Lighting up Einstein’s Dark Universe

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April 6, 2018
An impressive line-up of surveys …

Ongoing and upcoming wide field imaging and spectroscopic redshift surveys are in line to map more than a 100 cubic-billion-light-year of the Universe: exquisite measurements of expansion rate & reconstruction of lensing potentials and cosmic structure growth rate to ~ O(1)% over wider redshift range.
… and a quite dark Universe!
While GR has so far passed all experimental tests, it has yet to be accurately tested on scales beyond our solar system.

1. test the consistency with LCDM (GR)

2. explore the parameter space allowed to alternative models
... and a quite dark Universe!

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1. test the consistency with LCDM (GR)

2. explore the parameter space allowed to alternative models
Cosmic functions of interest

\[ ds^2 = -a^2(\tau) \left[ (1 + 2\Psi(\tau, \vec{x}')) d\tau^2 - (1 - 2\Phi(\tau, \vec{x}')) d\vec{x}'^2 \right] \]

Expansion history: \( a(\tau) \)

Non-relativistic dynamics (growth of structure, pec. vel.): \( \Psi(\tau, \vec{x}') \)

Relativistic dynamics (weak lensing, ISW): \( (\Phi + \Psi)(\tau, \vec{x}') \)
The Standard Model of Cosmology

The standard model is based on GR: \[ G_{\mu\nu} = \frac{T_{\mu\nu}}{M_P^2} \]

Cosmic acceleration is sourced by the cosmological constant.

The energy-momentum tensor is characterized by: \[ \Delta \pi \ll \delta p \ll \delta \rho \]
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**LCDM:**

\[ w = -1 \quad \Phi = \Psi \quad \Psi = -\frac{a^2}{k^2} \frac{\rho \Delta}{2M_P^2} \]
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The growing mode goes like: \[ D_1(a) = \frac{5\Omega_m}{2} \frac{H(a)}{H_0} \int_0^a \frac{da'}{a' \left[ \frac{H(a')}{H_0} \right]^3} \]
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\]

- relativistic and non-relativistic probes respond to the same metric potential
- the growth of structure is scale-independent
- there is a consistency relation btw expansion history and growth of structure.
EUCLID: Mapping the geometry of the Universe

1. Together, dark matter and dark energy pose some of the most important questions in fundamental physics today.

Euclid is a high-precision survey mission optimised for two independent cosmological probes:

1. Weak Gravitational Lensing from a high-resolution imaging survey
2. Galaxy Clustering measured via a massive spectroscopic redshift survey

to be launched in 2021 ...
Beyond LCDM

Where do we stand on the theory side?
Beyond LCDM

Where do we stand on the theory side?

How can we optimally explore departures from the LCDM scenario?
The theoretical gravitational landscape

- Dynamical DE
  - Minimally coupled
    - Quintessence
    - K-essence
  - Non-minimally coupled
    - Quintessence
    - K-essence

- Massless spin-2 graviton
  - Scalar-tensor theories
    - F(R) theories
    - TEVES
  - Bi-metric theories
  - Galileons

- Modified gravity
  - Broken Lorentz-inv.
    - Ghost condensate
  - IR modifications
    - Degravitation

- Massive spin-2 graviton
  - Infinite extra-dimensions
    - KK
  - DGP
The theoretical gravitational landscape

PARAMETRIZED FRAMEWORKS
EFT of Dark Energy
A unified action

Jordan frame

\[ S = \int d^4x \sqrt{-g} \left\{ \frac{m_0^2}{2} \left[ 1 + \Omega(\tau) \right] R + \Lambda(\tau) - a^2 c(\tau) \delta g^{00} \right. \\
+ \frac{M_2^4(\tau)}{2} \left( a^2 \delta g^{00} \right)^2 - \frac{\tilde{M}_1^3(\tau)}{2} a^2 \delta g^{00} \delta K_{\mu}^{\mu} - \frac{\tilde{M}_2^2(\tau)}{2} \left( \delta K_{\mu}^{\mu} \right)^2 \\
- \frac{\tilde{M}_3^2(\tau)}{2} \delta K_{\nu}^{\mu} \delta K_{\mu}^{\nu} + \frac{a^2 \tilde{M}_1^2(\tau)}{2} \delta g^{00} \delta R^{(3)} + \ldots \right\} + S_m[g_{\mu\nu}] \]

IN UNITARY GAUGE
A unified action

m matter follows geodesic of the metric

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IN UNITARY GAUGE
A unified action

The presence of an extra dynamical scalar d.o.f. wrt the standard model of cosmology naturally singles out a preferred foliation of space-time.

