# Primordial Black Holes as (part of the) dark matter

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**Lecture 1: Motivation**

**Formation:** **collapse of large (inflationary) density perturbations**
**other mechanisms**

**Mass function**

Lecture 2: Constraints

Application to extended mass functions

For further details on these topics (and also PBH binary mergers as source of GWs) see recent review by Sasaki, Suyama, Tanaka & Yokoyama arXiv:1801.05235.

# Motivation

Cosmological observations indicate that dark matter (DM) has to be cold and non-baryonic.

Primordial Black Holes (PBHs) form in the early Universe and are therefore non-baryonic.

PBHs evaporate (Hawking radiation), lifetime longer than the age of the Universe for $M > 10^{15}$ g.

A DM candidate which (unlike WIMPs, axions, sterile neutrinos,…) isn’t a new particle (however their formation does usually require Beyond the Standard Model physics).

LIGO has detected gravitational waves from mergers of 10+ $M_{sun}$ BHs. Could be formed by astrophysical processes, but a large population of such massive BH binaries was possibly somewhat unexpected (stellar winds from progenitors must be weak & hence metallicity low + natal kicks must be small).

Could PBHs be the CDM?
(and potentially also the source of the GW events??)

# Formation

During radiation domination an initially large (at horizon entry) density perturbation can collapse to form a PBH with mass of order the horizon mass.[$] Zeldovich & Novikov; Hawking; Carr & Hawking

For gravity to overcome pressure forces resisting collapse, size of region at maximum expansion must be larger than Jean's length.

## Simple analysis:

Carr; see Harada, Yoo & Kohri for refinements

density contrast: $$\delta \equiv \frac{\rho - \bar{\rho}}{\bar{\rho}}$$

threshold for PBH formation: $$\delta \geq \delta_{\rm c} \sim w = \frac{p}{\rho} = \frac{1}{3}$$

PBH mass: $$M \sim w^{3/2} M_{\rm H} \qquad M_{\rm H} \sim 10^{15}\,{\rm g} \left( \frac{t}{10^{-23}\,{\rm s}} \right)$$

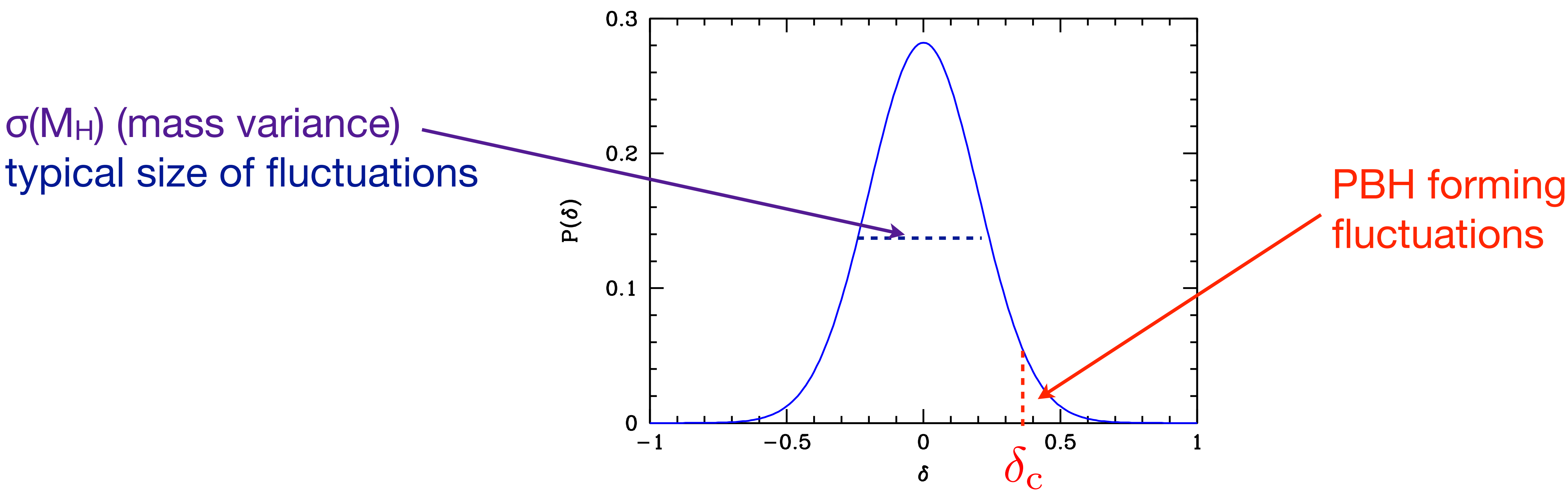
[$] Other formation mechanisms include collapse of cosmic string loops, bubble collisions, fragmentation of inflaton/scalar condensate into oscillons.

initial PBH mass fraction (fraction of universe in regions dense enough to form PBHs):

$$\beta(M) \sim \int_{\delta_{\rm c}}^{\infty} P(\delta(M_{\rm H}))\, {\rm d}\delta(M_{\rm H})$$

assuming a gaussian probability distribution:

$$\beta(M) = {\rm erfc}\left(\frac{\delta_{\rm c}}{\sqrt{2}\sigma(M_{\rm H})}\right)$$




If $\sigma(M_{\rm H})$ is independent of mass, PBHs have a power law mass function Carr.

Otherwise most PBHs form on scale(s) where perturbations are largest.

# PBH abundance

Since PBHs are matter, during radiation domination the fraction of energy in PBHs grows with time:

$$\frac{\rho_{\rm PBH}}{\rho_{\rm rad}} \propto \frac{a^{-3}}{a^{-4}} \propto a$$

Relationship between **PBH initial mass fraction, β**, and **fraction of DM in form of PBHs, f**:

$$\beta(M) \sim 10^{-9} f \left( \frac{M}{M_{\odot}} \right)^{1/2}$$

i.e. initial mass fraction must be small, but non-negligible.

On CMB scales the primordial perturbations have amplitude $\sigma(M_{\rm H}) \sim 10^{-5}$

If the primordial perturbations are very close to scale-invariant the number of PBHs formed will be completely negligible:

$$\beta(M) \sim {\rm erfc}(10^5) \sim 10^5 \exp\left[ -(10^5)^2 \right]$$

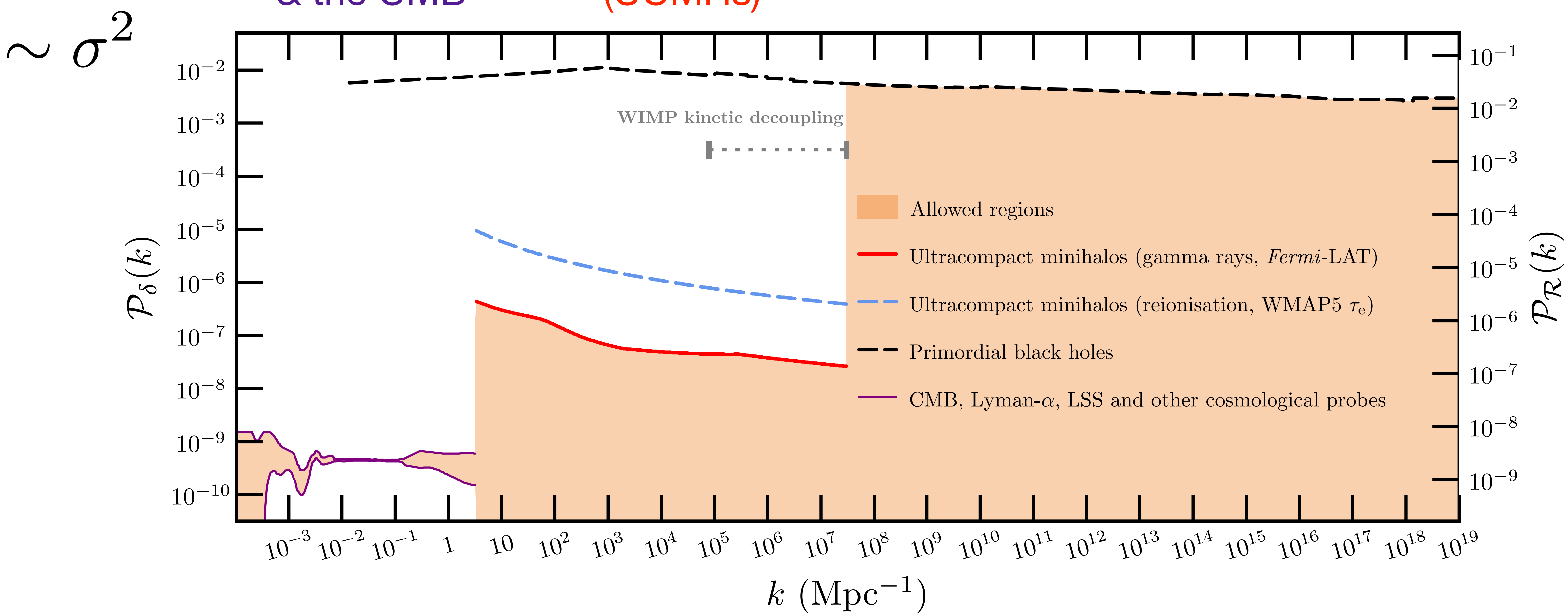
To form an interesting number of PBHs the primordial perturbations must be significantly larger ($\sigma(M_{\rm H})$~0.01) on small scales than on cosmological scales.

# Constraints on the primordial power spectrum

Large scale structure & the CMB

Ultracompact minihalos (UCMHs)*

Primordial Black Holes




Bringmann, Scott & Akrami

* UCMH constraints only hold if most of the DM is WIMPs.

Also recent studies find UCMHs have shallower density profiles than assumed in this calc Gosenca et al., Delos et al. which will affect constraints.

## deviations from simple scenario:

## i) non-gaussianity

Since PBHs are formed from rare large density fluctuations, changes in the shape of the tail of the probability distribution (i.e. non-gaussianity) can significantly affect the PBH abundance. Bullock & Primack; Ivanov;… Byrnes, Copeland & Green;…

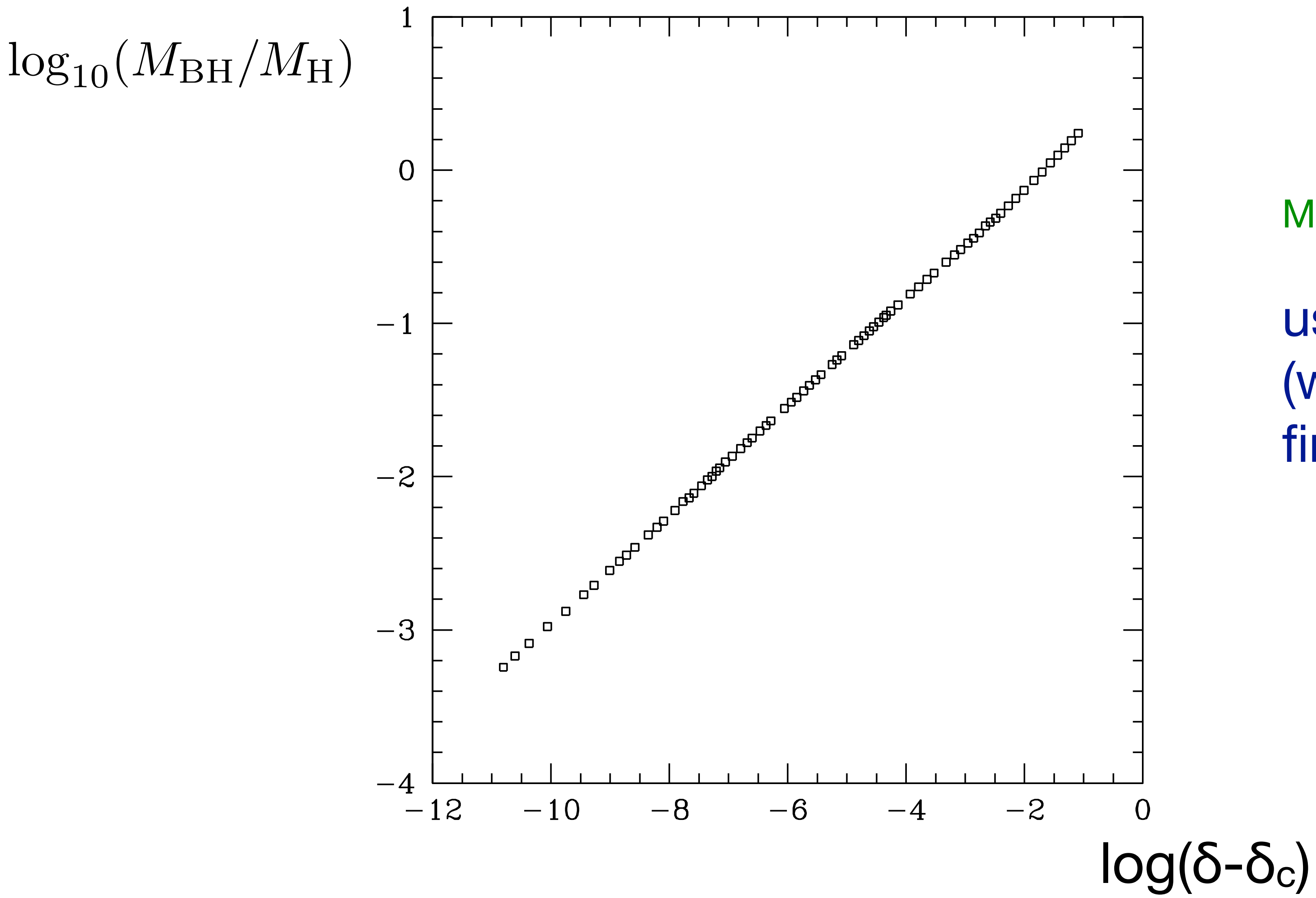
Franciolini, Kehagias, Matarrese & Riotto use a path integral formalism to derive an exact expression for the PBH abundance. However it involves all of the smoothed N-point connected correlation functions…

