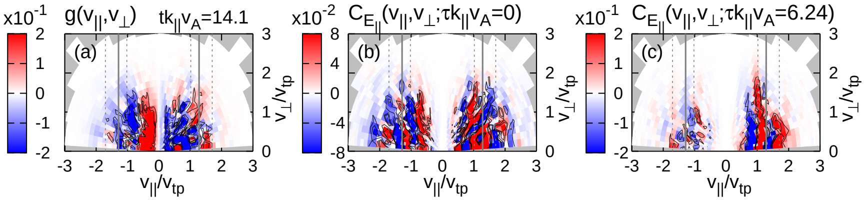
Application of Field-Particle Correlations to Space and Laboratory Plasmas



Kristopher G. Klein — April 11^{th}, 2017

with Gregory Howes (Iowa) Justin Kasper (Michigan)

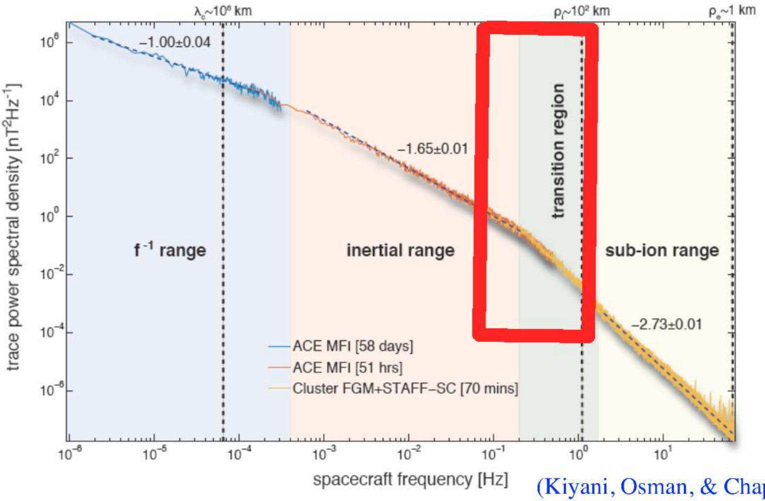
Jason TenBarge (Maryland) Chris Chen (Imperial College)

Francesco Valentini (Calabria)

Exploring the Physics of Space Plasmas in the Laboratory

Support from NASA HSR NNX16AM23G and NNX16AG81G

Observed Turbulent Spectra (Largely) Agree with Theory



(Kiyani, Osman, & Chapman, 2015)

Unfortunately, spectra are insufficient to distinguish between competing dissipation mechanisms...

Mechanism(s) Damp **Collisionless** Turbulence

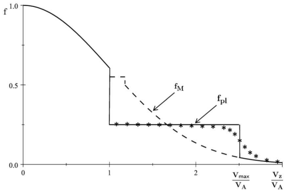
Collisionless Heating Mechanisms

- **Coherent** (Landau Damping)
- **Incoherent** (Stochastic Heating)
- **Intermittent Spatial Structure** (Magnetic Reconnection, Shocks)

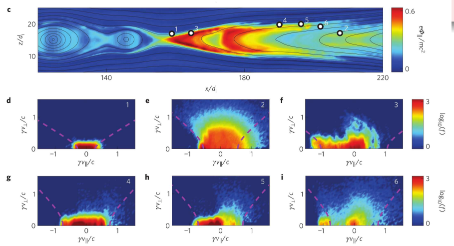
The Solar Wind is hot & diffuse

- $\nu_{\rm coll} \propto n/T^{3/2}$
- non local-thermodynamic equalibria persist

Each Mechanism Generates Characteristic Velocity Space Structure.



Voitenko & Pierrard 2013; Model

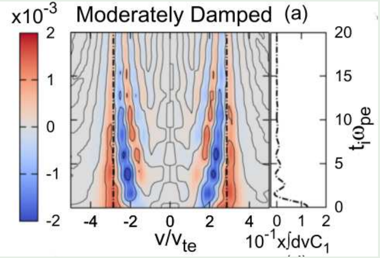


Egedal et al 2012; PIC Simulation

Measuring Collisionless Energy Transfer

- Rather than measure $f(\mathbf{v})$ and infer the nature of the energy transfer, we directly measure energy transfer between f and fields.
- The energy transfer is determined by the field-particle interaction term in the Vlasov Equation.
- The single point nature of in situ observations informs the structure of our measurement.