It is natural to employ the ADM (Arnowitt-Deser-Misner) decomposition of spacetime:

\[ ds^2 = -N^2 dt^2 + h_{ij} (dx^i + N^i dt)(dx^j + N^j dt) \]

lapse function  
3D metric  
shift vector

If we define:

\[ n_\mu \equiv N t;_\mu \quad h_{\mu\nu} = g_{\mu\nu} + n_\mu n_\nu \]

intrinsic curvature: \( R^{(3)}_{\mu\nu} \)
extrinsic curvature: \( K_{\mu\nu} = h^\lambda_\mu n_\nu;_\lambda \quad K_{ij} = \frac{1}{2N} (\partial_i h_{ij} - \nabla_i N_j - \nabla_j N_i) \)
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- intrinsic curvature:

\[ R^{(3)}_{\mu \nu} \]

- extrinsic curvature:

\[ K_{\mu \nu} = h^\lambda_{\mu} n_\nu ; \mu \quad K_{ij} = \frac{1}{2N} (\partial_i h_{ij} - \nabla_i N_j - \nabla_j N_i) \]

UNITARY GAUGE

\[ n_\mu \equiv \frac{\partial_\mu \phi}{\sqrt{(\partial_\mu \phi)^2}} \]

And the action is built out of all geometrical quantities that are invariant under the time-dependent 3D spatial diffeomorphisms.
A unified action

It is an interesting framework that offers both a model-independent parametrization of alternatives to LCDM and a unifying language to analyze specific DE/MG models.

pure EFT:

\[ \{ \Omega(\tau), \Lambda(\tau), M_2^4(\tau), \bar{M}_1^3(\tau)\bar{M}_2^2(\tau) \} \]

mapping EFT:

\[ f(R) \quad \Omega = f_R; \quad \Lambda = \frac{m_0^2}{2} [f - R f_R]; \quad c = 0 \]

minimally coupled quintessence

\[ \Omega = 0; \quad c - \Lambda = V(\phi); \quad c = \frac{\dot{\phi}^2}{2} \]
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## A unified action

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<th>$c$</th>
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From Theory to Observables

matter and metric perturbations + theory

EINSTEIN-BOLTZMANN SOLVER
e.g. CAMB
EFTCAMB STRUCTURE
(Main EFT flag: EFTflag)

0: GR code
1: pure EFT
2: designer mapping EFT
3: EFT alternative parametrization
4: full EFT mapping

Use some parametrized forms for the EFT functions
Use a theory whose background mimics exactly the one specified
Use a parametrization that is mapped to the EFT framework
Use a theory by specifying it completely and mapping it to the EFT framework

Background DE equation of state:
(Flag: EFTwDE)

0: LCDM
1: wCDM
2: CPL
3: JBP
4: Turning point
5: Taylor expansion
6: User defined

Pure EFT Omega model selection:
(Flag: PureEFTmodelOmega)

0: Zero
1: Constant
2: Linear model
3: Power law model
4: Exponential model
5: User defined

Pure EFT gamma_1 model selection:
(Flag: PureEFTmodelGamma1)

Pure EFT gamma_2 model selection:
(Flag: PureEFTmodelGamma2)

Pure EFT gamma_3 model selection:
(Flag: PureEFTmodelGamma3)

Pure EFT gamma_4 model selection:
(Flag: PureEFTmodelGamma4)

Pure EFT gamma_5 model selection:
(Flag: PureEFTmodelGamma5)

Pure EFT gamma_6 model selection:
(Flag: PureEFTmodelGamma6)

Pure EFT Horndeski:
(Flag: PureEFTHorndeski)

Restricts pure EFT models to Horndeski. Pure EFT choices for gamma_4, gamma_5, gamma_6 will be ignored and handled internally.

