

# Theoretical aspects of dark matter physics

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# Outline

## Lecture 1

- Evidence for dark matter
- Dark matter properties
- Production mechanisms and particle candidates

## Lecture 2

- Production mechanisms and particle candidates (cont)
- Interaction types: contact-type vs long-range
- Current topics

The material contained in these slides is the result of the work of many people. It is impossible to do justice to everybody's work in this limited space, and only very few references are provided. If you would like more references on a particular topic, please ask me later.

# I. Evidence for dark matter

# Overwhelming evidence

- **Rotation curves of galaxies and of galaxy clusters** cannot be explained by ordinary matter only.
- **Gravitational lensing** of background galaxies by clusters requires a significant dark matter component.
- 1E0657-56, aka the **Bullet Cluster**, shows separation of ordinary matter (gas) from dark matter.
- **Cosmic microwave background** radiation shows gravitational wells created by dark-matter clustering.

# Dark matter in galaxy clusters

In the 1930s, Fritz Zwicky measured the velocity dispersion of galaxies in the Coma Cluster, and inferred its total mass using the Virial theorem.

He discovered that the luminous matter, i.e. the galaxies in the cluster, accounted for far less mass!

Later on, measurements of the X-rays emitted by hot intra-cluster gas through bremsstrahlung, showed that the gas cannot account for the missing mass.

Virial theorem

$$2\langle K \rangle = -\langle V \rangle$$

$$\frac{1}{2} m 3\sigma^2 = \frac{GM_{\text{tot}}(r) m}{r}$$

velocity dispersion

Die Rotverschiebung von extragalaktischen Nebeln  
von F. Zwicky.  
(16. II. 33.)

Fritz Zwicky, Helvetica Physica Acta  
Vol.6 p.110-127, 1933

**Galaxy clusters are  
DM dominated!**

# Rotation curves of galaxies

In 1970, Vera Rubin discovered that the rotation curves of galaxies are approximately flat.

Newtonian dynamics

$$F = \frac{GMm}{r^2} = \frac{mv^2}{r}$$

Assuming only the star density, which falls exponentially with  $r$ ,

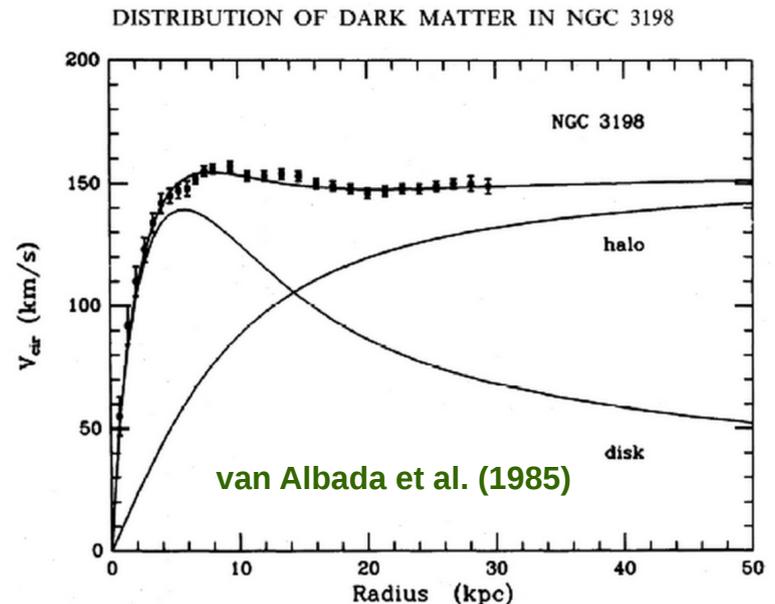
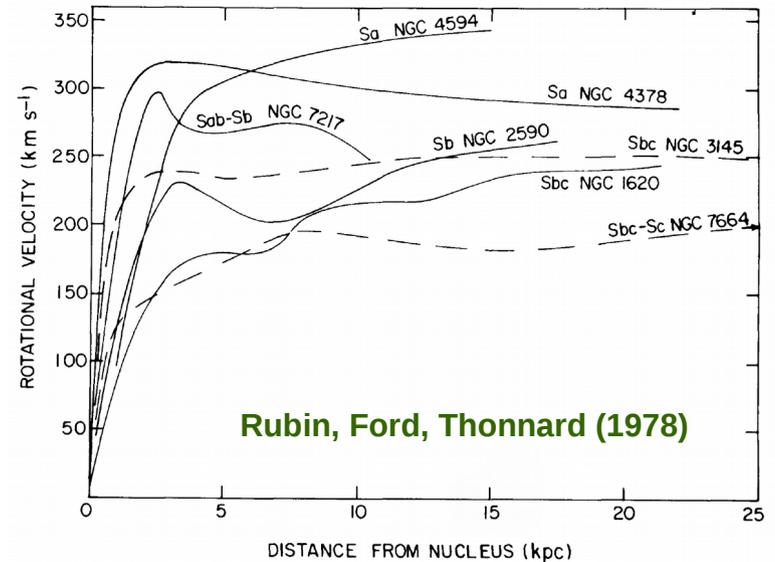
$$v \propto r^{-1/2}$$

For constant  $v$ ,

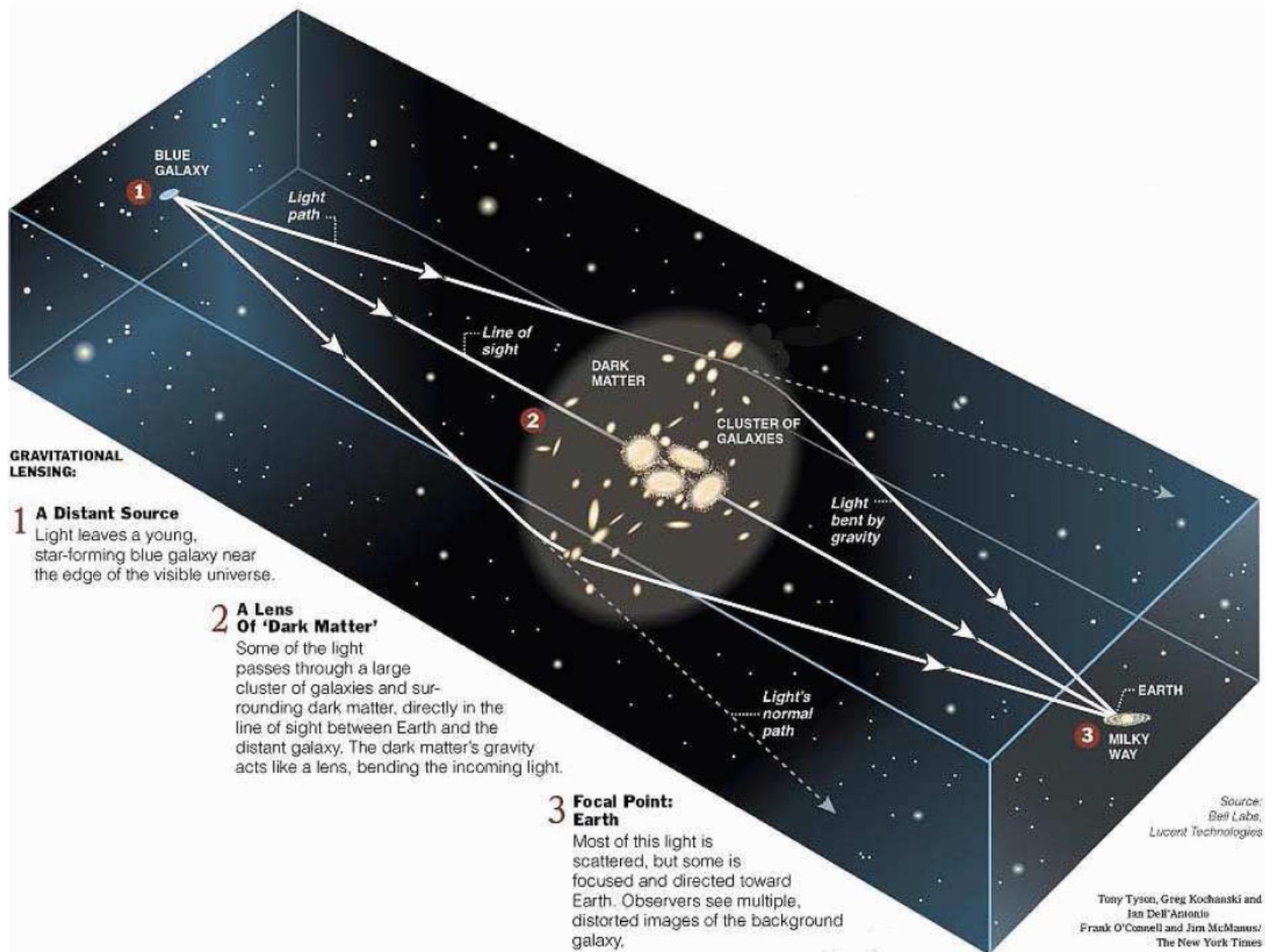
$$M(r) \propto r, \quad \rho(r) \propto r^{-2}$$



Extended DM halo!



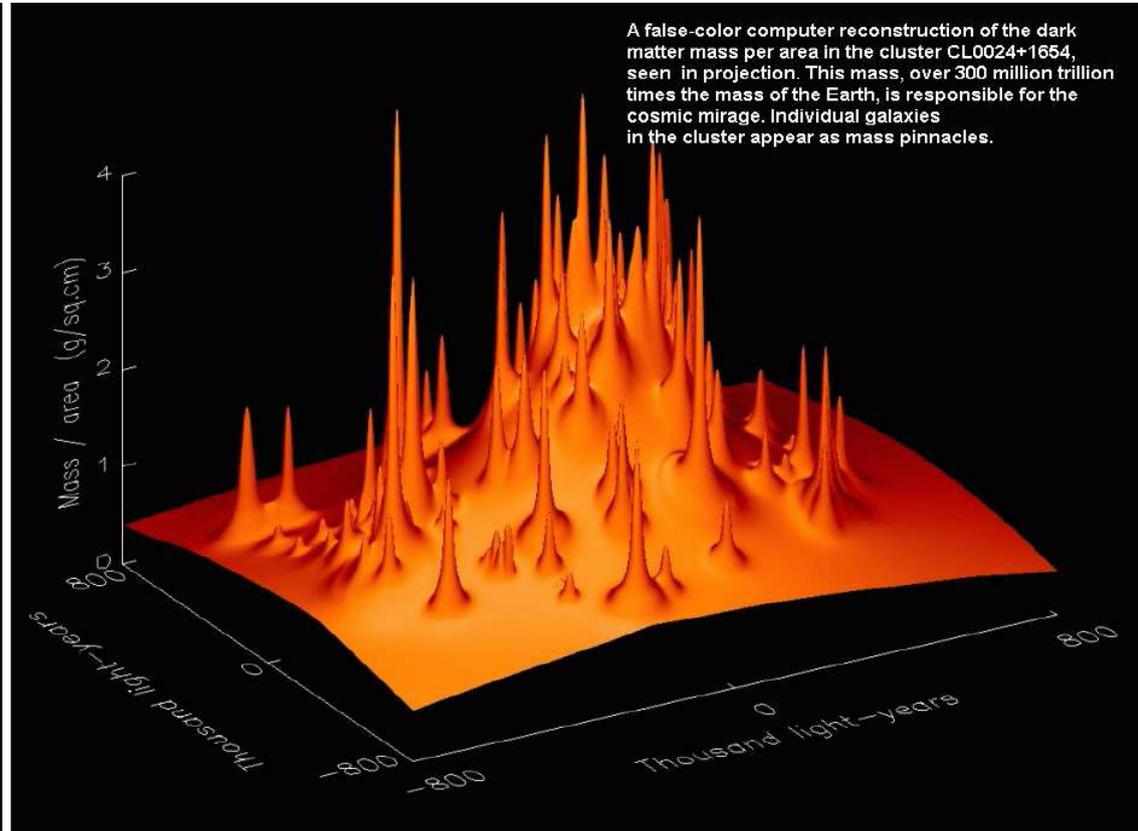
# Gravitational lensing



# Gravitational lensing



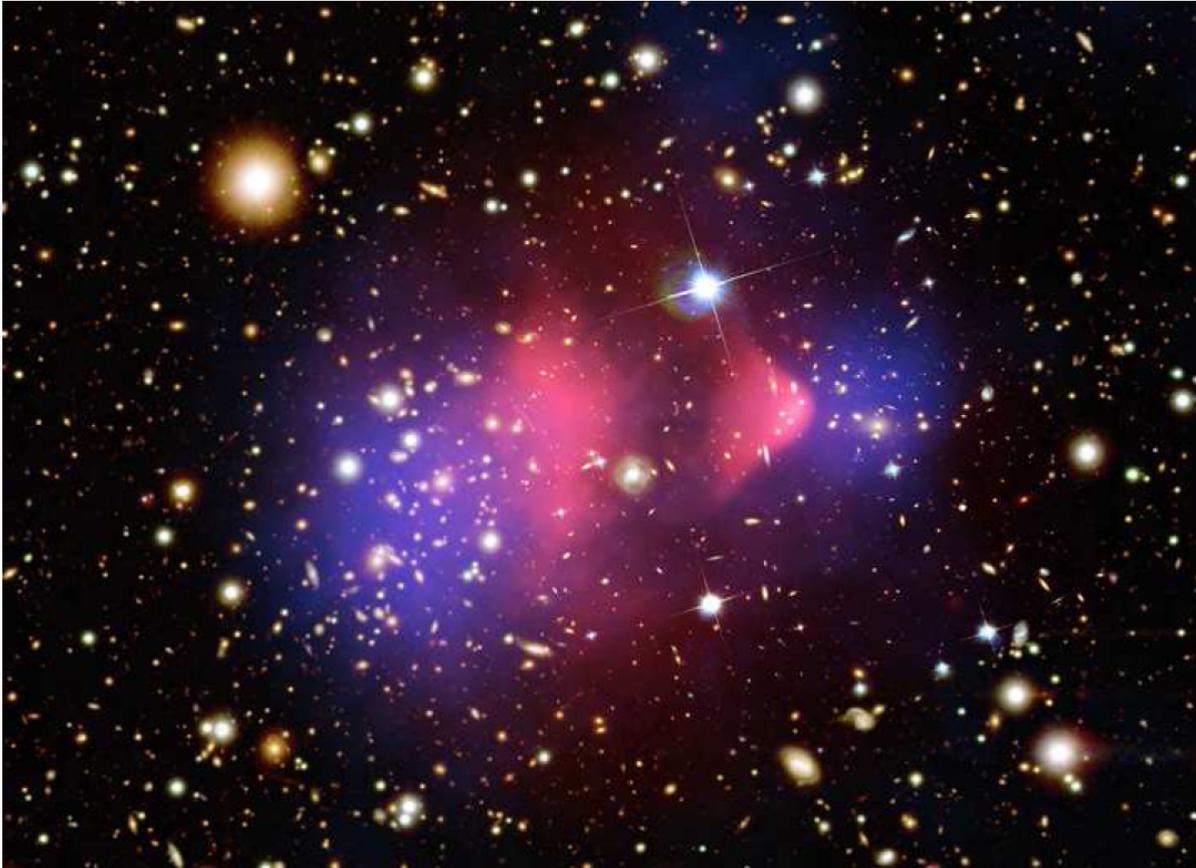
**Foreground cluster CL0024+1654**  
produces multiple images of  
a blue background galaxy.



A false-color computer reconstruction of the dark matter mass per area in the cluster CL0024+1654, seen in projection. This mass, over 300 million trillion times the mass of the Earth, is responsible for the cosmic mirage. Individual galaxies in the cluster appear as mass pinnacles.