- Develop Field-Particle Correlation C_F
- Apply to Simulations:
  - 1D1V Electrostatic (`vp`)
  - 3D2V Gyrokinetic (`AstroGK`)
  - 3D3V Hybrid Vlasov-Maxwell (`HVM`)
- Future Application to Observations



Klein & Howes, 2016; *vp*

Our Toy Model: Electrostatic Landau Damping

Consider $f_s(x, v, t)$ and
$E(x, t) = -\nabla\phi(x, t)$
governed by:

$$\frac{\partial f_s}{\partial t} + v\frac{\partial f_s}{\partial x} - \frac{q_s}{m_s}\frac{\partial \phi}{\partial x}\frac{\partial f_s}{\partial v} = 0$$

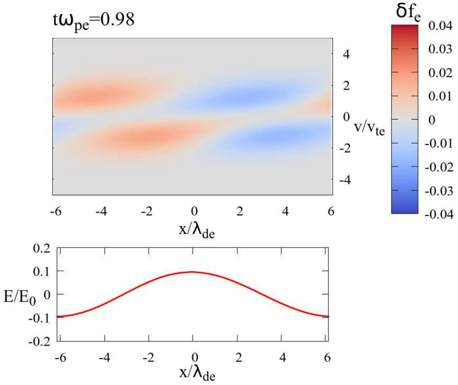
$$\frac{\partial^2 \phi}{\partial x^2} = -4\pi \sum_s \int dv q_s f_s$$

with conserved energies

$$W_\phi = \int dx \frac{E^2}{8\pi}$$

and

$$W_s = \int dx \int dv \frac{m_s v^2}{2} f_s$$



Can we characterize $\frac{\partial W_s}{\partial t}$ by measuring at only a single point in space?

Measuring Energy Transfer (c.f. Howes et al 2017)

Plugging the conserved energies into the Vlasov Equation yields

$$\frac{\partial W_s}{\partial t} = \int dx \frac{\partial}{\partial x} \int dv \frac{m_s v^3}{2} f_s - \int dx \int dv \frac{q_s v^2}{2} \frac{\partial f_s}{\partial v} E$$

$$= \int dx j_s(x) E(x,t)$$

- Energy is transferred via E (see Kellogg et al 2006)
- $\partial W_s / \partial t$ is observationally accessible only with full knowledge of $f_s(x,v,t)$ and $E(x,t)$.
- $\partial W_s / \partial t$ describes both oscillatory and secular energy transfer between waves and particles (see Bellan 2012).

We define the phase-space energy density:
$w_s(x,v,t) = \frac{m_s v^2}{2} f_s(x,v,t)$
whose rate of change is

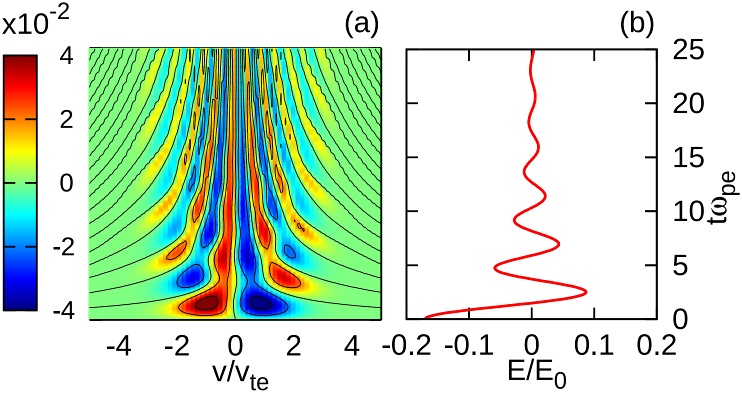
$$\frac{\partial w_s(x,v,t)}{\partial t} = -\frac{m_s v^3}{2} \frac{\partial \delta f_s}{\partial x} - \frac{q_s v^2}{2} \frac{\partial f}{\partial v} E(x,t).$$

The field-particle interaction terms are responsible for secular energy transfer.

Single Point Simulation Measurement of Energy Transfer

Using the 1D1V Code vp (Howes, Klein, & Li, 2017)
we construct 'measurements' of $f_e(x_0, v, t)$ and $\phi(x_0, t)$:

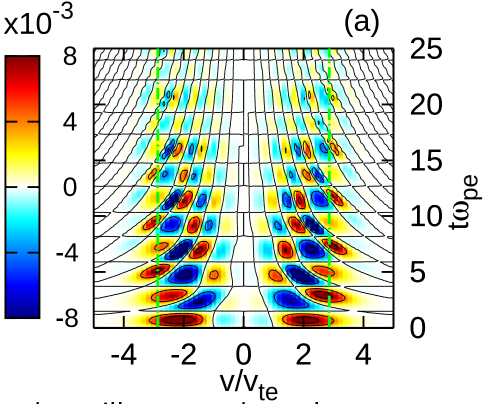
$\delta f_e(x = 0, v, t)$ $\qquad$ $E(x = 0, t)$