## ii) critical collapse

Choptuik; Evans & Coleman; Niemeyer & Jedamzik

BH mass depends on size of fluctuation it forms from:

$$M = k M_{\mathrm{H}} (\delta - \delta_{\mathrm{c}})^{\gamma}$$




Musco, Miller & Polnarev

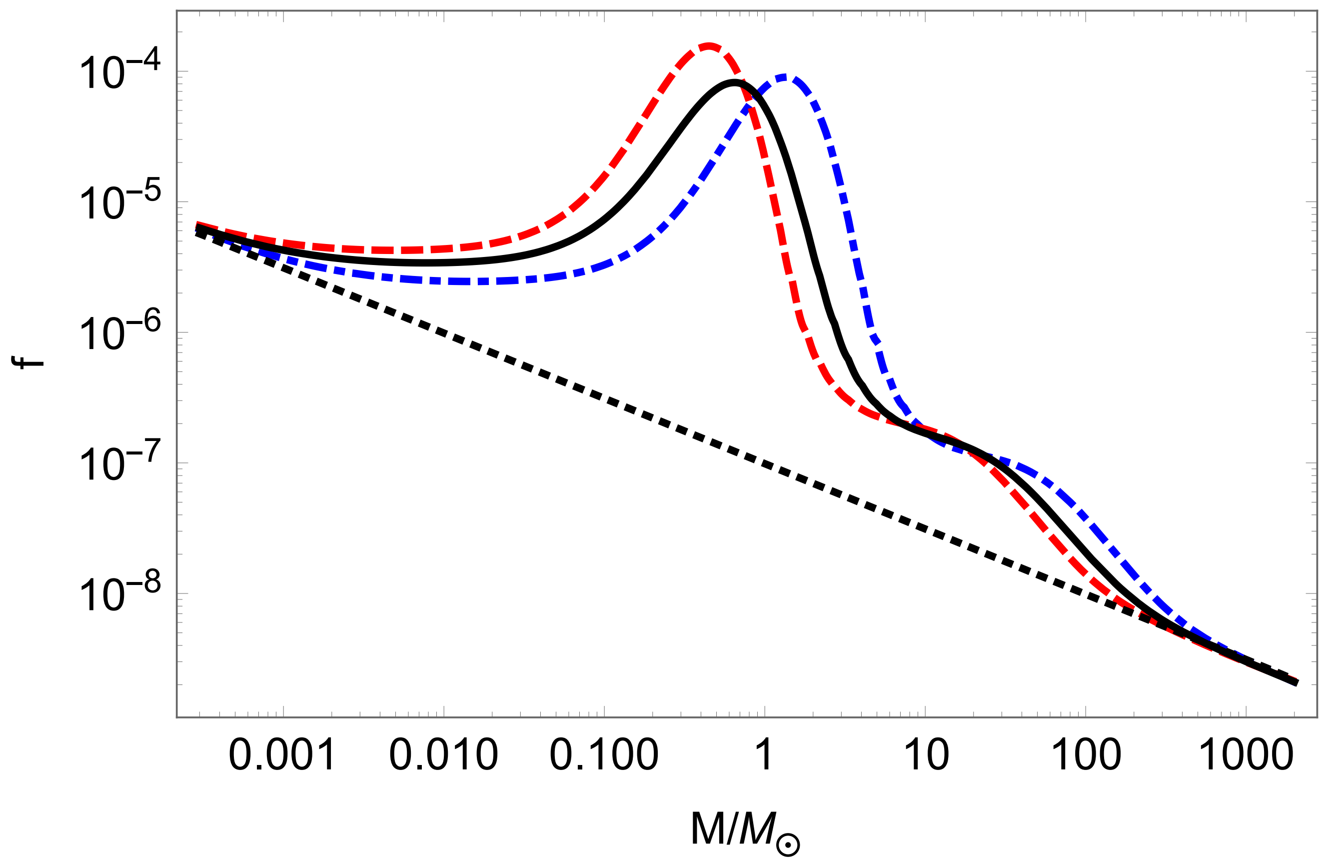
using numerical simulations (with appropriate initial conditions) find $k=4.02$, $\gamma=0.357$, $\delta_c = 0.45$

Get PBHs with range of masses produced even if they all form at the same time i.e. we don't expect the PBH MF to be a delta-function

## iii) phase transitions

Reduction in the equation of state parameter ($w=p/\rho$) at phase transitions decreases the threshold for PBH formation $\delta_c$ and enhance the abundance of PBHs formed on this scale. (Horizon mass at QCD phase transition is of order a solar mass.) Jedamzik

Using new lattice calculation of QCD phase transition Byrnes et al. transition find a 2 order of magnitude enhancement in $\beta$ (but still need a mechanism for amplifying the primordial perturbations):




Byrnes et al.

# A brief introduction to inflation

Inflation: A period of accelerated expansion ($\ddot{a} > 0$ ) in the early Universe.

Problems with the Big Bang:

Flatness: if universe isn't exactly flat density evolves away from critical density (for which geometry is flat), to be so close to critical density today requires fine tuning of initial conditions.

Horizon: regions that have never been in causal contact have the same Cosmic Microwave Background temperature and anisotropy distribution.

Monopoles/massive relics: formed when symmetry breaks, would dominate the density of the Universe today.

Inflation solves these problems by:

driving 'initial' density extremely close to critical density

allowing currently observable universe to originate from small region (originally in causal contact)

diluting monopoles

It can also generate density perturbations:

Quantum fluctuations

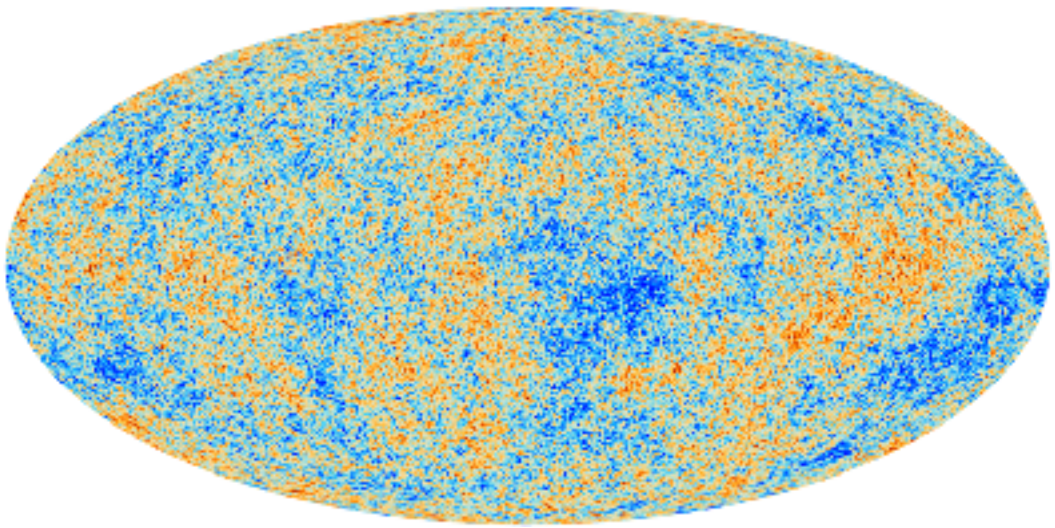
↓

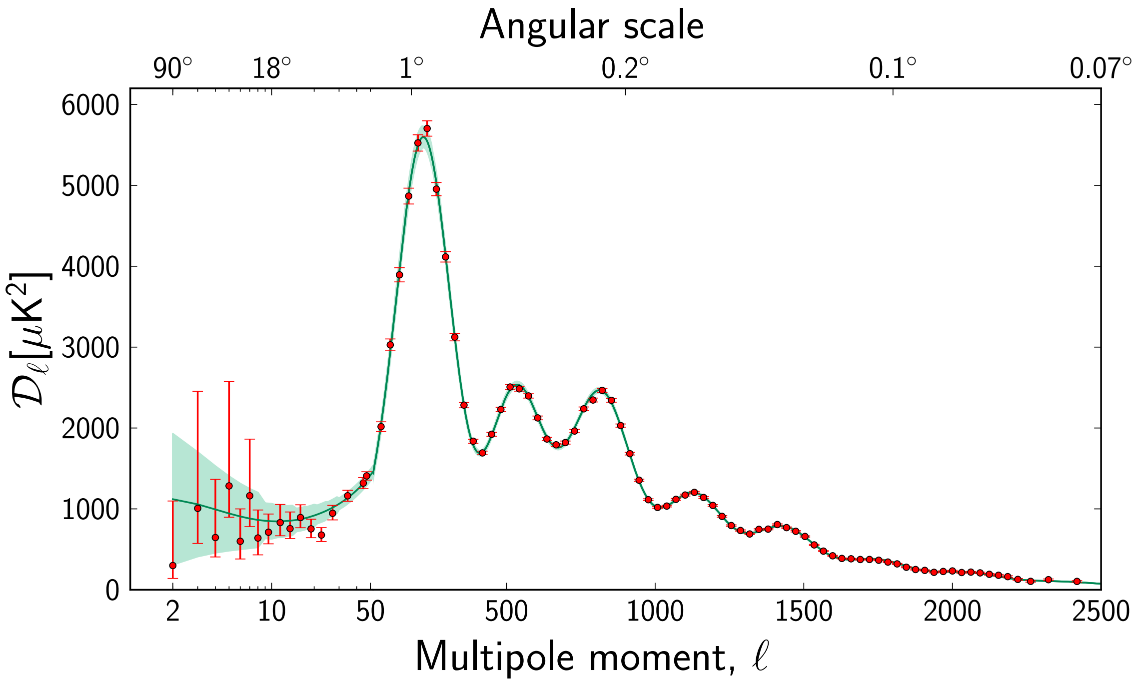
Curvature perturbations

↓

Density & temperature perturbations

which are close to scale-invariant and hence consistent with the temperature anisotropies in the cosmic microwave background radiation.






Planck

## What drives inflation?

what do we need to get $\ddot{a} > 0$ ?

Fluid equation: $$\dot{\rho} + 3H(\rho + p) = 0$$

Acceleration equation: $$\frac{\ddot{a}}{a} = -4\pi G(\rho + 3p)$$

$$\ddot{a} > 0 \quad \Rightarrow \quad p < -\frac{1}{3}\rho$$

i.e. negative pressure!

**Scalar field:**

spin zero particle (unchanged under co-ordinate transformations)

required for spontaneous symmetry breaking

common in ‘beyond standard model’ particle theories

$$\rho = \frac{1}{2}\dot{\phi}^2 + V(\phi) \qquad p = \frac{1}{2}\dot{\phi}^2 - V(\phi)$$

if potential dominates: $$\rho \approx -p \approx V(\phi)$$

## Scalar field dynamics-a quick overview

Friedman equation: $$H^2 = \frac{8\pi G}{3}\left(V + \frac{1}{2}\dot{\phi}^2\right)$$

Fluid equation: $$\ddot{\phi} + 3H\dot{\phi} = -\frac{dV}{d\phi}$$

[c.f. a ball rolling down a hill, with the expansion of the Universe acting as friction]

## Slow roll approximation

Slow roll parameters: $$\epsilon = \frac{m_{\rm Pl}^2}{16\pi}\left(\frac{V'}{V}\right)^2$$ slope of potential

$$\eta = \frac{m_{\rm Pl}^2}{8\pi}\frac{V''}{V}$$ curvature of potential

If $\varepsilon, |\eta| \ll 1$ $$H^2 \approx \frac{8\pi}{3m_{\rm Pl}^2}V \qquad 3H\dot{\phi} \approx -\frac{\mathrm{d}V}{\mathrm{d}\phi}$$

$$\varepsilon < 1 \quad \Longrightarrow \quad \ddot{a} > 0$$

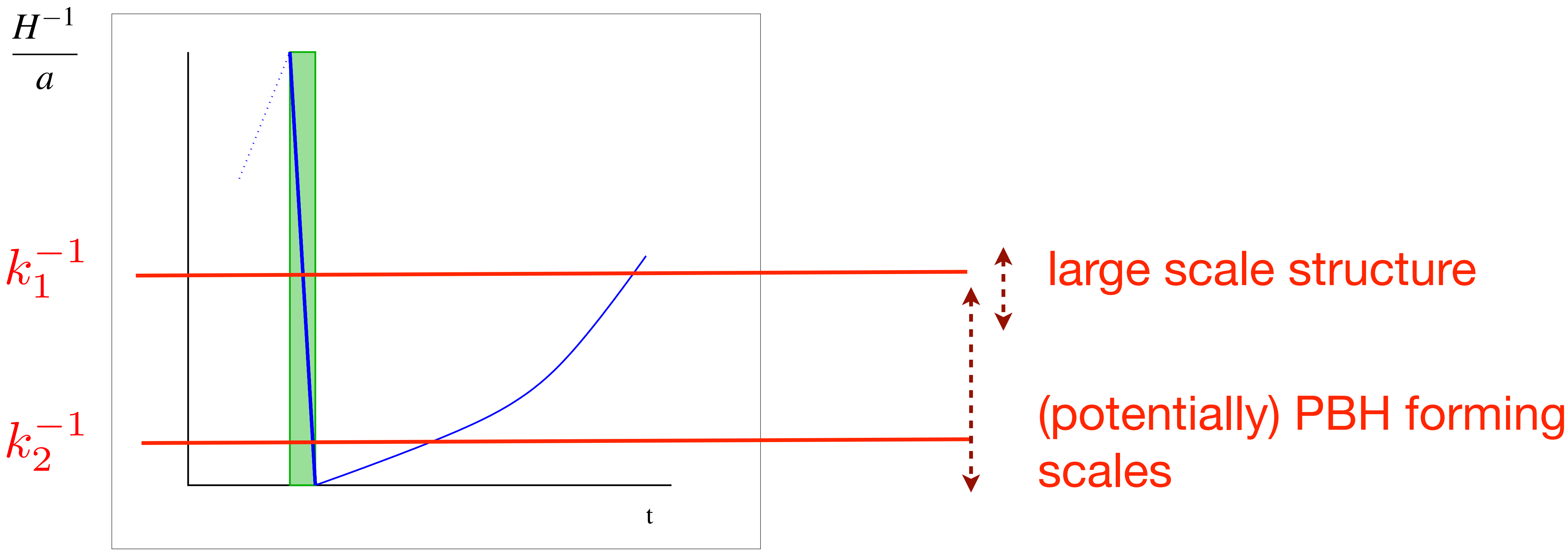
Inflation ends when potential becomes too steep: $\varepsilon \approx 1$

Field oscillates around minimum of potential.

Inflaton field decays creating radiation dominated Universe (reheating).

$$\ddot{a} > 0 \quad \Rightarrow \quad \frac{\mathrm{d}(H^{-1}/a)}{\mathrm{d}t} < 0$$

i.e. comoving Hubble radius decreases during inflation




A scales exits the horizon during inflation when k = aH, re-enters when k = aH again (and if fluctuations are sufficiently large they collapse to form PBH soon afterwards).

CMB & LSS probe scales: $k \sim 1 - 10^{-3}\,\mathrm{Mpc}^{-1}$

PBHs can form on scales: $k \sim 10^{-2} - 10^{23}\,\mathrm{Mpc}^{-1}$

Lower limit on PBH mass set by reheat temperature at the end of inflation:

$$M \sim M_{\mathrm{H}} = 10^{18}\,\mathrm{g}\left(\frac{10^{7}\,\mathrm{GeV}}{T}\right)^{2}$$

amplitude of fluctuations: $$\sigma^2(M_{\rm H}) \propto \frac{V^3}{(V')^2}$$

during slow-roll: $$\sigma^2(M_{\rm H}) \propto k^{n_s - 1}$$

$$n_{\rm s} = 1 - 6\epsilon + 2\eta + ...$$

observations (CMB + large scale structure):

$$\sigma \approx 10^{-5} \qquad n_{\rm s} = 0.9655 \pm 0.0062$$

on scale: $k_0 = 0.002\,{\rm Mpc}^{-1}$

n.b. power law expansion of power spectrum

$$\ln \sigma^2(k) \approx \ln \sigma^2(k_0) + (n_{\rm s}(k_0) - 1) \ln\left(\frac{k}{k_0}\right) + \frac{1}{2}\frac{{\rm d}\ln n_{\rm s}}{{\rm d}\ln k}(k_0) \ln^2\left(\frac{k}{k_0}\right) + ...$$

is only valid over small range of k (fine for CMB/LSS, but not for extrapolating down to PBH forming scales).