**Dark matter mass reconstruction**  
of CL0024+1654

# The Bullet Cluster



Stars: optical

Gas: X-rays

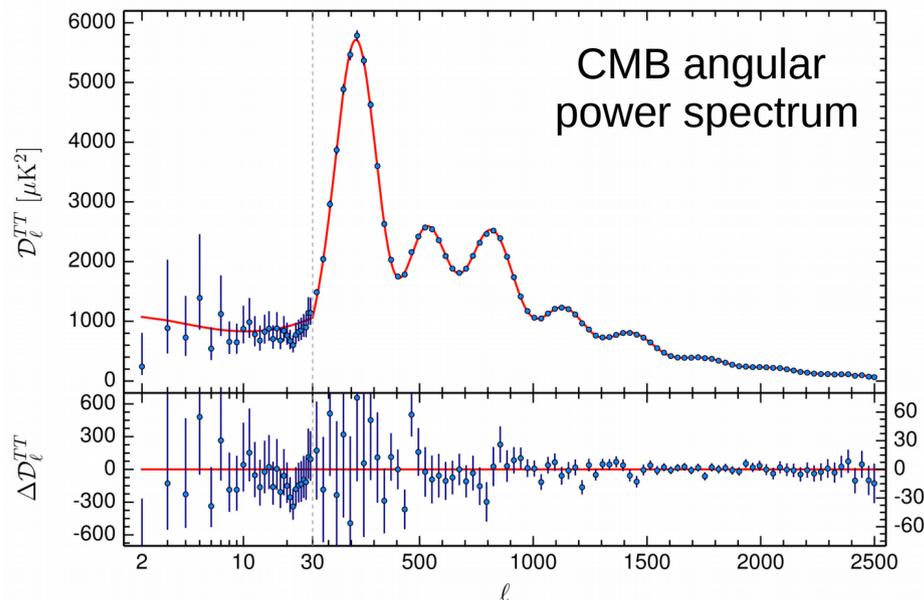
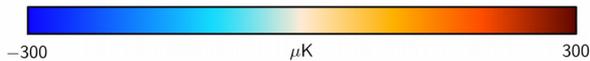
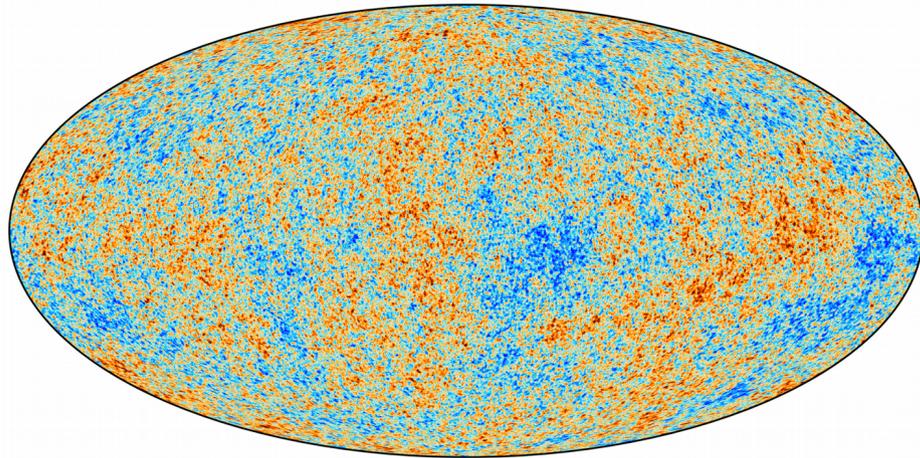
Total mass: gravitational lensing

## Separation of gas from the bulk of gravitating mass

- Strongly supports the DM hypothesis
- Sets upper limit on DM self-scattering,  
 $\sigma/m_{\text{DM}} < 1.3 \text{ barn/GeV} = 0.7 \text{ cm}^2/\text{g}$

# Cosmic Microwave Background

CMB temperature fluctuations



When the universe was at temperature  $T \sim \text{eV}$ , atoms formed and the universe became transparent to radiation.

The photon bath from that time has red-shifted into the microwave range,  $T = 2.725 \text{ K}$ .

Temperature fluctuations measure the gravitational wells created by DM clustering!

Decompose temperature field in spherical harmonics

$$T(\hat{n}) = \sum_{\ell=0}^{\infty} \sum_{m=-\ell}^{\ell} a_{T,\ell m} Y_{\ell m}(\hat{n})$$

$$C_{\ell}^{TT} = \frac{1}{2\ell + 1} \sum_{m=-\ell}^{\ell} a_{T,\ell m} a_{T,\ell m}^*$$

The CMB angular power spectrum depends on several parameters, including  $\Omega_B$ ,  $\Omega_M$ ,  $\Omega_{\Lambda}$

Particle dark matter or modified gravity?

# Modified Newtonian Dynamics (MOND)

In 1983, Milgrom proposed that Newtonian dynamics fails at very low accelerations.

Characteristic acceleration

$$a_0 = 1.2 \times 10^{-10} \text{ m s}^{-2}$$

$$a \gg a_0 : \quad a = GM/r^2$$

$$a \ll a_0 : \quad a^2/a_0 = GM/r^2$$

$$\Downarrow \quad a = v^2/r$$

$$v_\infty^4 = GM_{\text{tot}} a_0^2$$

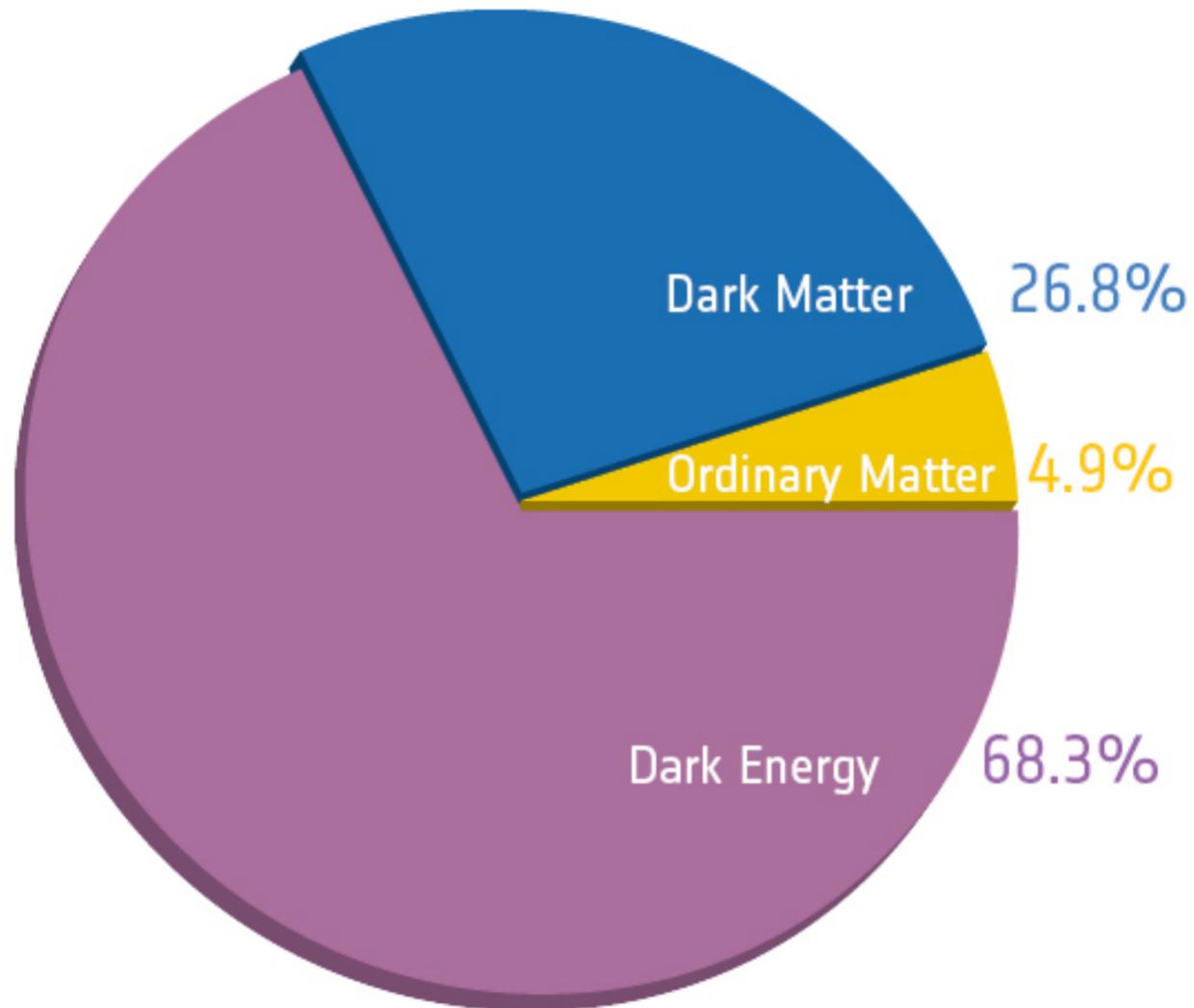
Explains flatness of rotational curves!

Very successful in galaxies.

However, MOND and relativistic generalizations

- fail at galaxy clusters;
- cannot explain the separation of the gas from the bulk of gravitating matter in the Bullet Cluster;
- cannot explain the CMB!

# Our Universe



## II. Dark matter properties

# The knowns

## The Standard Model of elementary particles

mass →	$\approx 2.3 \text{ MeV}/c^2$	$\approx 1.275 \text{ GeV}/c^2$	$\approx 173.07 \text{ GeV}/c^2$	0	$\approx 126 \text{ GeV}/c^2$
charge →	2/3	2/3	2/3	0	0
spin →	1/2	1/2	1/2	1	0
	<b>u</b> up	<b>c</b> charm	<b>t</b> top	<b>g</b> gluon	<b>H</b> Higgs boson
<b>QUARKS</b>	$\approx 4.8 \text{ MeV}/c^2$	$\approx 95 \text{ MeV}/c^2$	$\approx 4.18 \text{ GeV}/c^2$	0	
	-1/3	-1/3	-1/3	0	
	1/2	1/2	1/2	1	
	<b>d</b> down	<b>s</b> strange	<b>b</b> bottom	<b><math>\gamma</math></b> photon	
	$0.511 \text{ MeV}/c^2$	$105.7 \text{ MeV}/c^2$	$1.777 \text{ GeV}/c^2$	$91.2 \text{ GeV}/c^2$	
	-1	-1	-1	0	
	1/2	1/2	1/2	1	
	<b>e</b> electron	<b><math>\mu</math></b> muon	<b><math>\tau</math></b> tau	<b>Z</b> Z boson	
<b>LEPTONS</b>	$< 2.2 \text{ eV}/c^2$	$< 0.17 \text{ MeV}/c^2$	$< 15.5 \text{ MeV}/c^2$	$80.4 \text{ GeV}/c^2$	
	0	0	0	$\pm 1$	
	1/2	1/2	1/2	1	
	<b><math>\nu_e</math></b> electron neutrino	<b><math>\nu_\mu</math></b> muon neutrino	<b><math>\nu_\tau</math></b> tau neutrino	<b>W</b> W boson	
				<b>GAUGE BOSONS</b>	

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## Dark Matter

- Stable or long-lived,  $\tau \gg t_U$
- Not electrically charged
- Non-relativistic today

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## Dark Matter

- Stable or long-lived,  $\tau \gg t_U$
- Not electrically charged
- Non-relativistic today
- $\Omega_{DM} \approx 26\%$  of our universe
- Large-scale gravitational clustering: **not hot**  
[hot = relativistic when gravitational clustering started]

**DM: not a known particle!**

# The canonical paradigm of structure formation

## Collisionless Cold Dark Matter ( $\Lambda$ CDM)

→ James Bullock's lecture

- **Cold DM**

DM particles were non-relativistic when gravitational clustering started, at the time of matter-radiation equality (redshift  $z \sim 3400$ ,  $T \sim \text{eV}$ )

- **Collisionless DM**

(a) No significant self-scattering.

(b) Early kinetic decoupling from the relativistic plasma.

- **Reproduces the large-scale structure ( $> \text{Mpc}$ ) very well!**

- At galactic and sub-galactic scales, gravity-only simulations of collisionless  $\Lambda$ CDM predict too rich structure.

Possible discrepancies with observations: “cusps vs cores”, “missing satellites”, “too-big-to-fail” → **too much matter in the centers of galaxies.**

# Small-scale structure

## Possible resolutions

- **Baryonic physics**

Hydrodynamic simulations: modeling of star feedback. Successes and failures. Very active field.

- **Shift in the DM paradigm**

Aim to retain success of collisionless  $\Lambda$ CDM at large scales, suppress structure at small scales

- **Warm DM**

Suppresses structure below a scale (free-streaming length).

- **Self-interacting DM**

Self-scatterings inside halos redistribute energy and momentum and heat up low-entropy material  $\rightarrow$  overdensities get smoothed out, star-formation rate is suppressed.

# Self-interacting dark matter

- Cross-section needed to affect galactic structure

$$\sigma_{\text{self-scatt}}/m_{\text{DM}} \sim \text{barn/GeV} \sim \text{cm}^2/\text{g}$$



at dwarf-galaxy scales,  $v_{\text{DM}} \sim 20$  km/s.

- Upper limit from Bullet Cluster is of the same order, at  $v_{\text{DM}} \sim 1000$  km/s.
- No tension between the two, if  $\sigma_{\text{self-scatt}}$  decreases with increasing  $v_{\text{DM}}$   
 $\Rightarrow$  Light mediators, long-range interactions!

## **III. Production mechanisms and particle candidates**

# Production mechanisms ↔ Candidates

thermal history

- **Freeze-out**

- Standard Model plasma
- Dark-sector plasma

- **Asymmetry generation & freeze-out**

- **Freeze-in**

- **Collapse of density perturbations**

- **Scalar condensates**

- Fragmentation
- Misalignment

non-thermal history

- WIMPs (e.g. neutralino in SUSY, minimal DM)
- Particles coupled to SM via heavy mediators (e.g.  $Z'$ , Higgs portal);
- Dark U(1), dark QCD

Mirror DM, technicolor dark QCD, dark U(1)

Sterile neutrino, gravitino

Primordial black holes

→ Anne Green's lecture

Q-balls (solitonic DM)

Axions

### **III. Production mechanisms and particle candidates**

- Non-relativistic thermal freeze-out
- Asymmetry generation & freeze-out
- Freeze-in

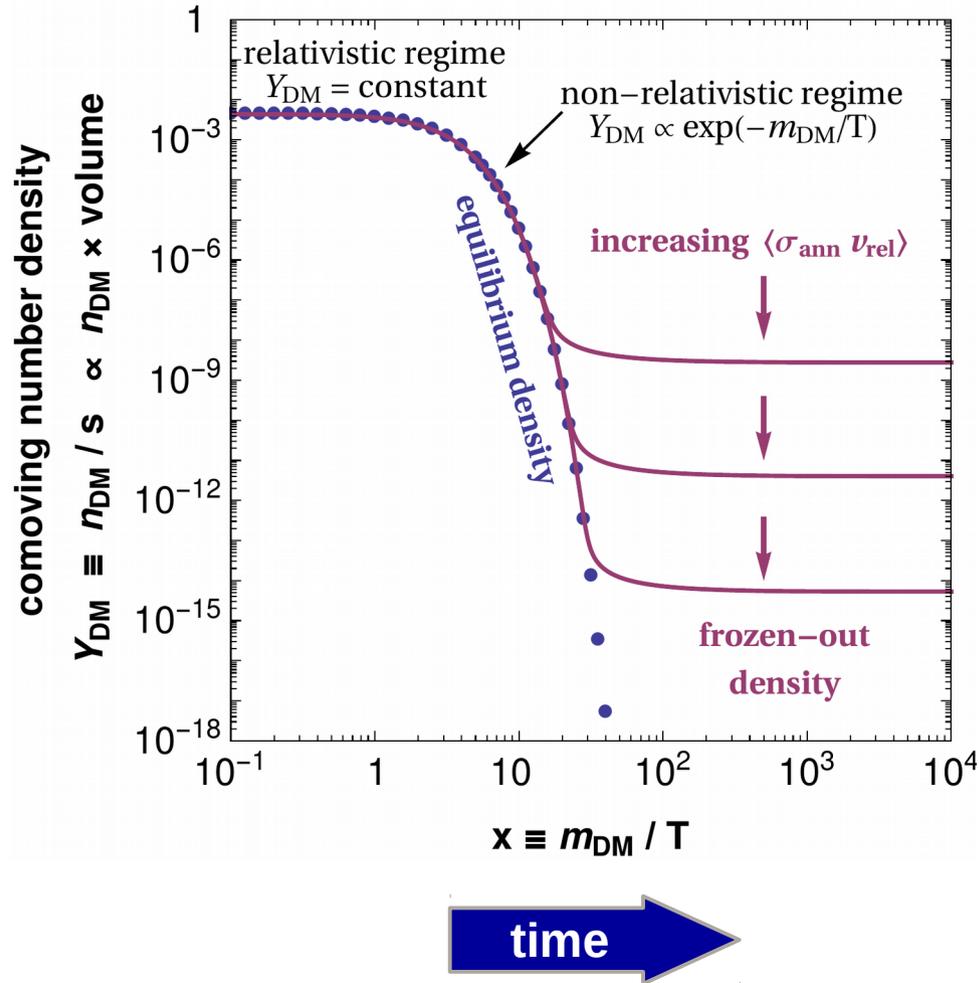
# Non-relativistic thermal freeze-out

## Motivation

**Very generic possibility  
within the standard cosmological story!**

# Non-relativistic thermal freeze-out

## How does it work?



- **At  $T > m_{\text{DM}} / 3$ :**

DM is kept in chemical & kinetic equilibrium with the plasma, via



$$n_{\text{DM}} \sim T^3 \quad \text{or} \quad Y_{\text{DM}} = \text{const.}$$

- **At  $T < m_{\text{DM}} / 3$ :**  $Y_{\text{DM}} \propto \exp(-m_{\text{DM}} / T)$ , while still in equilibrium