Mapping EFT model selection:
(Flag: MappingEFTmodel)

1: f(R)
2: minimally coupled quintessence
3: non-minimally coupled quintessence
4: k-essence
5: Brans-Dicke
6: ...

Background DE equation of state:
(Flag: EFTwDE)

0: LCDM
1: wCDM
2: CPL
5: ...
CosmicFish

observables:
Cosmic Microwave Background
Weak Lensing tomography
Galaxy Clustering
Supernovae
Redshift drift

models:
CAMB
MGCAMB
EFTCAMB

https://cosmicfish.github.io/

Raveri, Martinelli, Zhao, Wang, arXiv:1606.06273
CosmicFish

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- Cosmic Microwave Background
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- CAMB
- MGCAMB
- EFTCAMB

Model cumulative Information Gain

Raveri, Martinelli, Zhao, Wang, arXiv:1606.06273

https://cosmicfish.github.io/
Redshift drift to access the DE redshift desert

Looking at how the redshift of a source changes over an observed time interval $\Delta t_0$:

$$\Delta z_s \approx \frac{\dot{a}(t_0) - \dot{a}(t_s)}{a(t_s)} \Delta t_0.$$ 

This translates into a spectroscopic velocity shift:

$$\Delta v = \frac{c \Delta z_s}{1 + z_s}$$

that can be used to probe $H(z)$:

$$\frac{\Delta v}{c} = H_0 \Delta t \left[ 1 - \frac{E(z_s)}{1 + z_s} \right]$$

Corasaniti, Huterer, Melchiorri, PRD 2007
It is becoming real! Some forecasts

Upcoming experiments such as the E-ELT and SKA will achieve, through different means, high enough spectroscopic sensitivity to measure this velocity shift.

The E-ELT’s high-resolution optical spectrograph will be able to measure the shift in the spectroscopic velocity, observing the Lyman $\alpha$ absorption lines of distant quasar systems, in a redshift range $2 < z < 5$. 

Martinelli et al., PRD86 (2012), Martins et al., PRD94 (2016)
Since we start from an action, we can identify general, model-independent viability conditions that are well motivated theoretically and ensure also numerical stability.

This can significantly restrict the parameter space, and represents a powerful tool for the advocated open-minded approach to cosmological tests of GR.

The world is beautiful outside only if it is stable inside.
Expanding the given action up to second order in the perturbations, and removing spurious DOFs, we can inspect the dynamics of perturbations; in this case one scalar field, i.e. $\xi$, and the tensor:

$$S^{(2)}_{\xi,h} = \frac{1}{(2\pi)^3} \int d^3k \, dt \, a^3 \left\{ \left[ \mathcal{L}_{\xi\dot{\xi}} \dot{\xi}^2 - k^2 G \xi^2 \right] + \frac{A_T}{8} \left[ \left( \dot{h}_{ij} \right)^2 - \frac{c_T^2}{a^2} \left( h_{ij} \right)^2 \right] \right\}$$

E.g., avoidance of ghost and gradient instabilities translate into the following set of conditions:

$$\mathcal{L}_{\xi\dot{\xi}} > 0$$

$$c_s^2 = \frac{G}{\mathcal{L}_{\xi\dot{\xi}}} > 0$$

$$A_T > 0$$

$$c_T^2 > 0$$

very general conditions in terms of the EFT functions, that constitute a stability check to be run at the very beginning!
Theoretical priors & Observational Constraints

CPL DE: \( w = w_0 + w_a (1 - a) \)

Joudaki et al., arXiv:1610.04606
Theoretical priors & Observational Constraints

CPL DE: $w = w_0 + w_a (1 - a)$

 WHICH DE IS IT? 