'Instantaneous' Energy Transfer

The energy transfer described by the product

$$-\frac{q_s v^2}{2}\frac{\partial \delta f_s(x,v,t_i)}{\partial v}E(x,t_i)$$



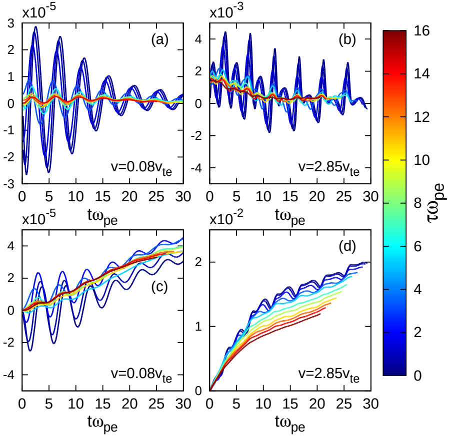
contains both oscillatory and secular energy components.

Removing Oscillatory Transfer

Averaging the product of $-\frac{q_s v^2}{2}\frac{\partial \delta f_s}{\partial v}$ and E over time interval $\tau = n_C dt$ removes the oscillatory energy transform, leaving only the net secular energy transfer at a given position x_0:

$$C_E(x_0, v_0, t, \tau)$$

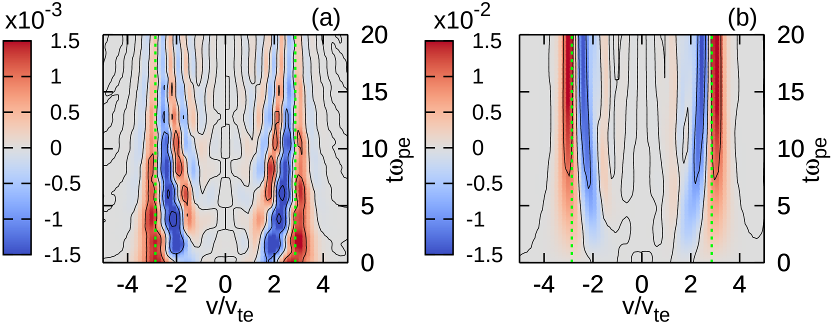
$$\int dt' C_E(x_0, v_0, t', \tau)$$



Velocity Dependent Secular Energy Transfer

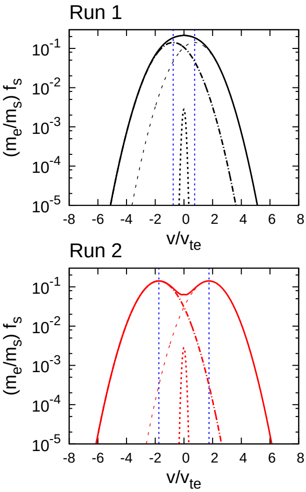
By calculating $C_E(v)$, the velocity-space signature of Landau damping is recovered from a single point observation. (Klein & Howes 2016)

$$C_E(x_0, v, t, \tau) \qquad \int dt' C_E(x_0, v, t', \tau)$$

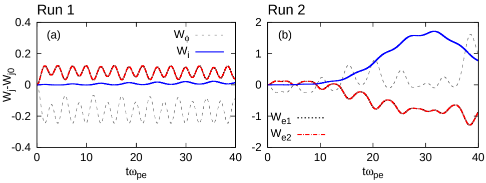


We have a velocity-dependent measurement of the secular transfer of energy between fields and particles using single point measurements.

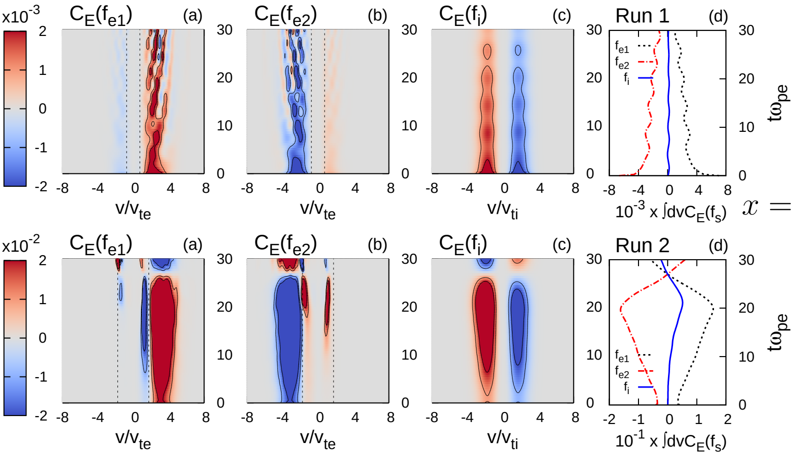
What is the signature of secular energy transfer for counter-streaming electron instabilities?