# Inflation models with (potentially) large perturbations on small scales

In single field models need to violate slow roll (and hence standard expressions for amplitude of fluctuations aren't valid).

Models which might naively be expected to produce large perturbations (e.g. potentials with an inflection point, $V'(\phi) \rightarrow 0$ 'ultra-slow-roll') don't. Kannike et al.; Germani & Prokopec; Motohashi & Hu; Ballesteros & Taoso

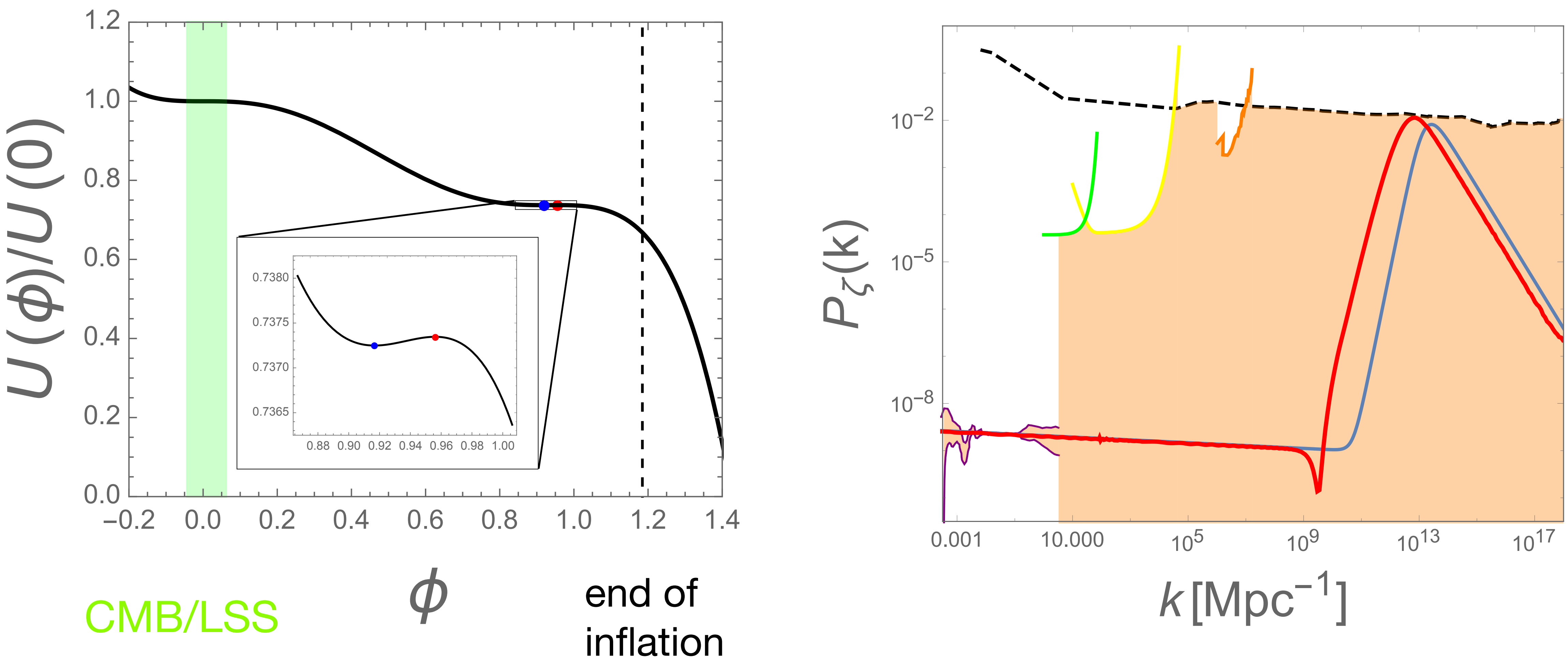
## i) models with a feature in the power spectrum