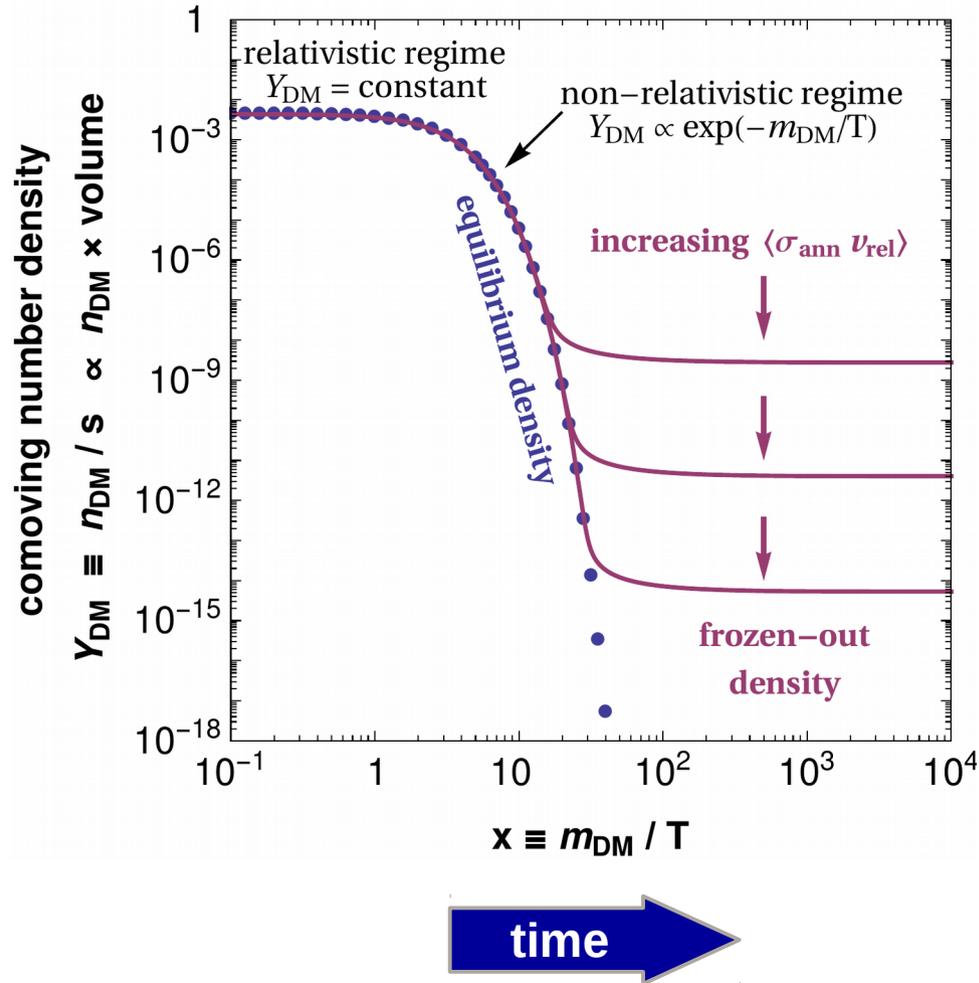
- **At  $T < m_{\text{DM}} / 25$ :** Density too small, annihilations stall  $\Rightarrow$  **Freeze-out!**

$$\Omega \simeq 0.26 \times \left( \frac{3 \times 10^{-26} \text{ cm}^3 / \text{s}}{\sigma_{\text{ann}} v_{\text{rel}}} \right)$$

“canonical”  
annihilation cross-section

# Non-relativistic thermal freeze-out

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# Non-relativistic thermal freeze-out

## How does it work?

[Gondolo, Gelmini (1990)]

- Boltzmann equation (no asymmetry,  $n_X = n_{\bar{X}}$ )

$$\frac{dn_X}{dt} + 3Hn_X = -\langle\sigma_{\text{ann}}v_{\text{rel}}\rangle (n_X^2 - n_X^{\text{eq}2})$$

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- Dilution due to the expansion of the universe

$$n_X \propto a^{-3} \quad \Rightarrow \quad \frac{1}{n_X} \frac{dn_X}{dt} = \frac{1}{a^{-3}} \frac{d(a^{-3})}{dt} = -\frac{3\dot{a}}{a} = -3H$$

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- Collision term: Annihilations  $X + \bar{X} \rightarrow f + \bar{f}$  ( $m_f < m_X$ )

$$\left. \frac{dN_X}{dtdV} \right|_{X\bar{X} \rightarrow f\bar{f}} = n_X n_{\bar{X}} \langle\sigma_{\text{ann}}v_{\text{rel}}\rangle = n_X^2 \langle\sigma_{\text{ann}}v_{\text{rel}}\rangle$$

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- Collision term: **Pair creation**  $f + \bar{f} \rightarrow X + \bar{X}$  ( $f, \bar{f}$  in equilibrium)

$$\begin{aligned} \left. \frac{dN_X}{dt dV} \right|_{f\bar{f} \rightarrow X\bar{X}} &= \langle\sigma_{f\bar{f} \rightarrow X\bar{X}}v_{\text{rel}}\rangle [n_f^{\text{eq}} n_{\bar{f}}^{\text{eq}}]_{E_f + E_{\bar{f}} = E_X + E_{\bar{X}}} \\ &= n_X^{\text{eq}} n_{\bar{X}}^{\text{eq}} \langle\sigma_{\text{ann}}v_{\text{rel}}\rangle \end{aligned}$$

using  
 $|\mathcal{M}|_{f\bar{f} \rightarrow X\bar{X}}^2 = |\mathcal{M}|_{X\bar{X} \rightarrow f\bar{f}}^2$

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Principle of detailed balance!

- Dilution due to the expansion of the universe

$$n_X \propto a^{-3} \Rightarrow \frac{1}{n_X} \frac{dn_X}{dt} = \frac{1}{a^{-3}} \frac{d(a^{-3})}{dt} = -\frac{3\dot{a}}{a} = -3H$$

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# Non-relativistic thermal freeze-out

## How does it work?

[Gondolo, Gelmini (1990)]

- **Initial conditions:** Thermal distribution

$$\text{At } T < m_X/3 : \quad n_X^{\text{eq}} = g_X \left( \frac{m_X T}{2\pi} \right)^{3/2} e^{-m_X/T}$$

Note: Boltzmann equation is self-correcting  $\Rightarrow$  It will restore  $n_X$  to the equilibrium distribution, even if it starts from non-equilibrium initial conditions, ***provided that collision terms are sufficiently large.***

That is, ***freeze-out has no memory*** (contrast with freeze-in later on).

# Non-relativistic thermal freeze-out

## How does it work?

[Gondolo, Gelmini (1990)]

- Factor out the expansion of the universe:

In thermal equilibrium, the total entropy  $S$  is conserved.  
The entropy density  $s = S/a^3$  follows

$$ds/dt = -3Hs$$

$$\dot{n}_x + 3Hn_x = s\dot{Y}_x$$

$$\dot{Y}_x = -s \langle \sigma v_{\text{rel}} \rangle (Y_x^2 - Y_x^{\text{eq}2})$$

$Y_x \equiv n_x/s \propto n_x a^3$   
co-moving number density

Boltzmann equation

The comoving density stays constant when annihilations become inefficient



freeze-out when  
 $n_x \langle \sigma_{\text{ann}} v_{\text{rel}} \rangle \lesssim H$

$$\Rightarrow \Omega_x = \frac{m_x Y_x s_0}{\rho_c} (\times 2)$$

entropy density of the universe today

if DM non-self-conjugate

# Non-relativistic thermal freeze-out

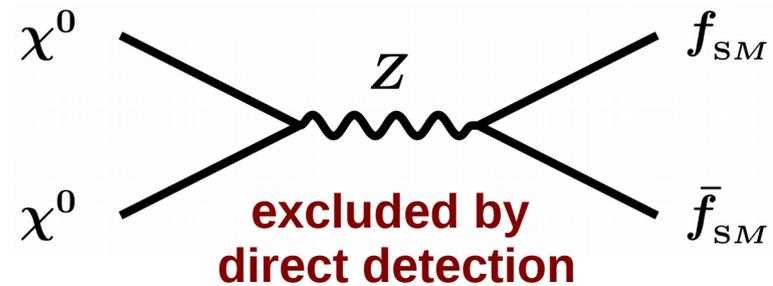
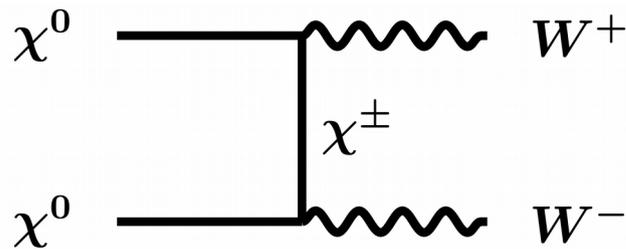
## Particle candidates

- Weakly interacting massive particles (WIMPs), coupled to the SM gauge interactions  $SU(2) \times U_Y(1)$ .

Motivated by the electroweak hierarchy problem.

⇒ Expect new physics at the electroweak scale.

Caveat: Not all WIMP scenarios address the hierarchy problem.



Expected annihilation cross-section:

$$\sigma_{\text{ann}} v_{\text{rel}} \sim \frac{\alpha_2^2}{m_W^2} \sim \mathcal{O}(pb) \quad [\text{if } m_{\text{DM}} \sim m_W]$$



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Caveat: Not all WIMP scenarios address the hierarchy problem.



Expected annihilation cross-section:

$$\sigma_{\text{ann}} v_{\text{rel}} \sim \frac{\alpha_2^2}{m_W^2} \sim \mathcal{O}(pb) \quad [\text{if } m_{\text{DM}} \sim m_W]$$

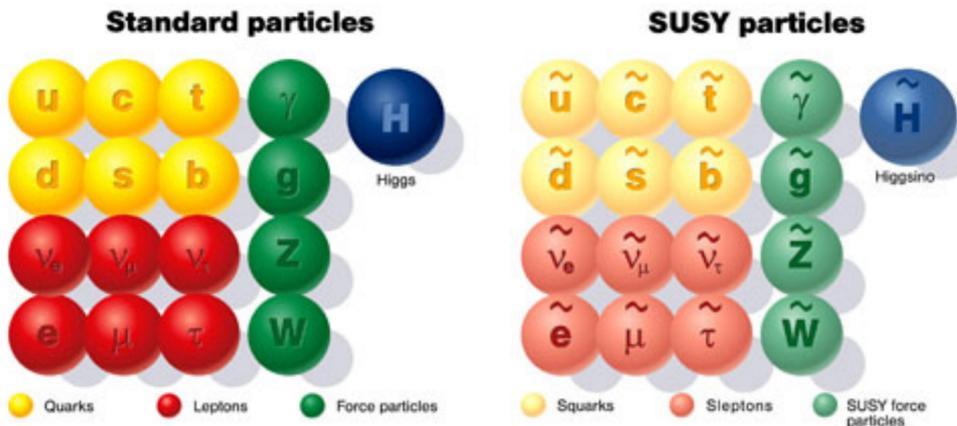


# Non-relativistic thermal freeze-out

## Particle candidates

- Weakly interacting massive particles (WIMPs), coupled to the SM gauge interactions  $SU(2) \times U_Y(1)$ .

### – Neutralino in SUSY models



Lightest SUSY particle  
stabilized by  $R$ -parity (not SUSY)

$$R = (-1)^{3(B-L)+2S}$$

SM particles: +1, SUSY particles: -1

The DM candidate is a linear combination of the neutral superpartners:

Bino, Wino, Higgsinos.

Plethora of annihilation channels.

Coannihilations potentially important, e.g. mass-degenerate neutralino-stop scenario.

Relic density within the MSSM parameter space varies by many orders of magnitude.

# Non-relativistic thermal freeze-out

## Particle candidates

- **Weakly interacting massive particles (WIMPs), coupled to the SM gauge interactions  $SU(2) \times U_Y(1)$ .**

- **Inert (Higgs) doublet models**

Stability due to  $Z_2$  symmetry. Mass  $\gtrsim 500$  GeV from freeze-out.

- **Minimal scenarios**

Neutral component of a pure  $SU(2)_L$  multiplet

$$\delta\mathcal{L} = \bar{X}(i\not{D} - m_x)X \quad \text{or} \quad (D_\mu X)(D^\mu X)^\dagger - m_x^2 |X|^2$$

Hypercharge determined by requiring one component to be neutral.

For stability against renormalizable and Planck-suppressed operators,  $X$  has to be a spin-(1/2) 5-plet (unless other physics invoked).

$m_x \sim \text{few TeV}$ , determined by freeze-out (Sommerfeld effect important [\*])

Pure-multiplet scenarios encountered also within the MSSM (e.g. Wino).

[\*] see discussion on long-range interactions

# Non-relativistic thermal freeze-out

## Particle candidates

- Massive particles interacting weakly with the SM, via new (non-SM) heavy mediators.

Such interactions are often parametrized with *effective operators*, e.g.

$$\delta\mathcal{L} \supset \frac{g_{XS}g_{qS}}{\Lambda_{\text{NP}}^2} \bar{X} X \bar{q} q, \quad \text{scalar med} \quad \frac{g_{XV}g_{qV}}{\Lambda_{\text{NP}}^2} \bar{X} \gamma^\mu X \bar{q} \gamma_\mu q, \quad \text{vector med}$$

(circled  $\Lambda_{\text{NP}}^2$ ) → scale of new physics

Many more effective operators. Useful for placing model-independent constraints. Caveats:

- EFT does not hold at high energies → important for LHC.
- UV-complete models may contain more than one of these interactions.

→ Marcela Carena's lecture

# Non-relativistic thermal freeze-out

## Particle candidates

- **Thermal relics of dark sectors, annihilating into new light species**

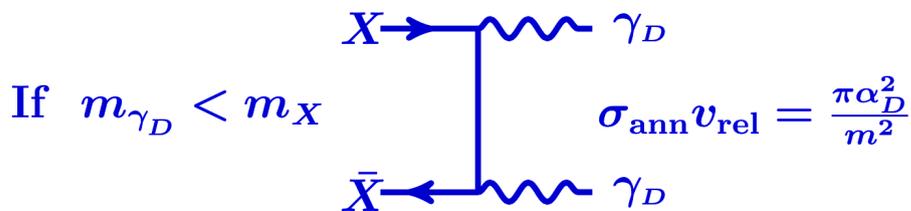
E.g. DM charged under a dark gauged  $U_D(1)$

$$\mathcal{L} = \bar{X}(i\not{D} - m_X)X - \frac{1}{4}F_D^{\mu\nu}F_{D\mu\nu} + m_{\gamma_D}^2 A_D^\mu A_{D\mu}$$

$$D^\mu = \partial^\mu + ig_D A_D^\mu$$

$$F_D^{\mu\nu} = \partial^\mu A_D^\nu - \partial^\nu A_D^\mu$$

Dark photon mass; generated via dark Higgs or Stueckelberg mechanisms



Freeze-out computation specifies  $\alpha_D = \alpha_D(m_X)$

The dark plasma may have different temperature than the SM. If in thermal contact early on,  $T_{\text{SM}}$  and  $T_D$  related via entropy conservation at later times

$$\frac{g_{*S}^D(T_D) T_D^3}{g_{*S}^{\text{SM}}(T_{\text{SM}}) T_{\text{SM}}^3} = \frac{g_{*S}^D(T_{\text{common}})}{g_{*S}^{\text{SM}}(T_{\text{common}})}$$

More to be said on this!