For $F_{s0}(v) = \frac{n_s}{\sqrt{2\pi}} \exp\left[\frac{-(v-v_{ds})^2}{2v_{ts}^2}\right]$ we calculate C_E for systems stable and unstable to the counter-streaming instability(Klein 2017).

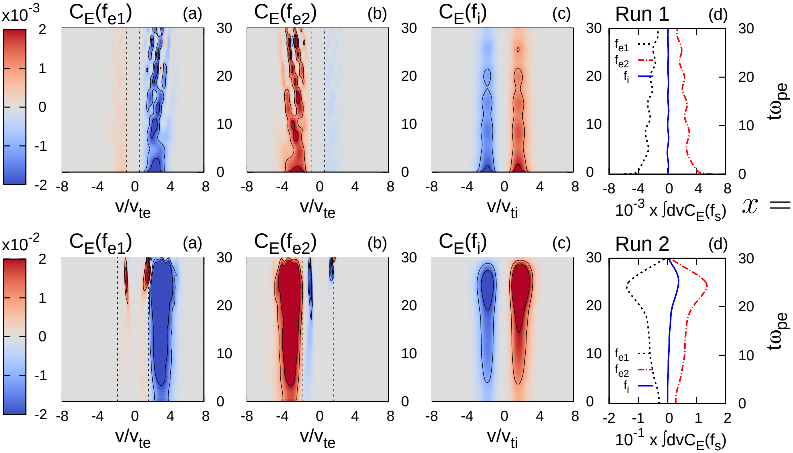


Correlation over $\tau = 2\pi/\omega_{\mathrm{Langmuir}}$ removes oscillatory transfer, revealing unstable growth.



$x = 0\lambda_{de}$

Correlation over $\tau = 2\pi/\omega_{\text{Langmuir}}$ removes oscillatory transfer, regardless of spatial location.



$x = -12.8\lambda_{de}$

What about Electromagnetic Turbulence?

The appropriate energy transfer for a general Vlasov Equation

$$\frac{\partial f_s}{\partial t} + \mathbf{v} \cdot \nabla f_s + \mathbf{a} \cdot \frac{\partial f_s}{\partial \mathbf{v}} = 0$$

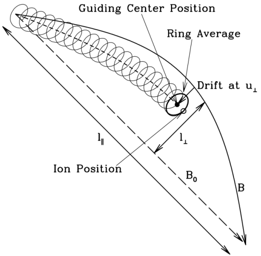
Takes the form of

$$C_{\text{General}}(\mathbf{v}, t_j, \tau) = -\frac{1}{n_C} \sum_{i=j}^{j+n_C} \frac{m_s v^2}{2} \mathbf{a}(x_0, t_i) \cdot \frac{\partial f_s(x_0, v, t_i)}{\partial \mathbf{v}}.$$

For $\mathbf{a} = \frac{q_s}{m_s} \left[\mathbf{E} + \frac{\mathbf{v}\times\mathbf{B}}{c}\right]$, the $\mathbf{v} \times \mathbf{B}$ term does no work and the transfer mediated by the electric field can be expressed as

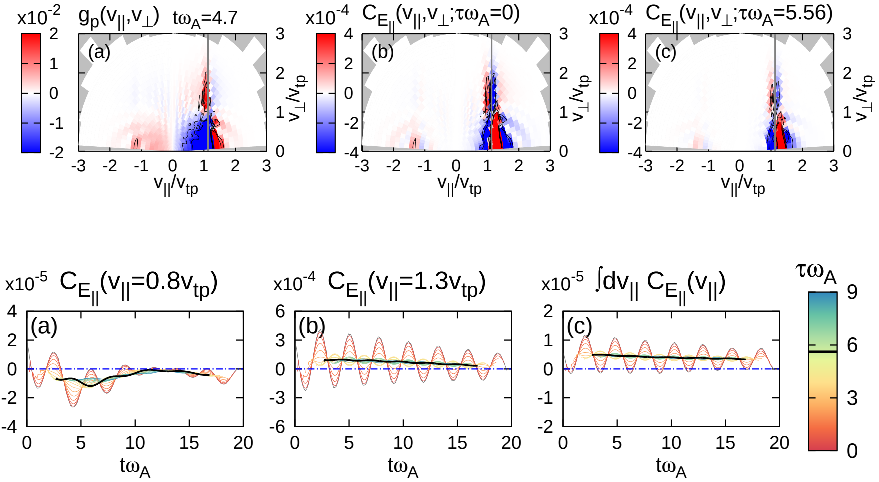
$$C_{E_l}(\mathbf{v}, t, \tau) = -\frac{1}{n_C} \sum_{i=j}^{j+n_C} \frac{q_s v_l^2}{2} \frac{\partial f_s(x_0, v, t_i)}{\partial v_l} E_l.$$