e.g. Ivanov, Naselsky & Novikov followed by many more

a) over-shoot a local minimum

Ballesteros & Taoso; Herzberg & Yamada

Potential fine-tuned so that field goes past local max, but with reduced speed

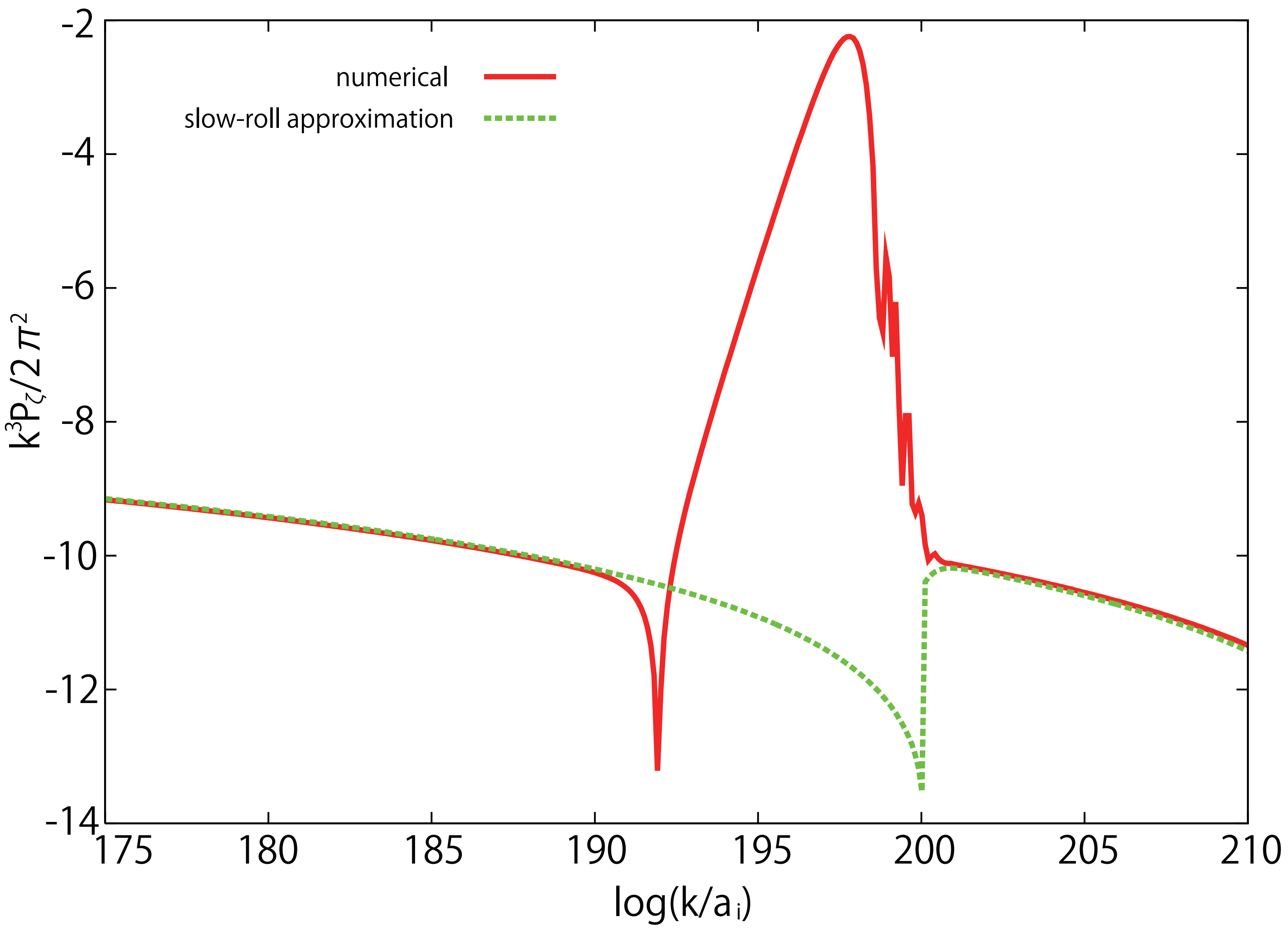



Can be done with quintic potential, with fine-tuning at $\sim 10^{-8.5}$ level…

b) double inflation

Saito, Yokoyama & Nagata; Kannike et al.

Perturbations on scales which leave the horizon close to the end of the 1st period, of inflation get amplified during the 2nd period.



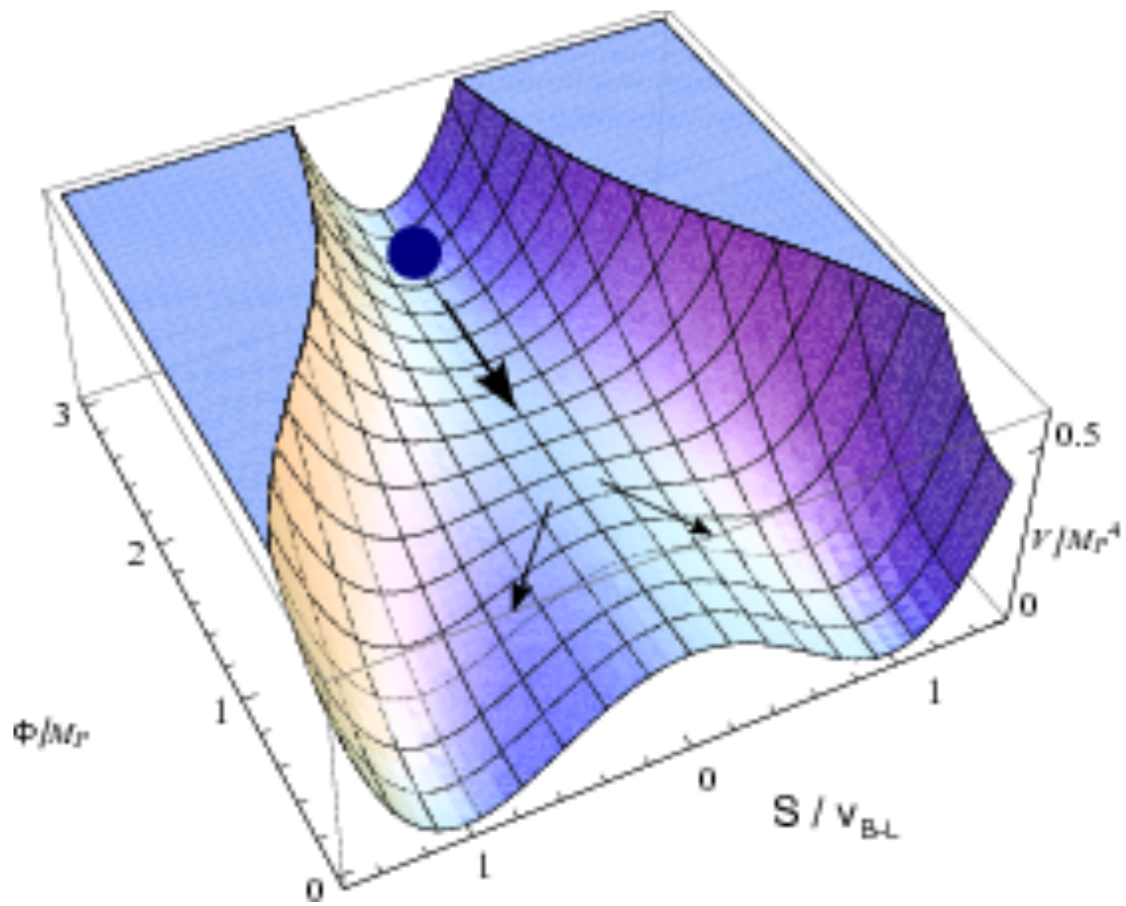

Also double inflation models where large scale perturbations are produced during 1st period, and small scale (PBH forming) perturbations during 2nd (Kawasaki et al.; Kannike et al.; Inomata et al. )