More complex dark sectors possible!

# Non-relativistic thermal freeze-out

## Particle candidates – summary

- **Weakly interacting massive particles (WIMPs)**, coupled to the SM gauge interactions  $SU(2) \times U_Y(1)$
- **Massive particles interacting weakly with & annihilating into the SM** via non-SM mediators
- **Thermal relics of dark sectors** [aka secluded, hidden/dark sector DM], annihilating into dark radiation

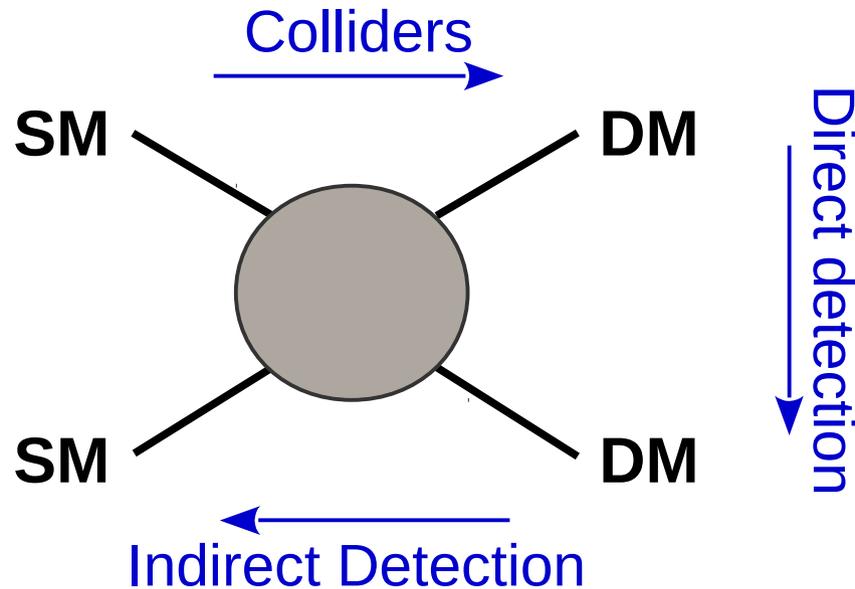
Often collectively referred to as  
“WIMPs”

*... but keep in mind ...*

they do NOT possess the same  
particle physics motivation,  
and may have very different pheno!

# Non-relativistic thermal freeze-out

## Main detection methods



See dedicated lectures by  
Matt Pyle,  
Christoph Weniger,  
Marcela Carena

- For secluded DM, detection requires that the dark mediators couple to the SM. In some models, cosmological considerations imply this must be so. Nevertheless, secluded models are harder to probe. Collider constraints in particular are much weaker.
- Same detection methods probe scenarios beyond WIMPs or thermal-relic DM.

## III. Production mechanisms and particle candidates

- Non-relativistic thermal freeze-out
- Asymmetry generation & freeze-out
- Freeze-in

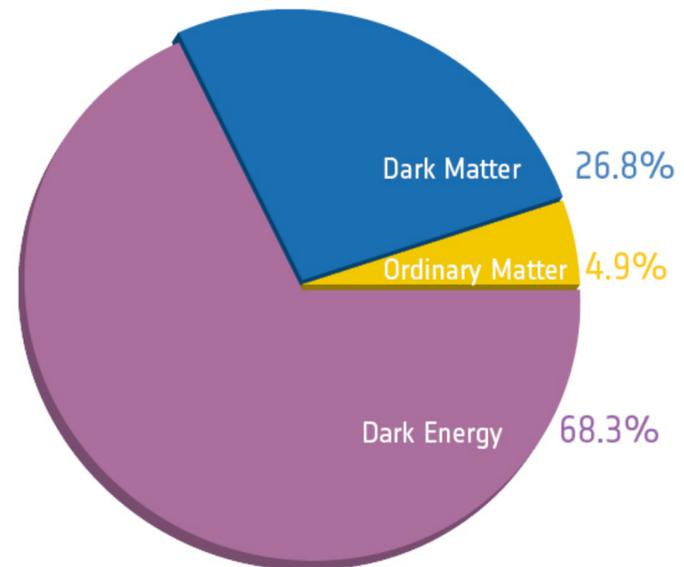
# Asymmetric dark matter

## Motivation

Reviews:  
arXiv:1305.4939  
arXiv:1308.0338

### A cosmic coincidence

Why  $\Omega_{\text{DM}} \sim \Omega_{\text{SM}}$  ?



- Unrelated mechanisms  $\rightarrow$  different parameters  
 $\rightarrow$  result expected to differ by orders of magnitude.
- Similarity of abundances hints towards **related physics** for OM and DM production.

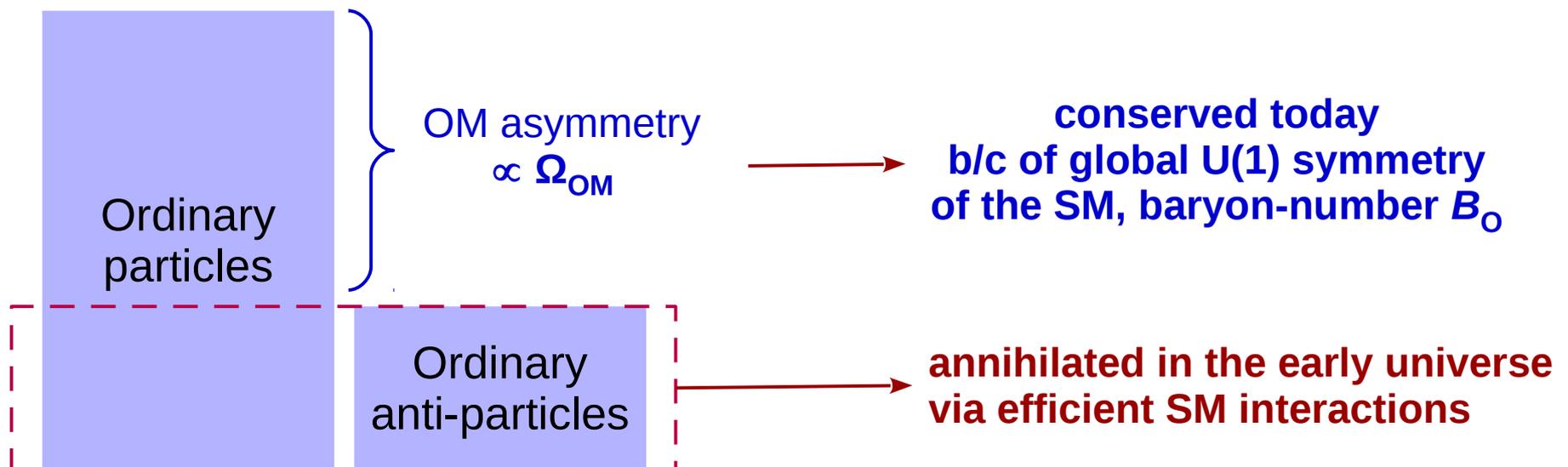
# Asymmetric dark matter

## Motivation

Reviews:  
arXiv:1305.4939  
arXiv:1308.0338

### A closer look at ordinary matter

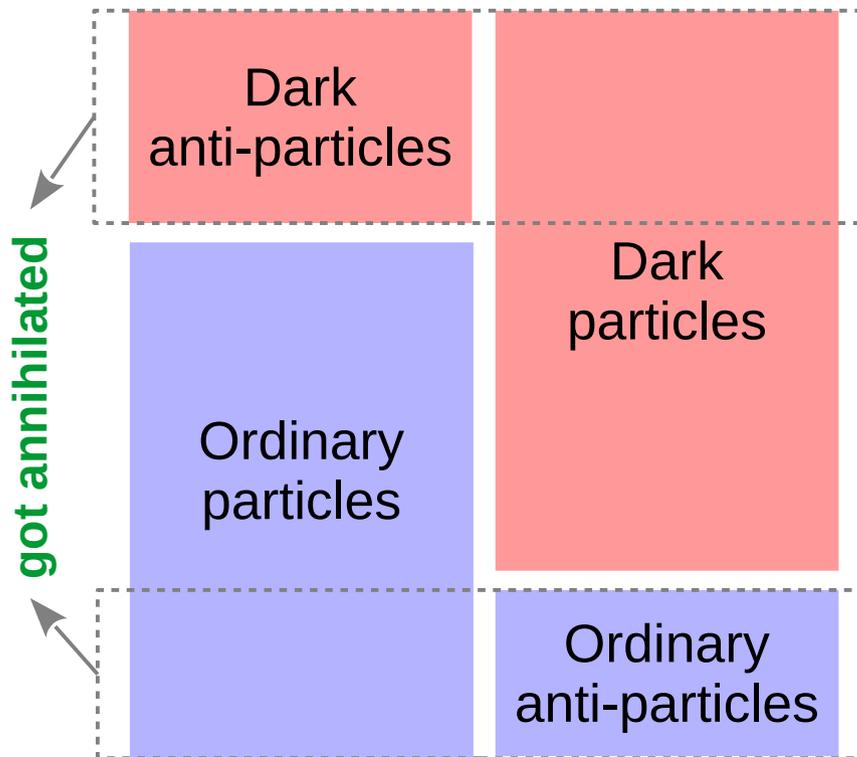
- **SM stable particles:**  $p, e, \gamma, \nu$
- **Protons** make up most of ordinary matter in the universe
- **Only  $p$ , no  $\bar{p}$**  present today: **matter-antimatter asymmetry**
  - Observational evidence: negligible antimatter in cosmic rays
  - Theoretical consistency:  $p - \bar{p}$  annihilation cross-section too large



# Asymmetric dark matter

## The proposal

Reviews:  
arXiv:1305.4939  
arXiv:1308.0338

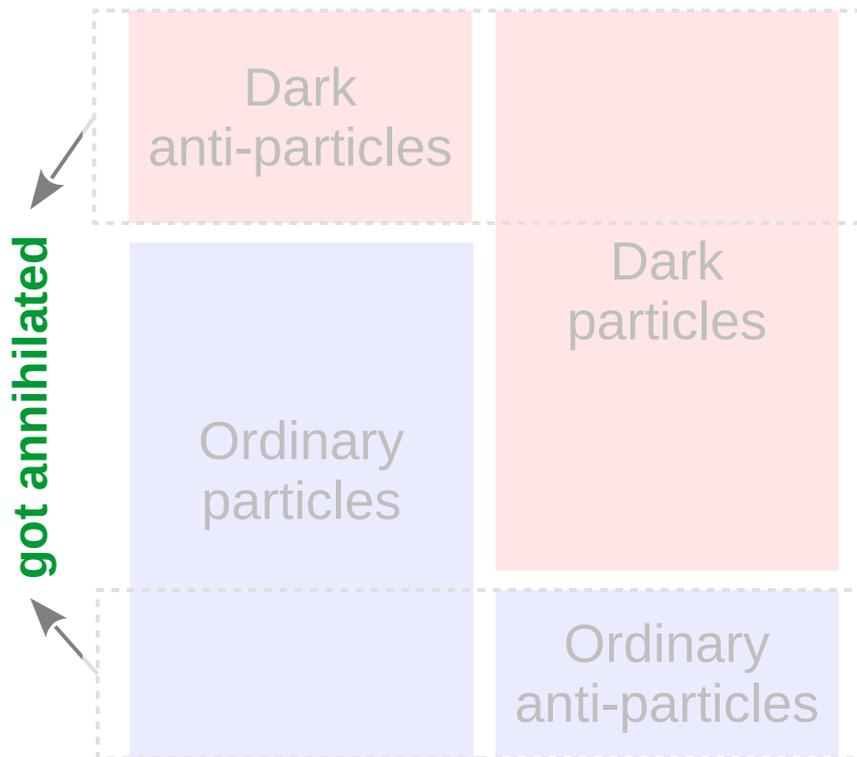


- DM relic density due to an excess of dark particles over antiparticles (asymmetry).
- Dark-ordinary asymmetries related dynamically, by processes which occurred in the early universe.
- Dark and visible asymmetries conserved separately today.

# Asymmetric dark matter

## The proposal

Reviews:  
arXiv:1305.4939  
arXiv:1308.0338



- DM relic density due to an excess of dark particles over antiparticles (asymmetry).

*Not necessary!  
ADM may be unrelated  
to ordinary matter*

- Dark and visible asymmetries conserved separately today.

# Asymmetry generation & freeze-out

## Ingredients of the scenario

Reviews:  
arXiv:1305.4939  
arXiv:1308.0338

- **Low-energy interactions** [operative at low temperatures]
  - Conserved particle numbers:  
Standard Model: Ordinary Baryon number  $B_{\text{SM}}$   
Dark sector: **Dark Baryon number  $B_{\text{D}}$  [global  $U(1)$  symmetry]**
  - **Interaction that annihilates efficiently the dark antiparticles.**  
How strong does it have to be?
- **High-energy interactions** [operative at high temperatures]
  - $B_{\text{SM}}$  violation + CP violation
  - $B_{\text{D}}$  violation + CP violation } if correlated  $\Rightarrow$  related  $\Delta B_{\text{SM}}$  &  $\Delta B_{\text{D}}$  asymmetries

# Asymmetry generation & freeze-out

## Structure

Reviews:  
 arXiv:1305.4939  
 arXiv:1308.0338



**CONNECTOR SECTOR**

particles with  $G_{SM}$ ,  $G_D$  and possibly  $G_{common}$

Interactions which break one linear combination of global symmetries:  
 conserved  $B_{SM} - B_D$ ; broken  $B_{SM} + B_D$   
 $\Rightarrow \Delta(B_V + B_D) = 2 \Delta B_{SM} = 2 \Delta B_D$

**STANDARD MODEL**

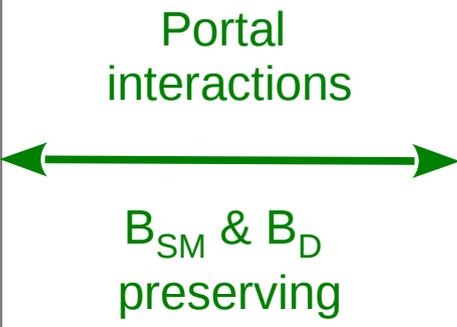
gauge group  
 $G_{SM} = SU(3)_C \times SU(2)_L \times U(1)_Y$

$\Rightarrow$  accidental global  $B_{SM}$   
 $\Rightarrow$  strong  $p\bar{p}$ ,  $n\bar{n}$  annihilation

**DARK SECTOR**

gauge group  $G_D$

$\Rightarrow$  accidental global  $B_D$   
 $\Rightarrow$  efficient annihilation



# Asymmetry generation & freeze-out

## How does it work?

Reviews:  
arXiv:1305.4939  
arXiv:1308.0338

### Order of events

- 
- **Asymmetry generation**, typically in heavy particles carrying  $B_D$ . Possible relation with  $B_{SM}$  generation.
  - $B_D$ -violating processes become inefficient.
  - **Cascade of  $\Delta B_D$  down to the lightest dark baryon (dark matter)**, via  $B_D$ -preserving decays and/or scattering processes.
  - **Annihilation of the symmetric component of dark matter.** Has to be efficient in order to
    - not produce too much DM
    - relate ordinary and dark matter number densities

How efficient does it have to be?

# Asymmetry generation & freeze-out

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Reviews:  
arXiv:1305.4939  
arXiv:1308.0338

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    - not produce too much DM
    - relate ordinary and dark matter number densitiesHow efficient does it have to be?

# Asymmetry generation & freeze-out

## How does it work?

Conditions for asymmetry generation [Sakharov 1967]

- **Violate global  $U(1)_B$  symmetry**  $\Rightarrow B_{\text{initial}} \neq B_{\text{final}}$
- **Violate CP symmetry** ( $C$  = charge conjugation,  $P$  = space parity)
  - process  $S$ :  $\Delta B$
  - process  $S$  conjugate:  $-\Delta B$ } have to happen at different rate  $\Rightarrow$  need CP violation
- **Occur out of equilibrium** (arrow of time, expansion of the universe)
  - process  $S$ :  $\Delta B$
  - process  $S$  inverse:  $-\Delta B$ } have to happen at different rate  $\Rightarrow$  out of equilibrium

These processes must be ineffective in the low-energy environment of today's universe, such that today  $\Delta B = \text{constant}$ .