Gyrokinetics

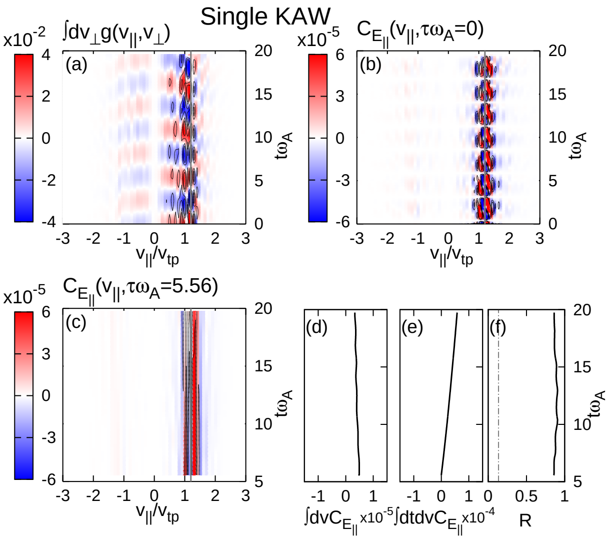


- Gyrokinetics is a rigorous, low-frequency approximation of the full Vlasov description (Frieman & Chen, 1982) which is derived by averaging over particle's gyromotion.
- We consider gyrokinetics for a simplified slab geometry (Howes et al 2006) using the `AstroGK` numerical simulation code (Numata et al 2010).

A Single Gyrokinetic KAW

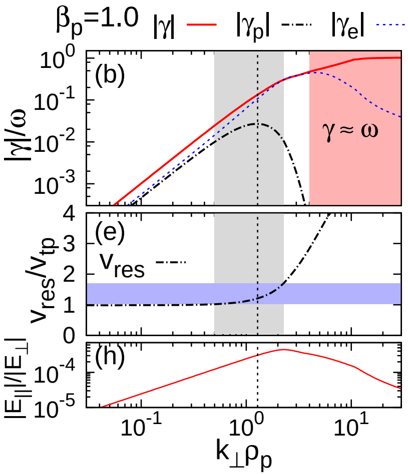


The Reduced Correlation $C_{E_\parallel}(v_\parallel)$



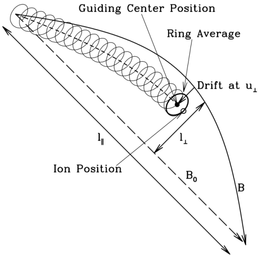
$$R \equiv \frac{\int_{v_1}^{v_2} dv_\parallel |C_{E_\parallel}(v_\parallel)|}{\int_{-4v_{tp}}^{4v_{tp}} dv_\parallel |C_{E_\parallel}(v_\parallel)|}$$ ≈ 0.90 of energy is transferred near the preferred $v_\parallel$

Should Broadband Turbulence Have a Clear Signature in v?



- The total damping rate γ/ω monotonically increases near $k_\perp \rho_p = 1.0$.
- γ_p/ω is strongly peaked near $k_\perp \rho_p = 1.0$.
- Proton damping will be dominated by modes with $k_\perp \rho_p \sim 1$, which have resonant velocities bound within a narrow region.

Gyrokinetic Turbulence



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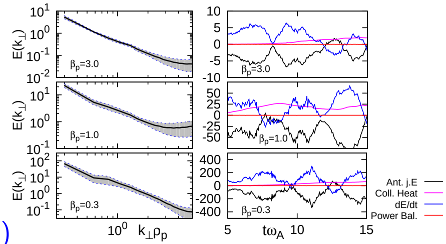
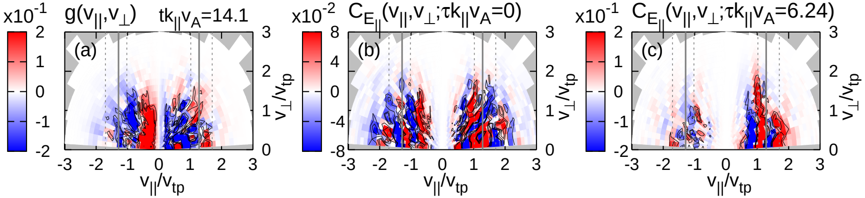
- We consider gyrokinetics for a simplified slab geometry (Howes et al 2006) using the `AstroGK` numerical simulation code (Numata et al 2010).