c) multi-field models

## hybrid inflation with a mild waterfall transition

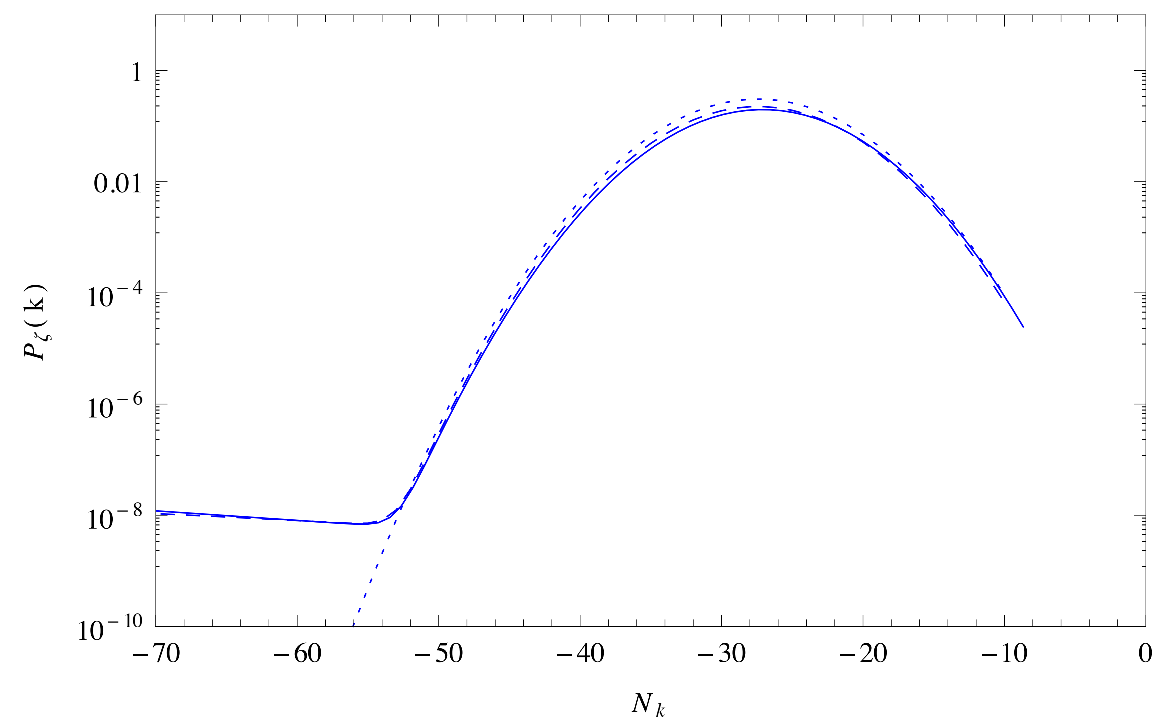
Garcia-Bellido, Linde & Wands

**potential**




Buchmuller

**primordial power spectrum**




Clesse & Garcia-Bellido

## axion-like curvaton

Kawasaki, Kitajima & Yanagida

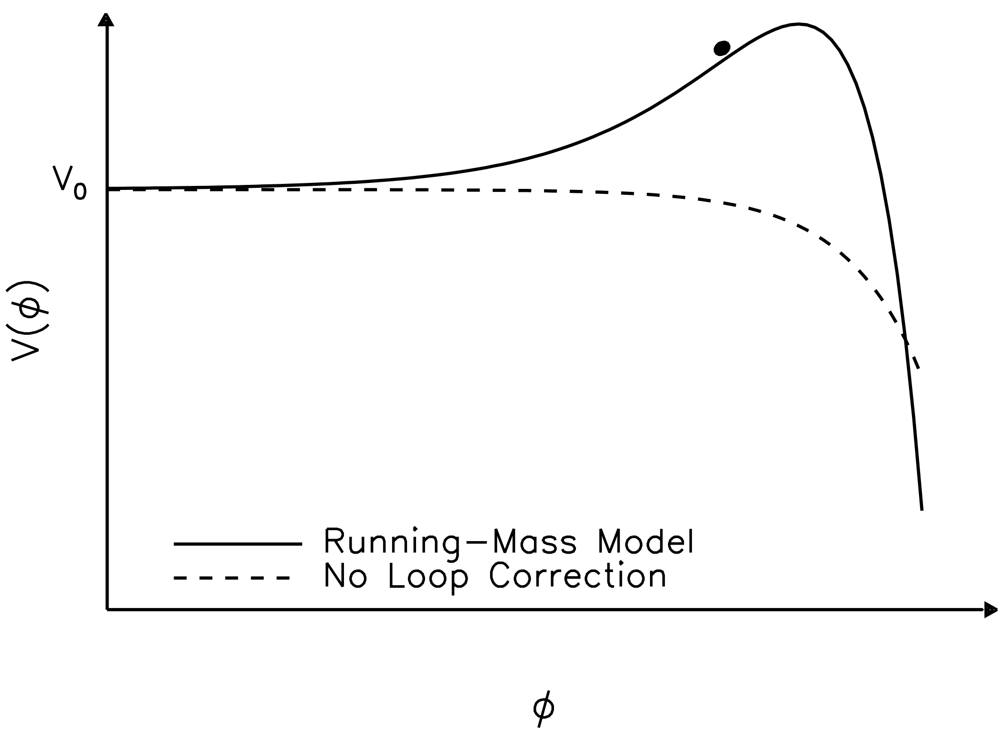
Large scale perturbations generated by inflaton, small scale (PBH forming) perturbations by curvaton (a spectator field during inflation gets fluctuations and decays afterwards producing perturbations Lyth & Wands)

ii) monotonically increasing power spectrum

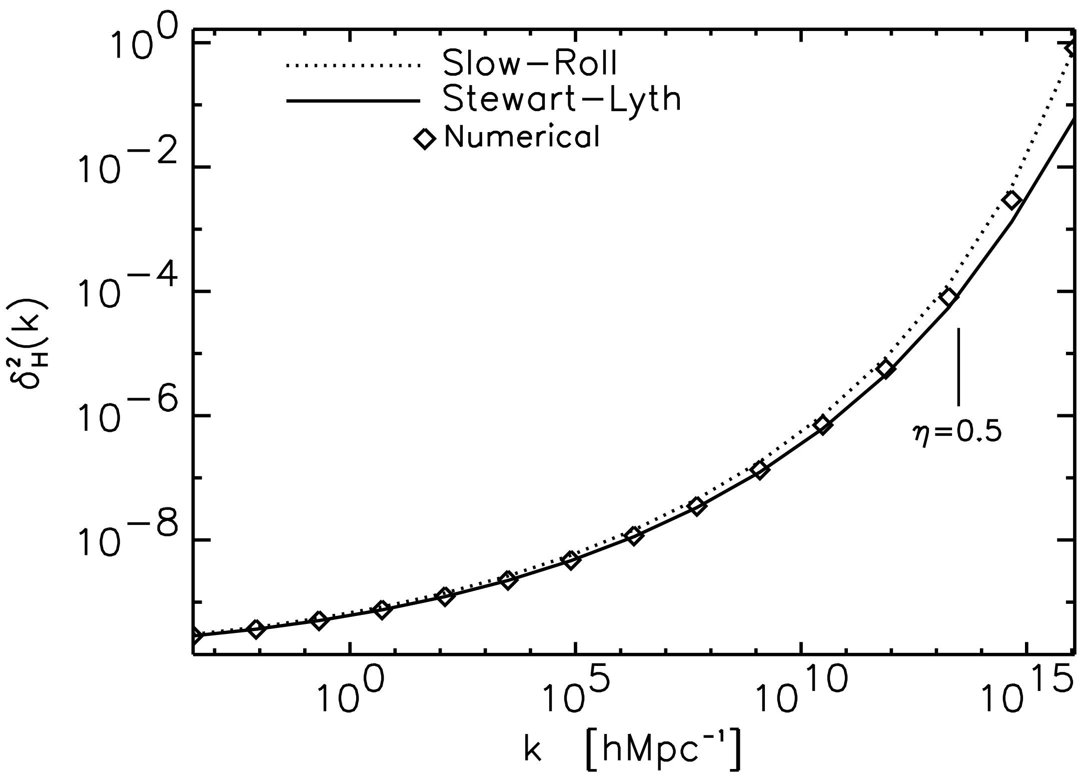
running-mass inflation Stewart

$$V(\phi) = V_0 + \frac{1}{2} m_\phi^2(\phi)\phi^2$$

potential




primordial power spectrum




Leach, Grivell, Liddle

## stochastically generated inflation models

Peiris & Easther

Generate inflation models stochastically using slow roll ‘flow equations’.

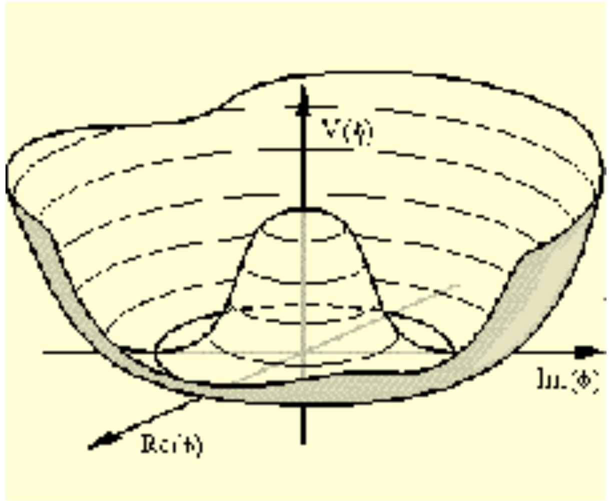
Get a class of models where inflation can continue indefinitely (and is assumed to be ended via an auxiliary mechanism). In these models the amplitude of fluctuations decreases with increasing k and can be large enough to form PBHs (while still satisfying cosmological constraints).

# Formation: other mechanisms

## Collapse of cosmic string loops Hawking; Polnarev & Zemboricz;

Cosmic strings are 1d topological defects formed during symmetry breaking phase transition.

String intercommute producing loops.