# Asymmetry generation & freeze-out

## How does it work?

### Freeze-out in the presence of an asymmetry

arXiv:1103.2771  
arXiv:1703.00478

$$\frac{dn_X}{dt} + 3Hn_X = -\langle\sigma_{\text{ann}}v_{\text{rel}}\rangle (n_X n_{\bar{X}} - n_X^{\text{eq}} n_{\bar{X}}^{\text{eq}})$$

$$\frac{dn_{\bar{X}}}{dt} + 3Hn_{\bar{X}} = -\langle\sigma_{\text{ann}}v_{\text{rel}}\rangle (n_X n_{\bar{X}} - n_X^{\text{eq}} n_{\bar{X}}^{\text{eq}})$$

where

$$n_X^{\text{eq}} = g_X \left(\frac{m_X T}{2\pi}\right)^{3/2} e^{-(m_X - \mu)/T} \quad n_{\bar{X}}^{\text{eq}} = g_X \left(\frac{m_X T}{2\pi}\right)^{3/2} e^{-(m_X + \mu)/T}$$

Conserved quantity

$$\eta_X \equiv \frac{n_X - n_{\bar{X}}}{s}$$

determined during  
asymmetry gen

(for ordinary baryons,  
 $\eta_B \approx 10^{-10}$ )

Need to compute

$$r(t) = \frac{n_{\bar{X}}}{n_X}$$

$r = 1$  : symm DM  
 $r \ll 1$  : ADM

Want to find

$$r_\infty = r(t \rightarrow \infty)$$

expectation:

large  $\sigma_{\text{ann}} \Rightarrow r_\infty \ll 1$   
small  $\sigma_{\text{ann}} \Rightarrow r_\infty \approx 1$

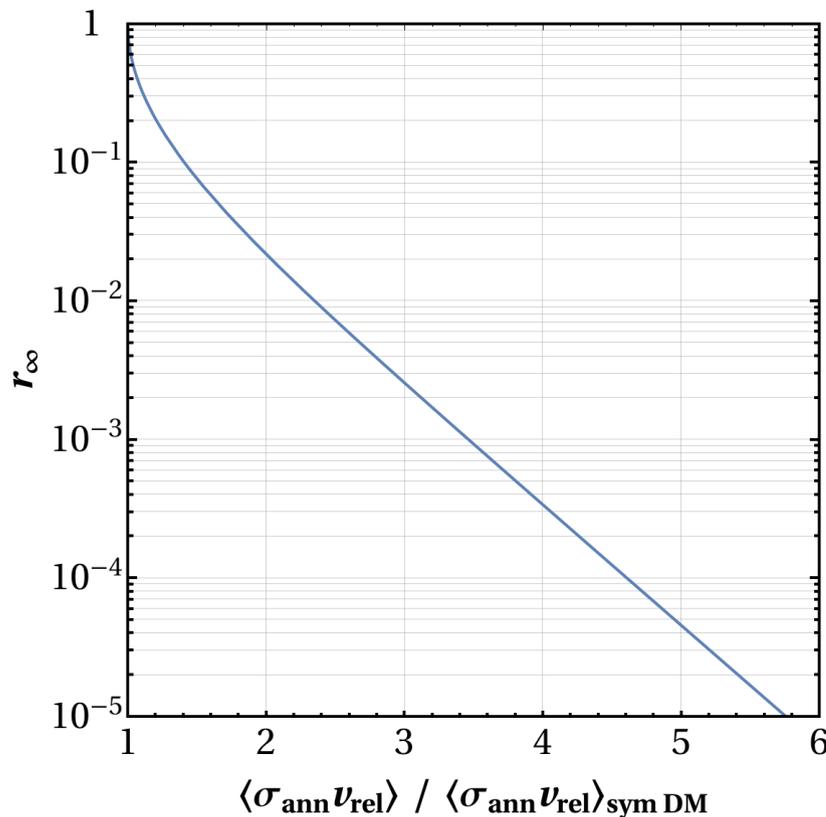
# Asymmetry generation & freeze-out

## How does it work?

### Freeze-out in the presence of an asymmetry

arXiv:1103.2771  
arXiv:1703.00478

$$r_\infty \approx \exp \left[ -2 \left( \frac{1 - r_\infty}{1 + r_\infty} \right) \left( \frac{\langle \sigma_{\text{ann}} v_{\text{rel}} \rangle}{\langle \sigma_{\text{ann}} v_{\text{rel}} \rangle_{\text{sym DM}}} \right) \right]$$



We need  $\sigma_{\text{ann}} > \sigma_{\text{ann, sym}}$ ,  
but not by much!

Even  $\sigma_{\text{ann}} / \sigma_{\text{ann, sym}} \simeq 2$ ,  
yields  $r_\infty \ll 1$ .

- ◇ Asymmetric WIMP DM possible!
- ◇ Lower, but no upper limit on  $\sigma_{\text{ann}}$ .

# Asymmetric dark matter

## Particle candidates

Reviews:  
arXiv:1305.4939  
arXiv:1308.0338

- Technicolor [Nussinov 1985; Sannino, Kouvaris et al. 2006]
  - Mirror DM: dark sector = a copy of the SM with different cosmology [1990s: Foot, Volkas; Mohapatra et al]
  - Q-balls (entirely non-thermal) [Kusenko 1998]
  - WIMP ADM
  - Various hidden sector models, e.g. dark U(1), dark QCD
- } Many papers in recent years

# Asymmetric dark matter

## Example: dark U(1) sector

$$G = G_{\text{SM}} \times U(1)_D \times U(1)_{B_{\text{gen}}}$$

Dark gauge force:  
provides efficient DM annihilation,  
and DM self-interactions

Generalized baryon number;  
same as  $(B-L)_{\text{SM}}$  for SM particles

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*Asymmetry and gauge invariance*  
imply that  
there must be at least **two dark species**  
whose net dark electric charges  
compensate each other



	gauged $D$	gauged $B_{\text{gen}}$
$p_D$	1	-2
$e_D$	-1	0

can be elementary

# Asymmetric dark matter

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	gauged $D$	gauged $B_{\text{gen}}$	accidental global $B_d$
$p_D$	1	-2	2
$e_D$	-1	0	0

can be elementary

accidental global  $(B - L)_{\text{SM}}$  &  $B_d$

$$\mathcal{L} = \mathcal{L}_{\text{SM}} - \frac{1}{4} F_D^{\mu\nu} F_{D\mu\nu} + \bar{p}_D (i\not{D} - m_{p_d}) p_D + \bar{e}_D (i\not{D} - m_{e_d}) e_D$$

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$$\mathcal{L} = \mathcal{L}_{\text{SM}} - \frac{1}{4} F_D^{\mu\nu} F_{D\mu\nu} + \bar{p}_D (i\not{D} - m_{p_d}) p_D + \bar{e}_D (i\not{D} - m_{e_d}) e_D$$

$$+ \frac{1}{\Lambda^8} (\bar{u}^c d \bar{s}^c u \bar{d}^c s) \bar{e}_D^c p_D + h.c.$$

preserves  $B_{\text{SM}} - B_d$

violates  $B_{\text{SM}} + B_d$

↓

may give rise to  $\Delta B_{\text{SM}} = \Delta B_d$

# Asymmetric dark matter

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can be elementary

accidental global  $(B - L)_{\text{SM}}$  &  $B_d$

$$\mathcal{L} = \mathcal{L}_{\text{SM}} - \frac{1}{4} F_D^{\mu\nu} F_{D\mu\nu} + \bar{p}_D (i\not{D} - m_{p_d}) p_D + \bar{e}_D (i\not{D} - m_{e_d}) e_D + \epsilon F_Y^{\mu\nu} F_{D\mu\nu}$$

$$+ \frac{1}{\Lambda^8} (\bar{u}^c d \bar{s}^c u \bar{d}^c s) \bar{e}_D^c p_D + h.c.$$

preserves  $B_{\text{SM}} - B_d$   
violates  $B_{\text{SM}} + B_d$

portal interaction:  
direct/indirect detection

may give rise to  $\Delta B_{\text{SM}} = \Delta B_d$

# Asymmetric dark matter

## Example: dark U(1) sector

### COSMOLOGY

[see e.g. arXiv:1209.5752]

Asymmetry generation	$T_{\text{asym}} \gg m_{p_D}, m_{e_D}$
Freeze-out of annihilations $p_D + \bar{p}_D \rightarrow \gamma_D \gamma_D$ $e_D + \bar{e}_D \rightarrow \gamma_D \gamma_D$	$T_{p_D}^{\text{FO}} \sim m_{p_D} / 25$ $T_{e_D}^{\text{FO}} \sim m_{e_D} / 25$
Dark recombination, $p_D + e_D \rightarrow H_D + \gamma_D$	$T_{\text{dark rec}} \sim \text{binding energy}$ $= \frac{1}{2} \left( \frac{m_{p_D} m_{e_D}}{m_{p_D} + m_{e_D}} \right) \alpha_D^2$

### Residual ionization fraction

$$f_{\text{ion}} \equiv \frac{n(p_D)}{n(p_D) + n(H_D)} \sim \max \left[ 1, \frac{10^{-10}}{\alpha_D^4} \left( \frac{m_{p_D} m_{e_D}}{\text{GeV}^2} \right) \right]$$

Similar situation if  $U(1)_D$  is mildly broken and  $\gamma_D$  has a small mass.

# Asymmetric dark matter

## Example: dark U(1) sector

### PHENOMENOLOGY

- **Self-interactions inside halos**

- Multicomponent DM:  $p_D, e_D, H_D \Rightarrow$  ion-ion, atom-atom & atom-ion collisions.
- Velocity-dependent scattering cross-sections.
- Interplay between cosmology and particle physics regulates  $\sigma_{\text{scatt}}$ :

Small  $\alpha_D \Rightarrow$  mostly ionized DM,  $\sigma_{\text{scatt}} \propto \alpha_D^2 / v_{\text{rel}}^4$

Large  $\alpha_D \Rightarrow$  mostly atomic DM, scattering screened.

- **Direct detection, via kinetic mixing**  $\delta\mathcal{L} = \epsilon F_Y^{\mu\nu} F_{D\mu\nu}$

Similar features as DM self-scattering

# Asymmetric dark matter

## Example: dark U(1) sector

### PHENOMENOLOGY

- **Indirect detection**, via kinetic mixing if dark photon is massive

- Annihilations of residual symmetric component,

$$p_D + \bar{p}_D \rightarrow \gamma_D + \gamma_D$$

$$\gamma_D \rightarrow f_{SM}^+ f_{SM}^-$$

Rate suppressed by asymmetry, but enhanced by Sommerfeld effect [\*]  
due to light dark photon. [\[arXiv:1703.00478,1712.07489\]](#)

- Radiative level transitions, e.g. dark atom formation from residual ionized component (also Sommerfeld-enhanced [\*])

$$p_D + e_D \rightarrow H_D + \gamma_D$$

$$\gamma_D \rightarrow f_{SM}^+ f_{SM}^-$$

[\[arXiv:1303.7294;](#)  
[arXiv:1404.3729;](#)  
[arXiv:1406.2276;](#)  
[arXiv:1502.01755\]](#)

[\*] see discussion on long-range interactions

# Asymmetric dark matter

## Example: dark U(1) sector

### PHENOMENOLOGY

- **Indirect detection**, via kinetic mixing if dark photon is massive
    - Annihilations of residual symmetric component,
$$p_D + p_D \rightarrow \gamma_D + \gamma_D$$
$$\gamma_D \rightarrow f_{SM}^+ f_{SM}^-$$

Rate suppressed by asymmetry, but enhanced by Sommerfeld effect due to residual symmetric component
    - Radiative level transitions from residual ionized component (also Sommerfeld enhanced)
$$p_D + e_D \rightarrow H_D + \gamma_D$$
$$\gamma_D \rightarrow f_{SM}^+ f_{SM}^-$$
- The features we saw in the context of atomic DM,
- multicomponent DM
  - velocity-dependent cross-sections
  - radiative level transitions
- are typical of asymmetric DM models coupled to light force mediators!

## III. Production mechanisms and particle candidates

- Non-relativistic thermal freeze-out
- Asymmetry generation & freeze-out
- Freeze-in

# Freeze-in

## Motivation

- What if DM couples very weakly to all other particles?  
⇒ Never in thermal equilibrium in the early universe
- Can it still be produced in sizable amounts?  
⇒ DM may be produced in various scattering/decay processes, without ever reaching a density so large that its self-annihilations destroy it.

# Freeze-in

## How does it work?

- Boltzmann equations**

scatterings of bath particles	$\frac{dn_X}{dt} + 3Hn_X = \langle \sigma_{\psi\bar{\psi} \rightarrow X\bar{X}} v_{\text{rel}} \rangle n_{\psi}^{\text{eq}2} - \langle \sigma_{X\bar{X} \rightarrow \psi\bar{\psi}} v_{\text{rel}} \rangle n_X^2$
$\psi + \bar{\psi} \rightarrow X + \bar{X}$	$= \langle \sigma_{X\bar{X} \rightarrow \psi\bar{\psi}} v_{\text{rel}} \rangle (n_X^{\text{eq}2} - n_X^2)$

decay of bath particles	$\frac{dn_X}{dt} + 3Hn_X = \langle \Gamma_{\psi \rightarrow X\bar{X}} \rangle n_{\psi} - \langle \sigma_{X\bar{X} \rightarrow \psi} v_{\text{rel}} \rangle n_X^2$
$\psi \rightarrow X + \bar{X}$	$= \langle \gamma_{\psi}^{-1} \rangle \Gamma_{\psi \rightarrow X\bar{X}}^0 \left[ n_{\psi} - n_{\psi}^{\text{eq}} (n_X^2 / n_X^{\text{eq}2}) \right]$

Many variations, e.g.  $\psi_1 + \psi_2 \rightarrow X + \psi_3 + \dots$ ,  $\psi_1 \rightarrow X + \psi_2$ , and/or  $\psi_j$  may be out of equilibrium with the plasma (but may have a thermal history). Boltzmann equations change accordingly.