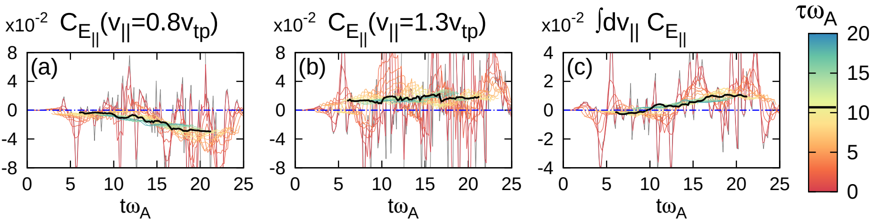
- Drive simulations with $\beta_p = 0.3,\ 1.0,\ 3.0.$
- $k_\perp \rho_p \in [0.25, 5.5]$
- Output perturbed distribution and fields at spatial points. (Klein, Howes, & TenBarge in prep)



The Gyrotropic Correlation $C_{E_\parallel}(v_\perp, v_\parallel)$



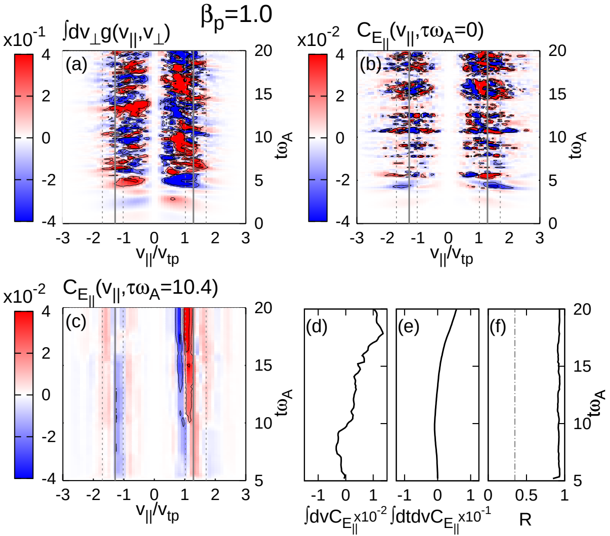
Selecting a Correlation Interval for Turbulence



Due to the spectra of frequencies ω_j governing the oscillatory energy transfer, need to select a $\tau > 2\pi/\omega_j$ for any particular ω_j.

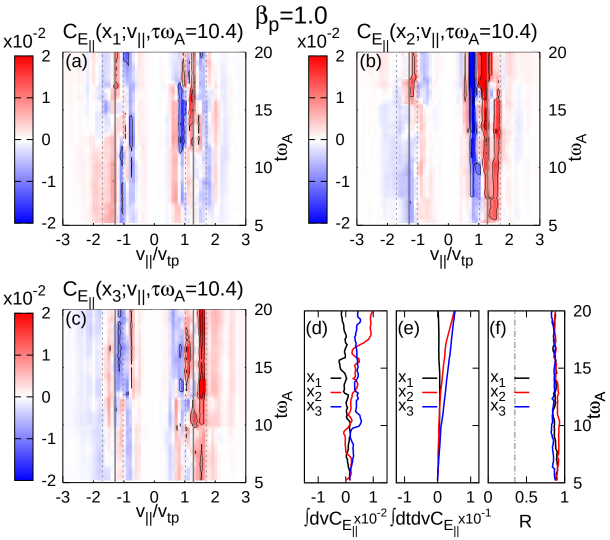
Choosing $\tau = 4\pi/\omega_{\rm peak}$ produces a nearly monotonic energy transfer.

The Reduced Correlation $C_{E_\parallel}(v_\parallel)$



$$R \equiv \frac{\int_{v_1}^{v_2} dv_\parallel |C_{E_\parallel}(v_\parallel)|}{\int_{-4v_{tp}}^{4v_{tp}} dv_\parallel |C_{E_\parallel}(v_\parallel)|} \approx 0.90 \text{ of energy is transferred near the preferred } v_\parallel$$

$C_{E_\parallel}(v_\parallel)$ Varies in Space...

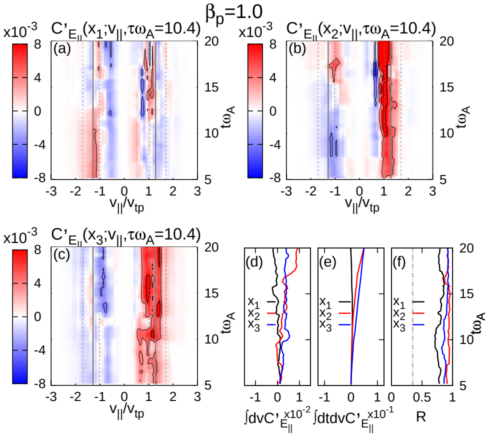


...but retains its resonant structure.

