z, \omega)}{dz} = -\frac{1}{4\mathcal{P}_q} \int \tilde{j}(\mathbf{r}, \omega) \cdot \tilde{E}_q^*(\mathbf{r}) dA
\]

\[
\tilde{c}_q^{\text{out}}(\omega) - \tilde{c}_q^{\text{in}}(\omega) = -\frac{1}{4\mathcal{P}_q} \int \tilde{j}(\mathbf{r}, \omega) \cdot \tilde{E}_q^*(\mathbf{r}) dV
\]
RF-Linac cavity can work for:

Electron Energy at the exit: 6 MeV
Macrobunch charge: 1 nC
Micropulse Rep. Rate: 1 THz
Macropulse Duration: 12 ps
Macropulse Rep. Rate: 100 Hz

See poster: A. Friedman, “Program and status of the THz Facility in Ariel University”
Tapered wiggler: fully trapped bunch
Tapered wiggler: maximal gain ST-SR
Concepts to be considered

- Fundamental processes of coherent spontaneous radiation emission from charged particles and bunched beam current.

- Radiation mode expansion formulation.

- Superradiant (SR), Stimulated Superradiant emission (ST-SR).

- Tapering Enhanced Superradiant (TES) and Stimulated Superradiance (TESSA)

- Nonlinear SR, ST-SR dynamics of a trapped bunched beam.

- Bunched-beam/radiation Self-interaction.
APPLICATION OF SUPERRADIANCE:
Taper enhanced superradiance (TES) and Taper enhanced stimulated superradiance Amplification (TESSA) [2]

\[
\tilde{C}_q(z) = \tilde{C}_q(0) - \frac{1}{8} I_0 F \frac{\tilde{E}_q(0)}{P_q} e^{i\varphi_b} \int_0^z \frac{a_w(z')}{\gamma(z') \beta_z(z')} |M_b(z')| e^{i \int_0^{z'} \theta^E(z'')dz''} dz'
\]

\[
\theta^E(v_z, z) = \frac{\omega_0}{v_z(z, E(z))} - k_w(z) - k_{zq}
\]

Expand around the phase velocity of the ponderomotive wave: \( v_{zd} = \omega_0 / (k_{zq} + k_w(z)) \):

\[
v_z = v_{zd} + \delta v_z(z, E_0) \quad \Rightarrow \quad \theta^E(z) = \theta(v_{zd}, z) - k_d \frac{\delta \beta_z}{\beta_{zd}^2(z)} \approx -k_0 \frac{\delta \gamma(z, E)}{\gamma_{zd}^2(z) \gamma_r(z)}
\]