# Freeze-in

## How does it work?

- Boltzmann equations**

scatterings of bath particles $\psi + \bar{\psi} \rightarrow X + \bar{X}$	$\frac{dn_X}{dt} + 3Hn_X = \langle \sigma_{\psi\bar{\psi} \rightarrow X\bar{X}} v_{\text{rel}} \rangle n_{\psi}^{\text{eq}^2} - \langle \sigma_{X\bar{X} \rightarrow \psi\bar{\psi}} v_{\text{rel}} \rangle n_X^2$ $= \langle \sigma_{X\bar{X} \rightarrow \psi\bar{\psi}} v_{\text{rel}} \rangle (n_X^{\text{eq}^2} - n_X^2)$
---	--

decay of bath particles $\psi \rightarrow X + \bar{X}$	$\frac{dn_X}{dt} + 3Hn_X = \langle \Gamma_{\psi \rightarrow X\bar{X}} \rangle n_{\psi} - \langle \sigma_{X\bar{X} \rightarrow \psi} v_{\text{rel}} \rangle n_X^2$ $= \langle \gamma_{\psi}^{-1} \rangle \Gamma_{\psi \rightarrow X\bar{X}}^0 \left[ n_{\psi} - n_{\psi}^{\text{eq}} (n_X^2 / n_X^{\text{eq}^2}) \right]$
--	--

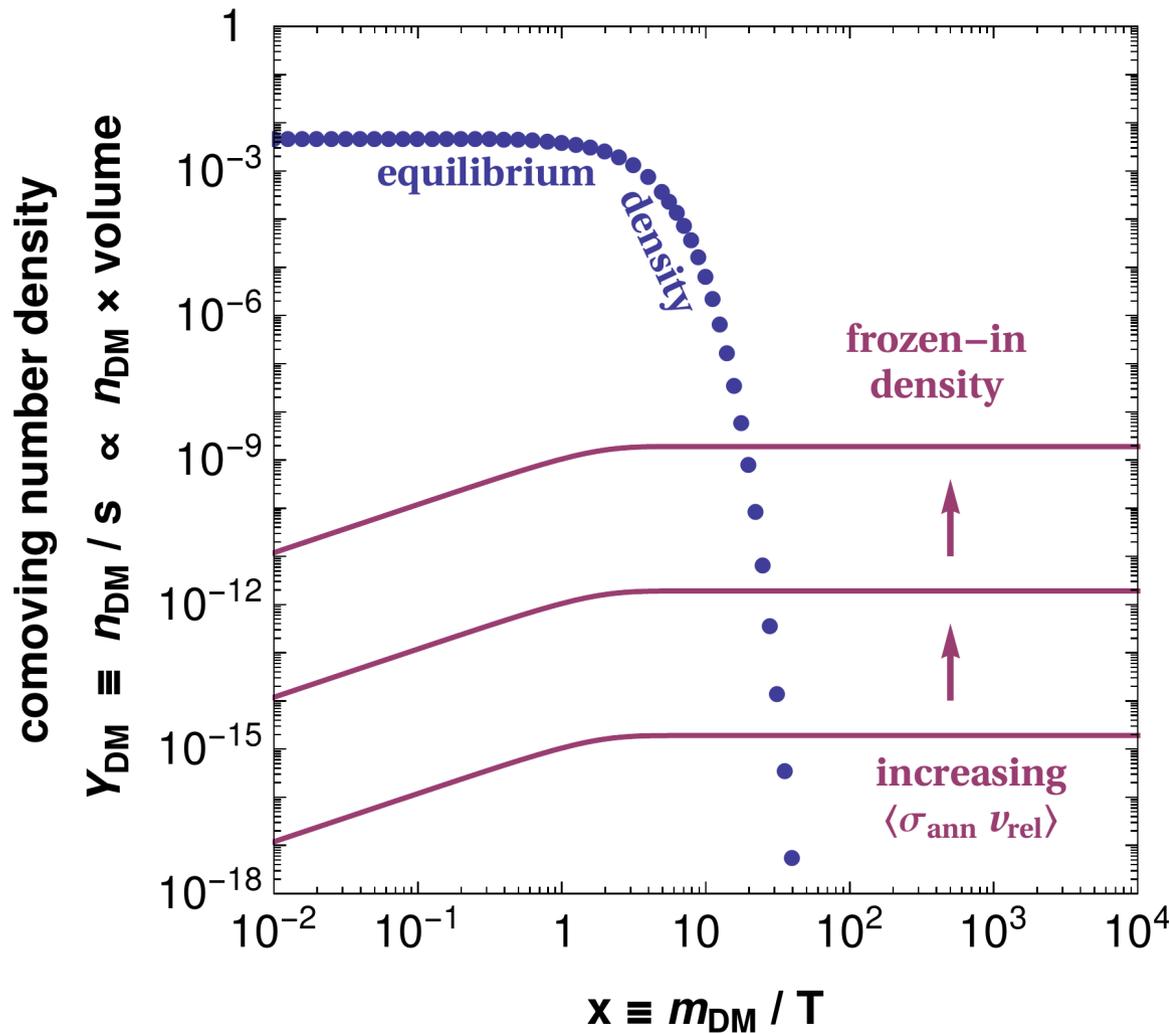
Many variations, e.g.  $\psi_1 + \psi_2 \rightarrow X + \psi_3 + \dots$ ,  $\psi_1 \rightarrow X + \psi_2$ , and/or  $\psi_j$  may be out of equilibrium with the plasma (but may have a thermal history). Boltzmann equations change accordingly.

- Initial conditions**

$$n_X \ll n_X^{\text{eq}} \quad \text{at high } T$$

# Freeze-in

## How does it work?



Relic density increases with cross-section / decay rate.

*Freeze-in has memory:*  
Initial DM density (produced during reheating) matters!

# Freeze-in Candidates

- Sterile neutrinos with mass  $M \sim \text{keV}$  and mixing  $\theta^2 \sim 10^{-10}$ 
  - Produced via oscillations from active neutrinos [\[arXiv:hep-ph/9303287\]](#)

$$\delta\mathcal{L} = \underbrace{-y_{\alpha a} \epsilon^{ij} (\bar{L}_\alpha)_i H_j N_a}_{\text{Neutrino mass mixing}} - \frac{M_a}{2} \bar{N}_a^c N_a + h.c. \Rightarrow \begin{array}{l} \text{mass eigenstates:} \\ m_\nu \sim y^2 v_H^2 / M, \quad M_N \sim M \\ \text{mixing: } \theta \sim y v_H / M \end{array}$$

# Freeze-in Candidates

- Sterile neutrinos with mass  $M \sim \text{keV}$  and mixing  $\theta^2 \sim 10^{-10}$

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- Produced in the decays of heavy scalars [\[arXiv:hep-ph/0609081\]](#)

$$\delta\mathcal{L} = \underbrace{-\frac{\lambda}{2} |S|^2 |H|^2}_{\text{Coupling to the Higgs brings } S \text{ in equilibrium with SM bath}} - \frac{f_a}{2} S \bar{N}_a^c N_a - \underbrace{y_{\alpha a} \epsilon^{ij} (\bar{L}_\alpha)_i H_j N_a}_{\text{Majorana mass } M_N \sim f \langle S \rangle, S \rightarrow NN \text{ decays produce DM.}}$$

# Freeze-in Candidates

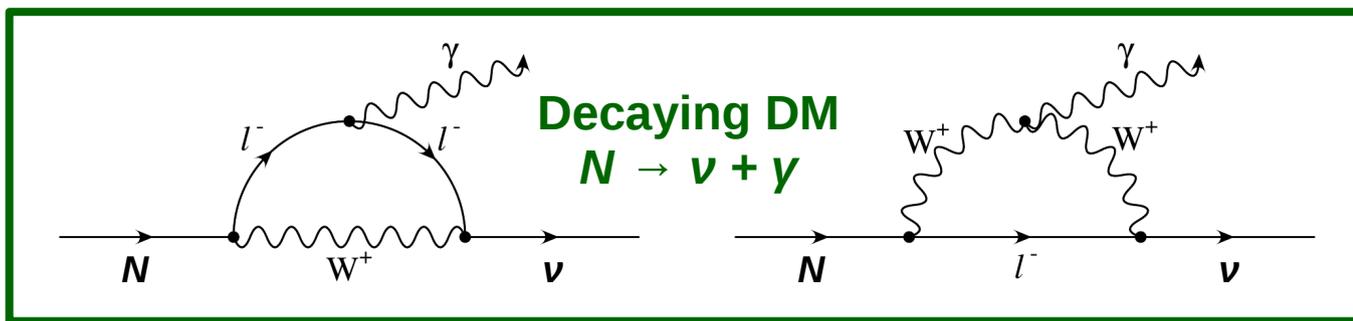
- Sterile neutrinos with mass  $M \sim \text{keV}$  and mixing  $\theta^2 \sim 10^{-10}$

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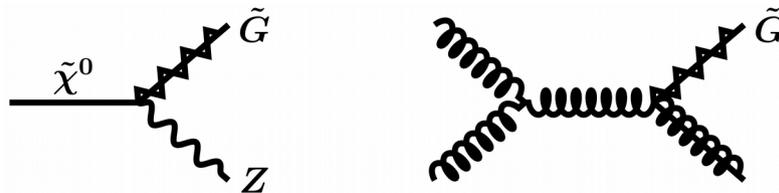
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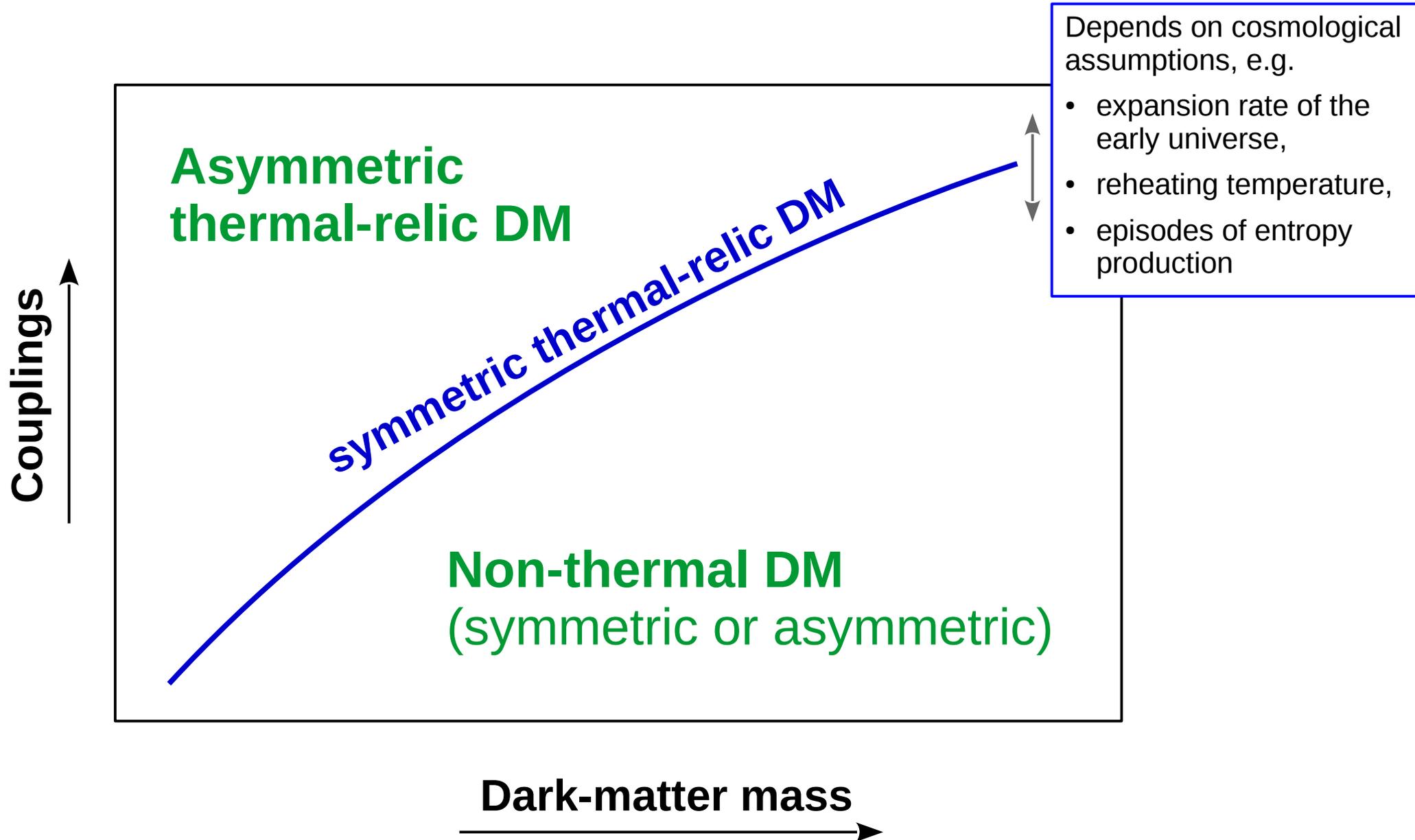
# Freeze-in Candidates

- **Gravitinos** produced in scatterings and decays involving SM and SUSY particles



Stabilized by  $R$ -parity. Typically gravitinos are the LSP in gauge-mediated SUSY-breaking scenarios.

# Phase space of a DM theory



## **IV. Interaction types**

# Contact-type vs long-range interactions

## Definition

- **Contact-type interactions:**  $m_{\text{DM}} \lesssim m_{\text{mediators}}$

E.g. in the prototypical WIMP scenario,  $m_{\text{DM}} \sim m_{\text{W,Z}} \sim 100 \text{ GeV}$

- **Long-range interactions:**  $m_{\text{DM}} \gg m_{\text{mediators}}$

How does the DM phenomenology look like?

**Non-perturbative effects:** Sommerfeld enhancement and bound states

⇒ New radiative processes and modified rates

# Light/Massless mediators

## Motivation

- Self-interacting DM
- Hidden-sector models inspired by GUTs or string theory
- Models designed to explain astrophysical anomalies (e.g. Pamela positrons, IceCube PeV neutrinos)

*... but even ...*

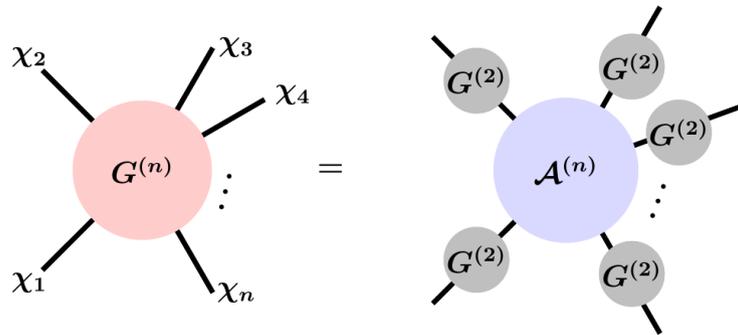
- WIMP DM with  $m_{\text{DM}} > \text{TeV}$
- WIMP DM with  $m_{\text{DM}} < \text{TeV}$  co-annihilating with colored/charged partners

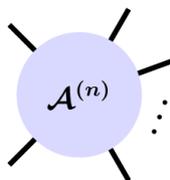
# *Contact-type vs long-range interactions*

## **Scattering processes**

# Contact-type vs long-range interactions

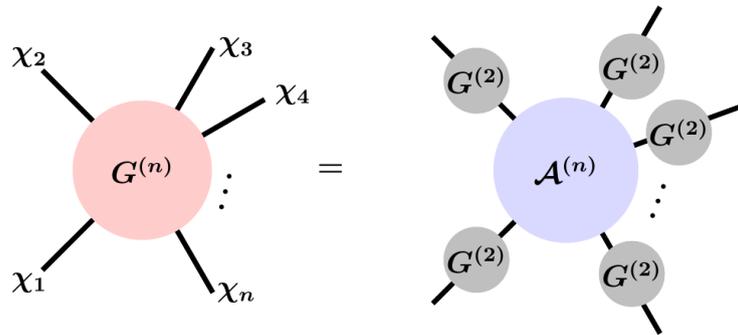
## Scattering processes

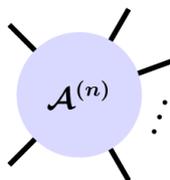


where  includes all connected diagrams with the 1PI factors amputated.

# Contact-type vs long-range interactions

## Scattering processes



where  includes all connected diagrams with the 1PI factors amputated.

The properties of the asymptotic states are determined by resumming the self-interactions at infinity, via the Dyson-Schwinger equation

$$\begin{aligned}
 \text{---} G^{(2)} \text{---} &= \text{---} + \text{---} \textcircled{1PI} \text{---} + \text{---} \textcircled{1PI} \textcircled{1PI} \text{---} + \dots \\
 &= \text{---} + \text{---} \textcircled{1PI} G^{(2)} \text{---} \\
 &= \frac{iZ_j}{p_j^2 - m_j^2}
 \end{aligned}$$

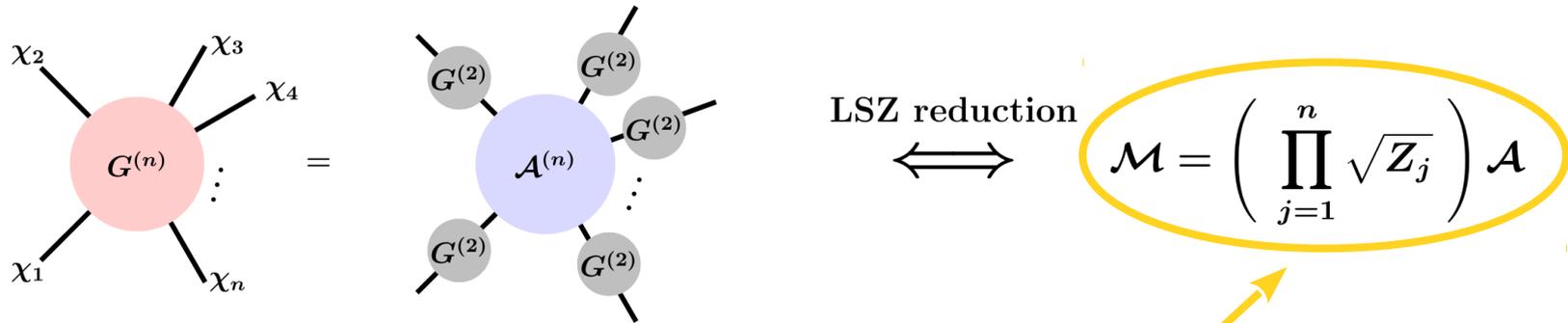
where e.g.  $\text{---} \textcircled{1PI} \text{---} = \text{---} \text{---}$

Field strength renormalization factor

Renormalized mass

# Contact-type vs long-range interactions

## Scattering processes



where  $\mathcal{A}^{(n)}$  includes all connected diagrams with the 1PI factors amputated.