$C'_{E_\parallel}(v_\parallel)$ as a Proxy for Energy Transfer

As $\partial f_s/\partial v$ can unreliable for for 'actual' data, consider the proxy C'_E, related to C_E by an integration by parts:

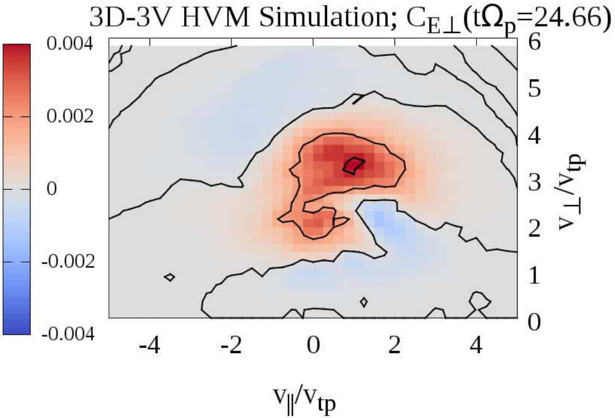
$$C'_E(x_0, v, t_i, \tau) \equiv \frac{1}{N} \sum_{j=i}^{i+N} q_s v f_s(x_0, v, t_j) E(x_0, t_j).$$



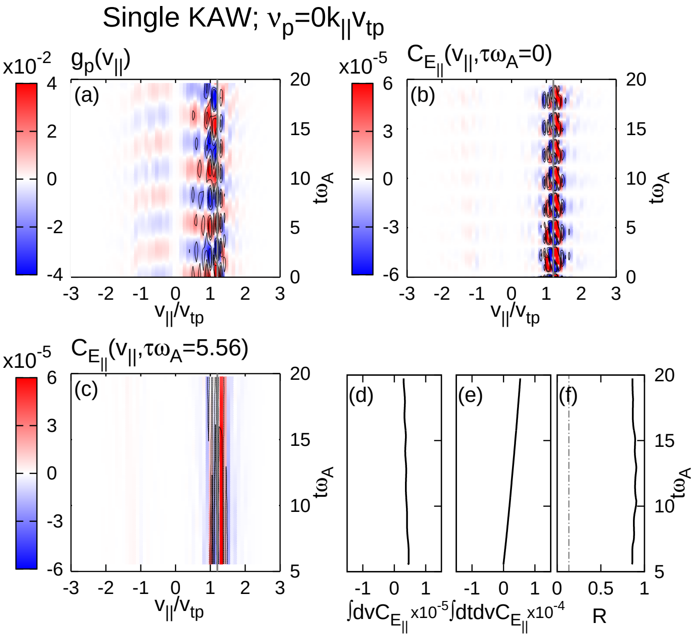
Preliminary Results using HVM

For a decaying turbulent Hybrid Vlasov-Maxwell system (kinetic ions, fluid electrons):

$$C'_{E_\perp}(\mathbf{v}, t, \tau) = \frac{1}{N_c} \sum_{t_i = t_0}^{t_0 + N_C} \left[\left(q_s v_{\perp,1} E_{\perp,1} + q_s v_{\perp,2} E_{\perp,2} \right) \delta f(\mathbf{v}, t_i) \right]$$



So...Collisions?



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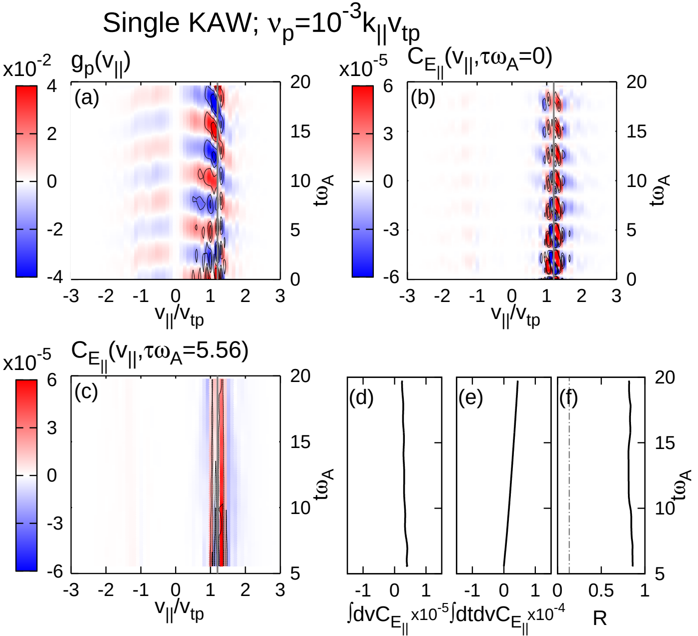