Joudaki et al., arXiv:1610.04606
Theoretical priors & Observational Constraints

\textbf{CPL DE:} \( w = w_0 + w_a (1 - a) \)

A stable Horndeski model will not give you that expansion!
Multifield scenarios shall be invoked…
Horndeski theoretical priors …

HORNDERSKI gravity \( \{ \Omega(a), \Lambda(a), M_{2}^{4}(a), \tilde{M}_{1}^{3}(a), \tilde{M}_{2}^{2}(a) \} \)

\[
f(a) = \sum_{n=0}^{N} \frac{\alpha_{n}}{n!} (a - a_{0})^{n}
\]

\[
f(a) = \sum_{n=0}^{N} \alpha_{n} (a - a_{0})^{n}
\]

\[
f(a) = \frac{\sum_{n=0}^{N} \alpha_{n} (a - a_{0})^{n}}{1 + \sum_{m=1}^{M} \beta_{m} (a - a_{0})^{m}}
\]

\( a_{0} = 0, 1 \)

\( \alpha_{n}, \beta_{m} \in [-1, 1] \)

\( M, N = 9 \)

\( O(10^{8}) \) models!
\[ H(a) \rightarrow w(a) = \frac{\rho_{\text{DE}}}{p_{\text{DE}}} \]

\[ \cdots \text{on DE equation of state} \]

1) Results for the quintessence class of models

2) Results for the general Horndeski class of models
… on DE equation of state

\[ H(a) \rightarrow w(a) = \frac{\rho_{DE}}{p_{DE}} \]
... on DE equation of state

\[ H(a) \rightarrow w(a) = \frac{\rho_{DE}}{p_{DE}} \]

\[ w(a) = w_0 + w_a (1 - a) \]
Towards correlation priors for non-parametric reconstructions

Horndeski gravity

correlation of \( \omega \) at a single scale factor against all other scale factors

Raveri, Bull, AS, Pogosian, PRD 2017
Towards correlation priors for non-parametric reconstructions

Horndeski gravity

correlation of w at a single scale factor against all other scale factors
... and on Large Scale Structure
Constraining Horndeski gravity with $\Sigma$ and $\mu$

Planck
Planck+BSh
Planck+WL
Planck+BAO/RSD
Planck+WL+BAO/RSD

… and on Large Scale Structure
Constraining Horndeski gravity with $\Sigma$ and $\mu$

Planck
Planck+BSH
Planck+WL
Planck+BAO/RSD
Planck+WL+BAO/RSD

... and on Large Scale Structure

Constraining Horndeski gravity with $\Sigma$ and $\mu$

Planck
Planck + BSH
Planck + WL
Planck + BAO/RSD
Planck + WL + BAO/RSD


FOM will increase by 2-3 orders of magnitude!
... and on Large Scale Structure

Constraining Horndeski gravity with $\Sigma$ and $\mu$

Santiago et al., Phys. Dark Univ. 2017
The framework: \((\mu, \Sigma)\)

Let us focus on linear scalar perturbations, described in Newtonian gauge by:

\[
ds^2 = -a^2(\tau) \left[ (1 + 2\Psi(\tau, x')) \, d\tau^2 - (1 - 2\Phi(\tau, x')) \, d\tilde{x}^2 \right]
\]

In a quite general manner, we can describe any deviation from LCDM in terms of two functions of time and scale:

**Poisson:**

\[
k^2 \Psi = -\mu(a, k) \frac{a^2}{2M_P^2} \rho \Delta
\]

**Lensing:**

\[
k^2 (\Phi + \Psi) = -\Sigma(a, k) \frac{a^2}{M_P^2} \rho \Delta
\]
$(\mu, \Sigma)$ in Horndeski
(μ,Σ) theoretical distribution functions

GBD

Horndeski + GWs

Espejo et al., in preparation
Correlation and Covariance prior matrices

GBD

Horndeski + GWs

Horndeski

Espejo et al., in preparation
Wrapping up!

This was a brief tale of the ongoing quest to test gravity on cosmological scales, eventually unveiling dark energy.

This is an exciting prospect that will be enabled by the SYNERGY of upcoming surveys.

I focused on the challenges, approaches and prospects on the theory side.

With a big effort we are making progress in terms of theoretical frameworks to interpret the data ... bare with us, next years will be exciting!
THANK YOU !