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$$\begin{aligned}
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 &= \text{---} + \text{---} \text{---} \text{1PI} \text{---} G^{(2)} \text{---} \\
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# *Contact-type vs long-range interactions*

## **Scattering processes**

The particles interact at very large distance. We cannot define the asymptotic states by isolating the particles at infinity.

What do we do?

# *Contact-type vs long-range interactions*

## **Scattering processes**

The particles interact at very large distance. We cannot define the asymptotic states by isolating the particles at infinity.

What do we do?

**Resum 2-particle interactions at infinity!**

# Contact-type vs long-range interactions

## Scattering processes

$$\begin{aligned}
 \text{---} G^{(2)} \text{---} &= \text{---} + \text{---} \textcircled{1\text{PI}} \text{---} + \text{---} \textcircled{1\text{PI}} \textcircled{1\text{PI}} \text{---} + \dots \\
 &= \text{---} + \text{---} \textcircled{1\text{PI}} G^{(2)} \text{---}
 \end{aligned}$$

where e.g.  $\text{---} \textcircled{1\text{PI}} \text{---} = \text{---} \text{---}$

$$\mathcal{M} = \left( \prod_{j=1}^n \sqrt{Z_j} \right) \mathcal{A}$$

$$\begin{aligned}
 \text{---} G^{(4)} \text{---} &= \text{---} + \text{---} \text{---} \text{---} \text{---} + \text{---} \text{---} \text{---} \text{---} \text{---} + \dots \\
 &= \text{---} + \text{---} \text{---} \text{---} G^{(4)} \text{---}
 \end{aligned}$$

where e.g.  $\text{---} \text{---} \text{---} \text{---} = \text{---} \text{---}$

$$\mathcal{M} = \int \frac{d^3q}{(2\pi)^3} \phi_{\vec{k}}(\vec{q}) \mathcal{A}(\vec{q})$$

field strength renormalization factors / form factors / wavefunctions

$$G^{(2)} \sim Z/\text{singularity} \leftrightarrow G^{(4)} \sim [\phi_{\vec{k}}]^2/\text{singularity}$$

# Contact-type vs long-range interactions

## Scattering processes

$$\begin{aligned}
 \text{---} G^{(2)} \text{---} &= \text{---} + \text{---} \textcircled{1\text{PI}} \text{---} + \text{---} \textcircled{1\text{PI}} \textcircled{1\text{PI}} \text{---} + \dots \\
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 \end{aligned}$$

where e.g.  $\text{---} \text{---} \text{---} = \text{---} \text{---} \text{---}$

Dyson eq for  
for  $G^{(4)}$   
↓  
Schrödinger eq  
for  $\phi_{\vec{k}}$

$$\mathcal{M} = \int \frac{d^3q}{(2\pi)^3} \phi_{\vec{k}}(\vec{q}) \mathcal{A}(\vec{q})$$

# Contact-type vs long-range interactions

## Scattering processes

$$\begin{aligned}
 \text{---} G^{(2)} \text{---} &= \text{---} + \text{---} \textcircled{1\text{PI}} \text{---} + \text{---} \textcircled{1\text{PI}} \textcircled{1\text{PI}} \text{---} + \dots \\
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where e.g.  $\text{---} \text{---} \text{---} = \text{---} \text{---}$

$$\mathcal{M} = \int \frac{d^3q}{(2\pi)^3} \phi_{\vec{k}}(\vec{q}) \mathcal{A}(\vec{q})$$

Expectation value of  
relative momentum

Relative momentum of  
interacting particles

No long-range force:  $\phi_{\vec{k}}(\vec{q}) = \delta^3(\vec{q} - \vec{k}) \quad \overset{\text{FT}}{\Leftrightarrow} \quad \tilde{\phi}_{\vec{k}}(\vec{r}) = e^{-i\vec{k}\cdot\vec{r}}$ .

In the presence of a long-range interaction:  $\tilde{\phi}_{\vec{k}}(\vec{r})$  is not a plane wave.

# Long-range interactions

## Phenomenological implications

### Case study: dark QED

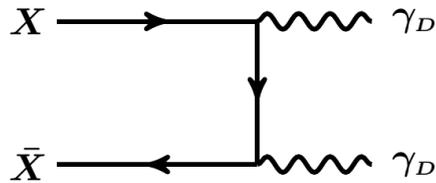
$$\mathcal{L} = \bar{X}(i\not{D} - m_X)X - \frac{1}{4}F_D^{\mu\nu}F_{D\mu\nu}$$
$$D^\mu = \partial^\mu + ig_D A_D^\mu; \quad \alpha_D \equiv g_D^2 / (4\pi)$$
$$F_D^{\mu\nu} = \partial^\mu A_D^\nu - \partial^\nu A_D^\mu$$

# Long-range interactions

## Modified interaction rates

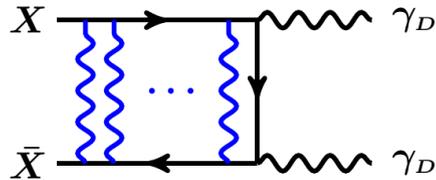
Annihilation:  $X\bar{X} \rightarrow \gamma_D\gamma_D$

Standard  
perturbative  
calculation



$$\sigma_{\text{ann}} v_{\text{rel}} = \frac{\pi \alpha_D^2}{m_X^2} \equiv \sigma_0$$

With  
Sommerfeld  
effect

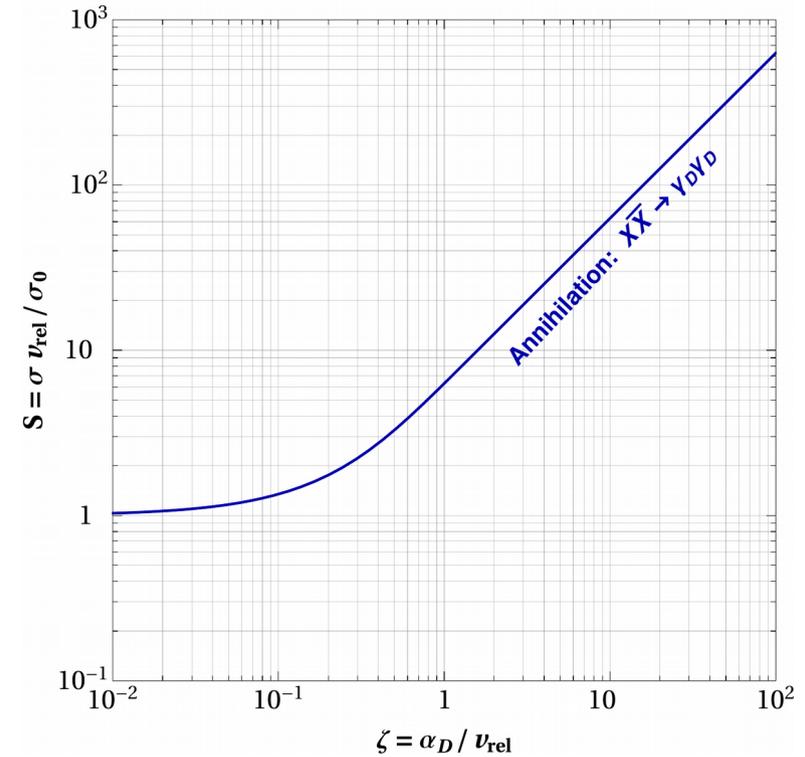


$$\sigma_{\text{ann}} v_{\text{rel}} = \sigma_0 \times S_{\text{ann}} \left( \frac{\alpha_D}{v_{\text{rel}}} \right)$$

$$S_{\text{ann}}(\zeta) = |\tilde{\phi}_{\vec{k}}(\vec{r}=0)|^2 = \frac{2\pi\zeta}{1 - e^{-2\pi\zeta}}; \quad \zeta = \alpha_D/v_{\text{rel}}$$

At  $\alpha_D \ll v_{\text{rel}}$  ( $\zeta \ll 1$ ):  $S_{\text{ann}} \simeq 1$

At  $\alpha_D \gtrsim v_{\text{rel}}$  ( $\zeta \gtrsim 1$ ):  $S_{\text{ann}} \simeq 2\pi \alpha_D/v_{\text{rel}} > 1$



cosmo time →

# Long-range interactions

## Scattering states and bound states

The Dyson-Schwinger equation with a Coulomb potential

where  
 $G^{(4)} \sim [\phi_{\vec{k}}]^2 / \text{singularity}$

### Solutions of the Schrödinger equation

continuous spectrum

$$\phi_{\vec{k}}(\vec{q}) \stackrel{\text{FT}}{\Leftrightarrow} \tilde{\phi}_{\vec{k}}(\vec{r})$$

$$\vec{k} = \mu \vec{v}_{\text{rel}}$$

$$E_{\vec{k}} = m_1 + m_2 + \vec{k}^2 / (2\mu)$$

discrete spectrum

$$\psi_{n\ell m}(\vec{q}) \stackrel{\text{FT}}{\Leftrightarrow} \tilde{\psi}_{n\ell m}(\vec{r})$$

$$\kappa_n = \mu\alpha / n$$

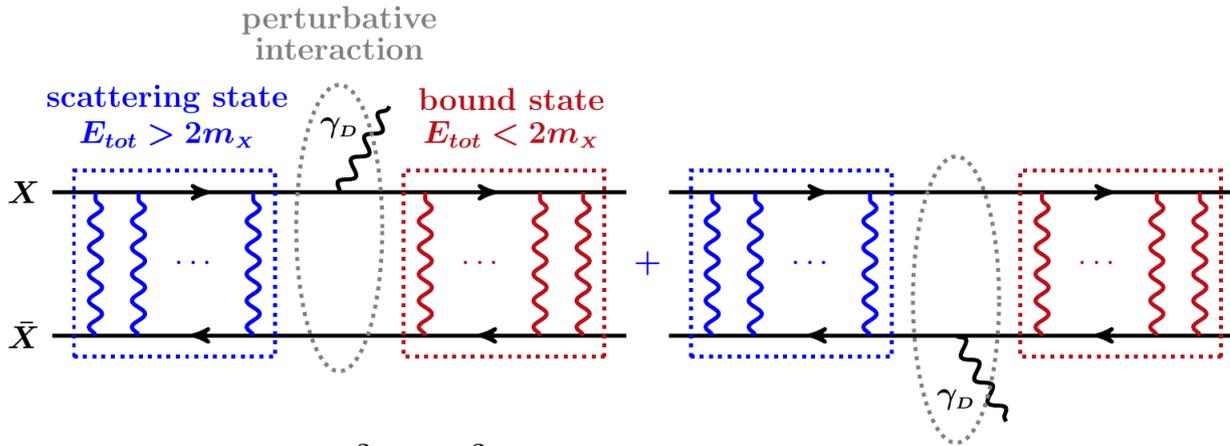
$$E_n = m_1 + m_2 - \kappa_n^2 / (2\mu)$$

where  $\mu \equiv m_1 m_2 / (m_1 + m_2)$  is the reduced mass

# Long-range interactions

## More radiative processes

Bound-state formation:  $X\bar{X} \rightarrow \mathcal{B}(X\bar{X}) + \gamma_D$



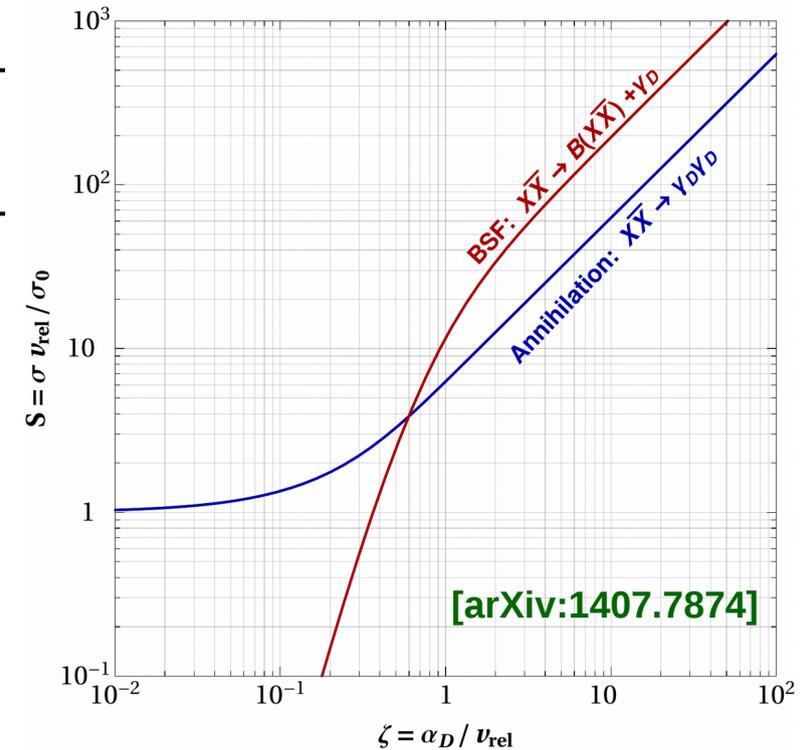
$$\mathcal{M}_{\text{BSF}} = \int \frac{d^3q}{(2\pi)^3} \frac{d^3p}{(2\pi)^3} \phi_{\vec{k}}(\vec{q}) \mathcal{A}(\vec{q}, \vec{p}) \psi_n^*(\vec{p})$$

$$\sigma_{\text{BSF}} v_{\text{rel}} = \frac{\pi \alpha_D^2}{m_X^2} \times S_{\text{BSF}}(\alpha_D / v_{\text{rel}})$$

$$S_{\text{BSF}}(\zeta) = \frac{2\pi\zeta}{1 - e^{-2\pi\zeta}} \frac{2^9 \zeta^4 e^{-4\zeta \operatorname{arccot}(\zeta)}}{3(1 + \zeta^2)^2}$$

At  $\alpha_D \ll v_{\text{rel}}$  ( $\zeta \ll 1$ ):  $S_{\text{BSF}} \simeq (2^9/3)\zeta^4 \ll 1$

At  $\alpha_D \gtrsim v_{\text{rel}}$  ( $\zeta \gtrsim 1$ ):  $S_{\text{BSF}} \simeq 3.13 \times (2\pi \alpha_D / v_{\text{rel}}) > 1$



cosmo time

# Long-range interactions

## DM relic density

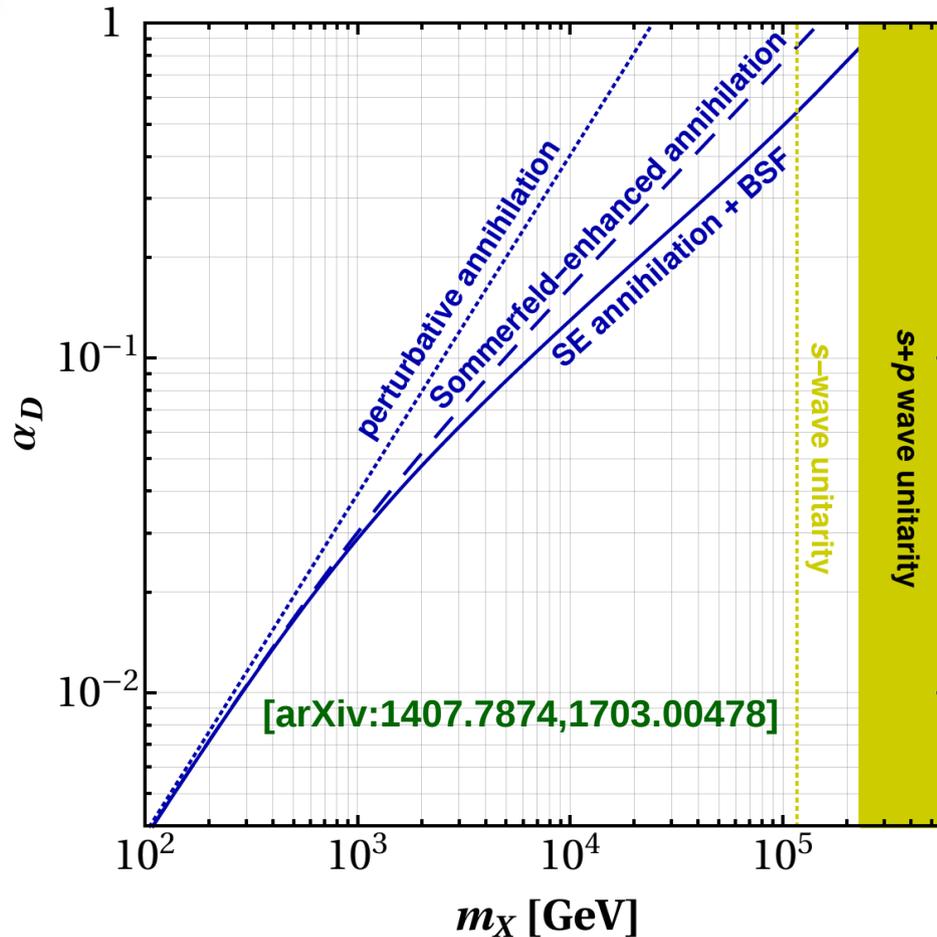
Direct Annihilation:

$$X\bar{X} \rightarrow \gamma_D \gamma_D$$

Bound-state formation and decay:

$$X\bar{X} \rightarrow \mathcal{B}(X\bar{X}) + \gamma_D$$

$$\mathcal{B}(X\bar{X}) \rightarrow 2\gamma_D \text{ or } 3\gamma_D$$



# Long-range interactions

## DM relic density

Direct Annihilation:

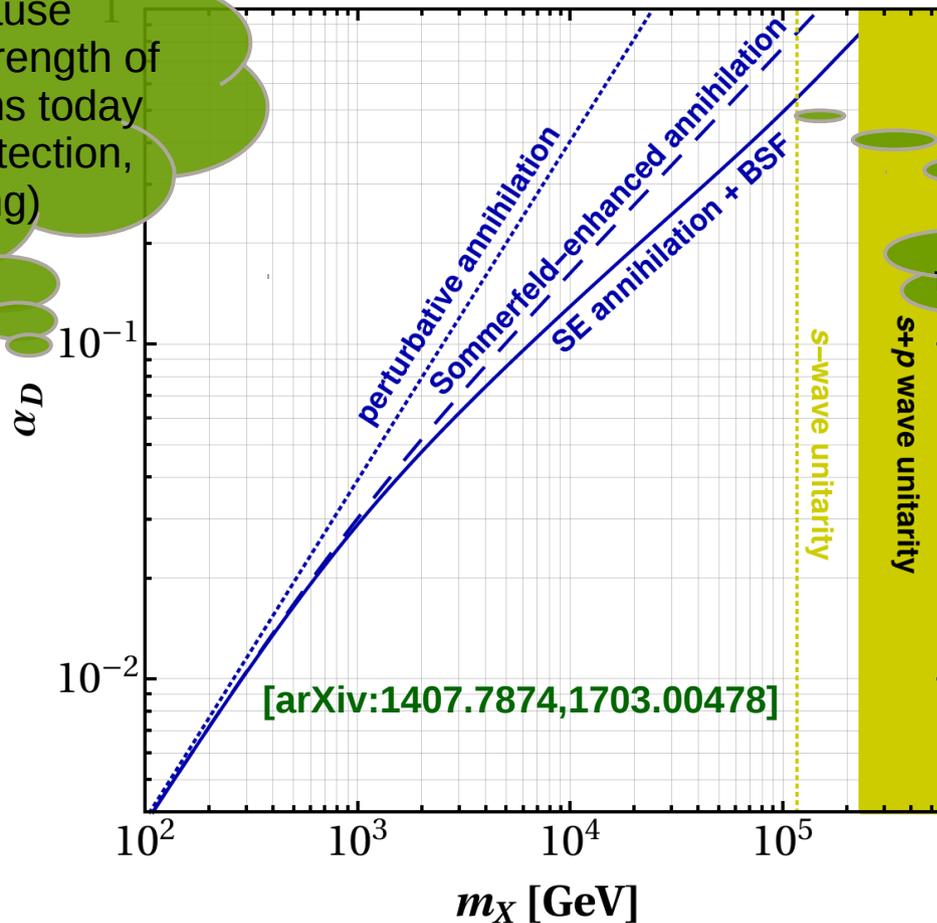
$$X\bar{X} \rightarrow \gamma_D \gamma_D$$

Bound-state formation and decay:

$$X\bar{X} \rightarrow \mathcal{B}(X\bar{X}) + \gamma_D$$

$$\mathcal{B}(X\bar{X}) \rightarrow 2\gamma_D \text{ or } 3\gamma_D$$

Important because it determines the strength of the DM interactions today (direct, indirect detection, self-scattering)



Shows how the early universe tames late-time DM interactions

# Long-range interactions

## Massive *but still light* mediators

### How light is light?

$$V(r) = -\alpha \frac{e^{-m_\varphi r}}{r}$$

$\mu v_{\text{rel}}$  : Average momentum transfer in non-relativistic scattering

$\mu\alpha, (\mu\alpha)^{-1}$  : Bohr momentum, Bohr radius

$m_\varphi^{-1}$  : Range of the potential

where  $\mu \equiv m_1 m_2 / (m_1 + m_2)$  is the reduced mass

Wavefunctions determined by two parameters

$$\zeta \equiv \frac{\mu\alpha}{\mu v_{\text{rel}}} = \frac{\alpha}{v_{\text{rel}}},$$

$$d \equiv \frac{\mu\alpha}{m_\varphi}$$

Sommerfeld effect  
important for  $\zeta \gtrsim \mathcal{O}(1)$ ,  
i.e. at low momentum transfer

Potential manifests as  
long-range if  $d \gtrsim \mathcal{O}(1)$

Bound-state levels  
open up for  $d \gtrsim n^2$   
( $n = 1, 2, \dots$ )

# Long-range interactions

## WORD OF CAUTION:

The thresholds (maximum  $m_\phi$  or minimum  $\mu\alpha$ )  
for the existence of bound states  
give rise to

**parametric resonances in the scattering-state wavefunction**  
that affect the annihilation cross-section.

These resonances – often described as the  
formation of bound states of zero binding energy –

**do NOT account for the  
formation of actual (above threshold) bound states!**

where  $\mu \equiv m_1 m_2 / (m_1 + m_2)$  is the reduced mass

Wavefunctions determined by two parameters

$$\zeta \equiv \frac{\mu\alpha}{\mu v_{\text{rel}}} = \frac{\alpha}{v_{\text{rel}}},$$

Sommerfeld effect  
important for  $\zeta \gtrsim \mathcal{O}(1)$ ,  
i.e. at low momentum transfer

$$d \equiv \frac{\mu\alpha}{m_\phi}$$

Potential manifests as  
long-range if  $d \gtrsim \mathcal{O}(1)$

Bound-state levels  
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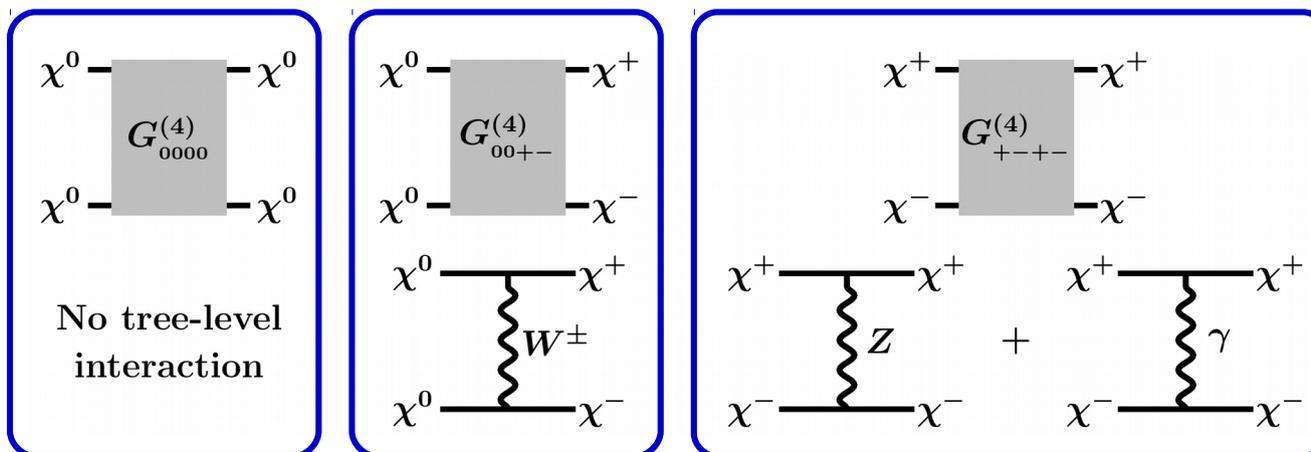
# Long-range interactions and good old WIMPs

- The Weak interactions of the SM have been the prototype of contact interactions in particle physics. True for all SM particles. However

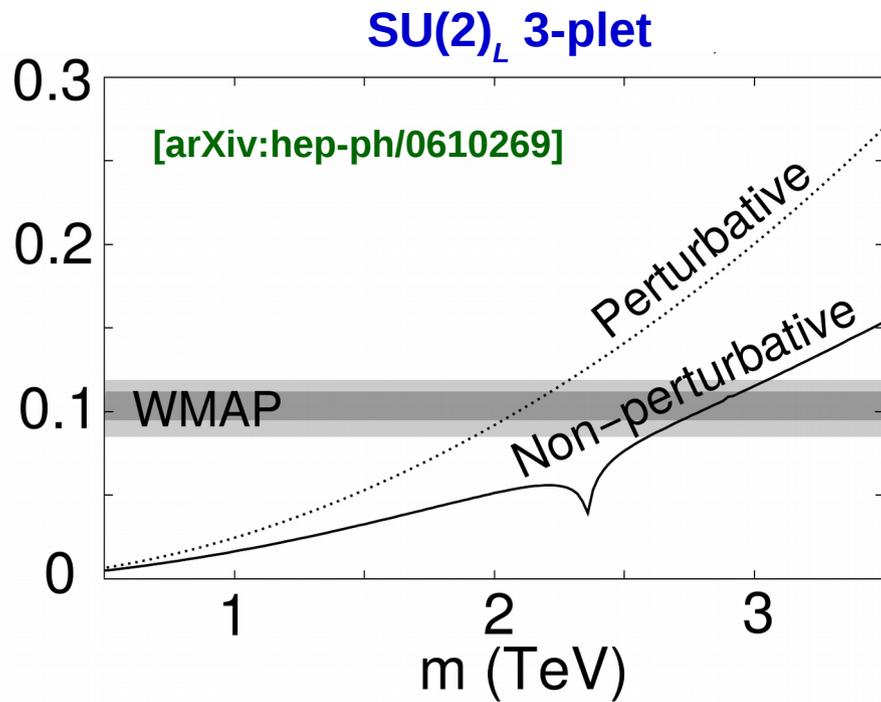
$$d_w = \frac{(m_x/2)\alpha_w}{m_w} \gtrsim \mathcal{O}(1) \quad \text{for} \quad m_x \gtrsim \text{TeV}$$

The Weak interactions manifest as long range!

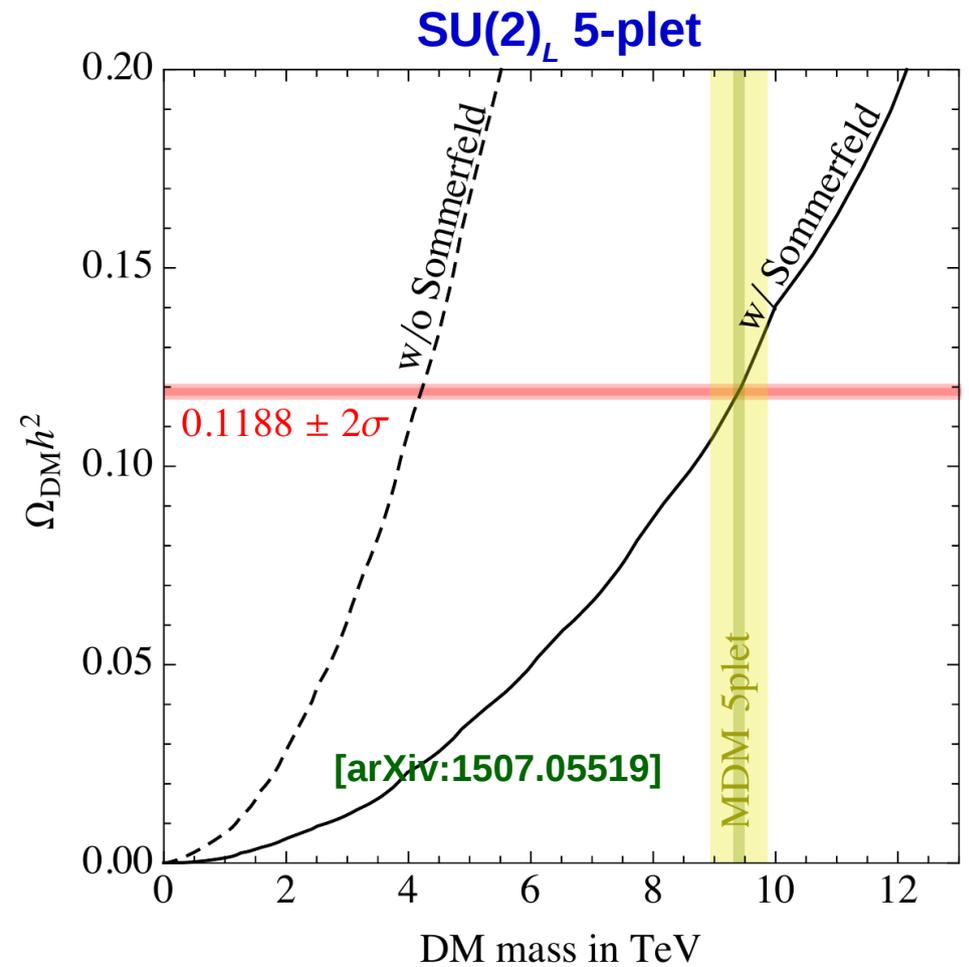
- Non-Abelian nature of WIMPs makes problem more complex. Consider e.g. an SU(2) triplet (Wino-like DM). During freeze-out, the neutral-charged component mass difference is less than the kinetic energy, thus:



# Long-range interactions and good old WIMPs



Bound-state formation  
not included



# Long-range interactions

## Implications for symmetric and asymmetric DM

- **Relic density**
  - Sommerfeld effect (SE) and BSF imply lower DM coupling to force mediators  
⇒ affects the expected signals
- **Indirect detection**
  - Enhanced annihilation rates inside halos
  - Signals from radiative level transitions (BSF and other excitations/de-excitations)
  - Significant signals from asymmetric DM due to both of the above
- **Direct detection**
  - Velocity dependence of DM-nucleon cross-sections alters interpretation of results
  - Stable bound states (asymmetric DM) screen DM-nucleon scattering.
- **Self-interactions inside galaxies**
  - SE implies that  $\sigma_{\text{scatt}}$  decreases with  $v_{\text{rel}}$  (e.g. Rutherford scattering:  $\sigma_{\text{scatt}} \sim 1/v_{\text{rel}}^4$ ).  
⇒ significant effect on small scales, negligible effect on large scales  
⇒ light mediators well-suited for self-interacting DM
  - Stable bound states (asymmetric DM) screen/curtail DM self-scattering.

## **V. Current topics**

Very very very partial list

# Current directions in dark matter research

- Galactic structure ↔ DM properties
  - Observations: Better understanding of small scales
  - Simulations: Modeling of baryonic physics and DM interactions
  - Theory: Scattering properties of self-interacting DM, input for simulations
- Light dark matter,  $m_{\text{DM}} < \text{few GeV}$ 
  - Experiment: New direct-detection technologies
  - Theory: Motivation, interaction strength, benchmark models
- Heavy dark matter,  $m_{\text{DM}} > \text{few TeV}$ 
  - Experiment: Indirect probes, e.g. IceCube, CTA, Hawk
  - Theory: Non-perturbative effects, radiative transitions, the role of substructure
- Metastable mediators
  - Displaced vertices at the LHC
  - Indirect signals from the sun
- Primordial black holes

# Conclusion

**Dark matter is an intriguing and complex problem that requires diverse expertise.**



**Many things to  
debate and calculate!**