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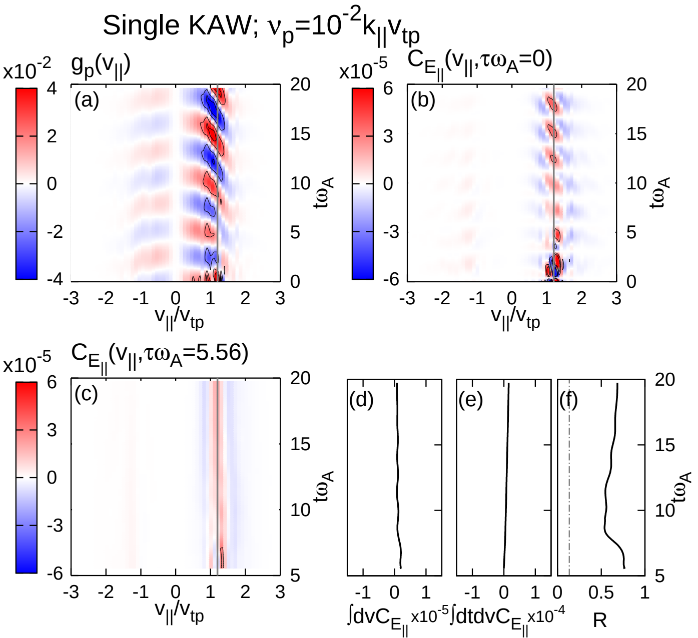

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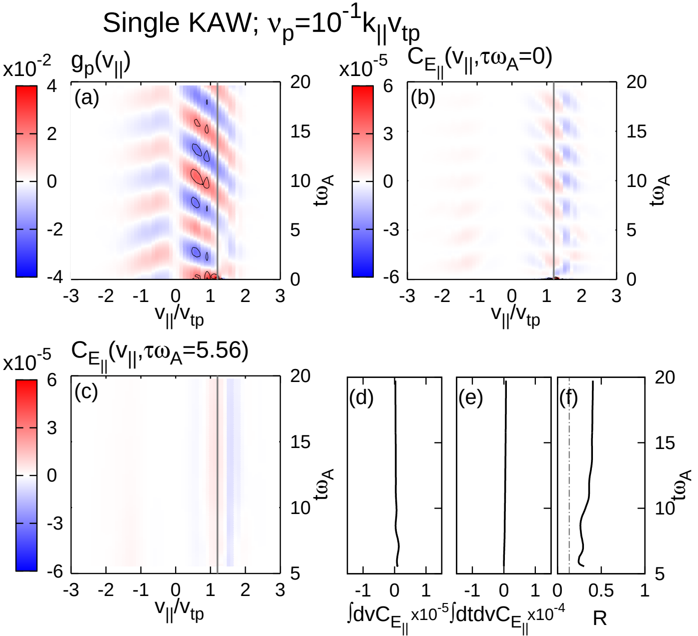

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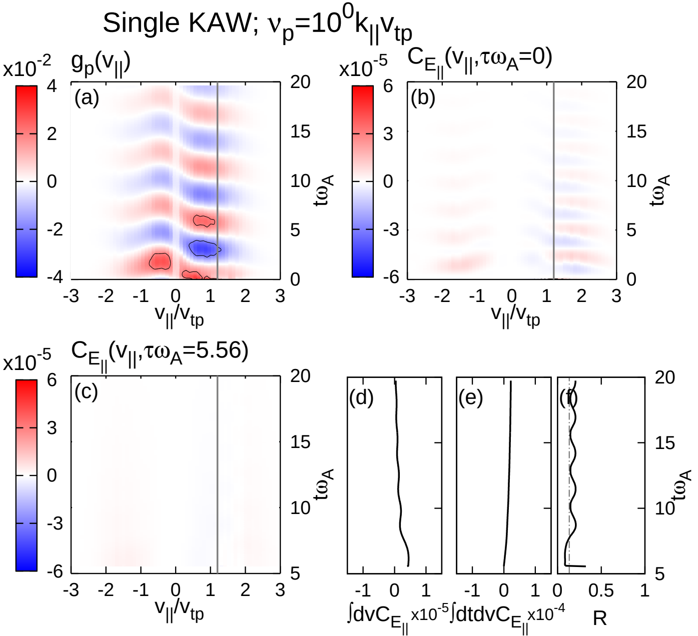

So...What do We Need to Measure C_E?

Trivially, $f_s(\mathbf{v}, \mathbf{x}, t)$ and $\mathbf{E}(\mathbf{x}, t)$.

But, remember:

- To remove oscillations, need measurements at several discrete times within a given period.
- If sweeping through $\mathbf{v}$, ensure the correct $\mathbf{E}$ is correlated with a given piece of velocity phase space.
- C' may be a good enough proxy if $\partial_v f$ is unreliable.

If $C_E(\mathbf{v})$ can be measured, it may be able to definitively identify the mechanisms which govern field-particle energy transfer in a plethora of plasma systems.