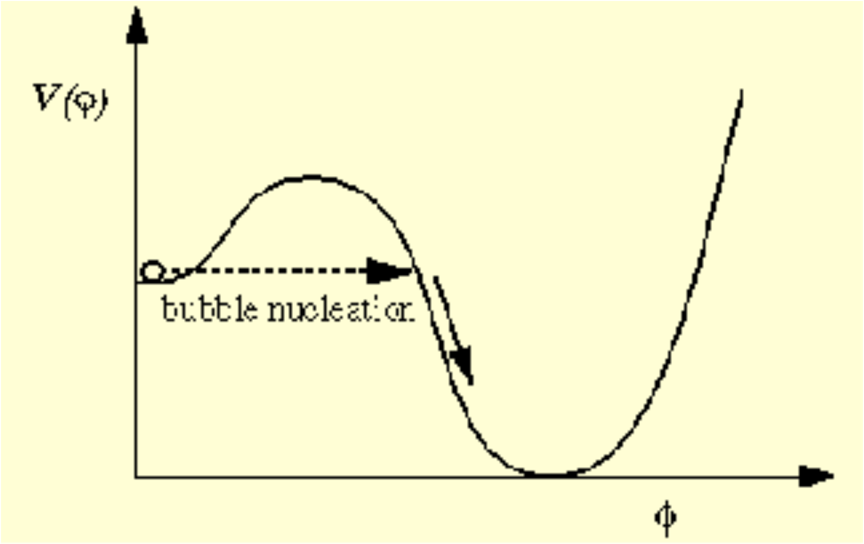


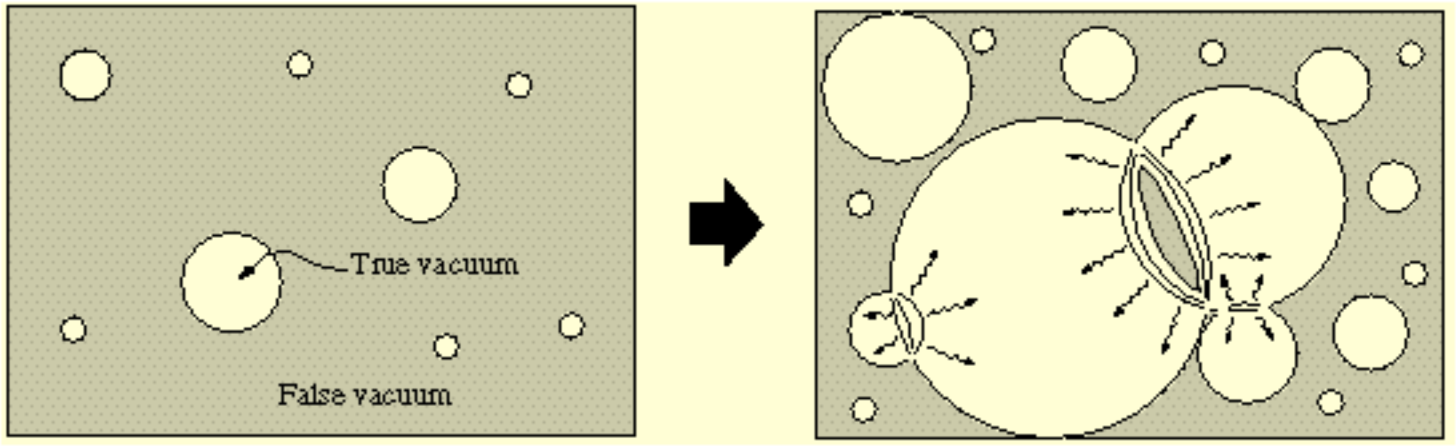
Small probability that loop will get into configuration where all dimensions lie within Schwarzschild radius (and hence collapse to from a PBH with mass of order the horizon mass at that time).

Probability is time independent, therefore PBHs have extended mass spectrum.

# Bubble collisions Hawking

1st order phase transitions occur via the nucleation of bubbles.




PBHs can form when bubbles collide (but bubble formation rate must be fine tuned).

PBH mass is of order horizon mass at phase transition.

# Fragmentation of inflaton scalar condensate into oscillons/Q_balls

Cotner & Kusenko; Cotner, Kusenko & Takhistov

# Mass function

## scale-invariant primordial density perturbation power spectrum

Carr

For PBH formation during radiation domination:

$$\frac{\mathrm{d}n}{\mathrm{d}M} \propto M^{-5/2}$$

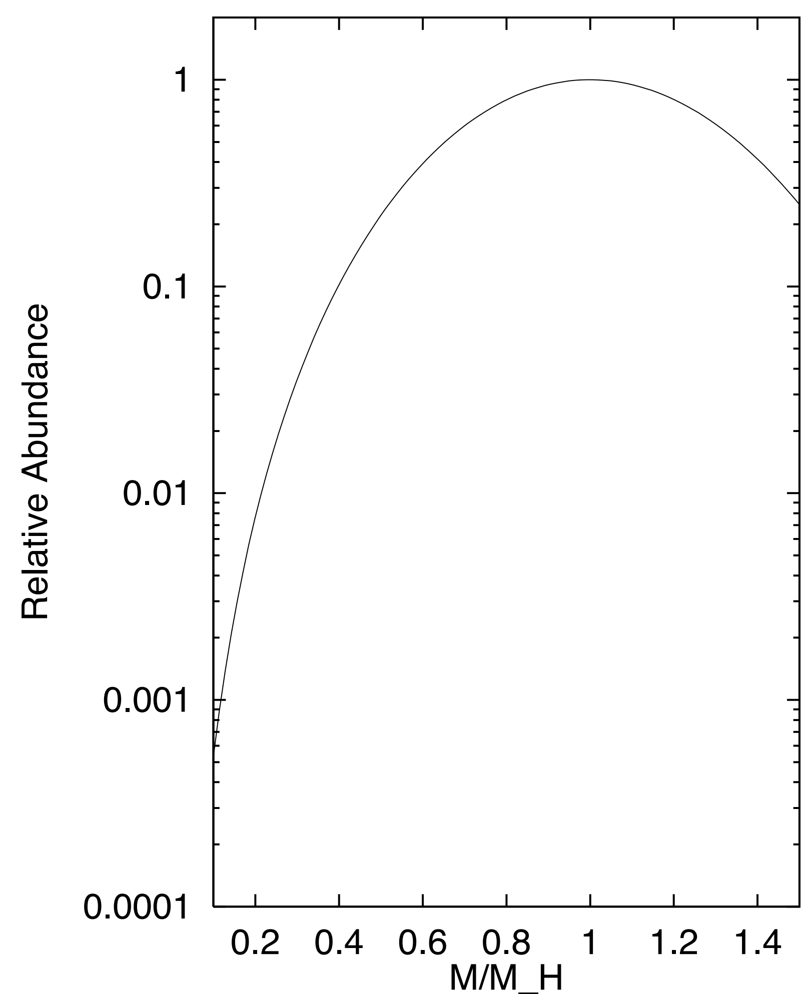
However if power spectrum is completely scale-invariant, then the number of PBHs formed is negligible.

PBHs which form from the collapse of cosmic string loops would also have a power-law mass function.

delta-function primordial density perturbation power spectrum, taking into account critical collapse $M = kM_{\rm H}(\delta - \delta_{\rm c})^{\gamma}$

Niemeyer & Jedamzik

$$\frac{{\rm d}n}{{\rm d}M} \propto \left(\frac{M}{M_{\rm H}}\right)^{(-1+1/\gamma)} \exp\left[-(1+\gamma)\left(\frac{M}{M_{\rm H}}\right)^{(1/\gamma)}\right]$$

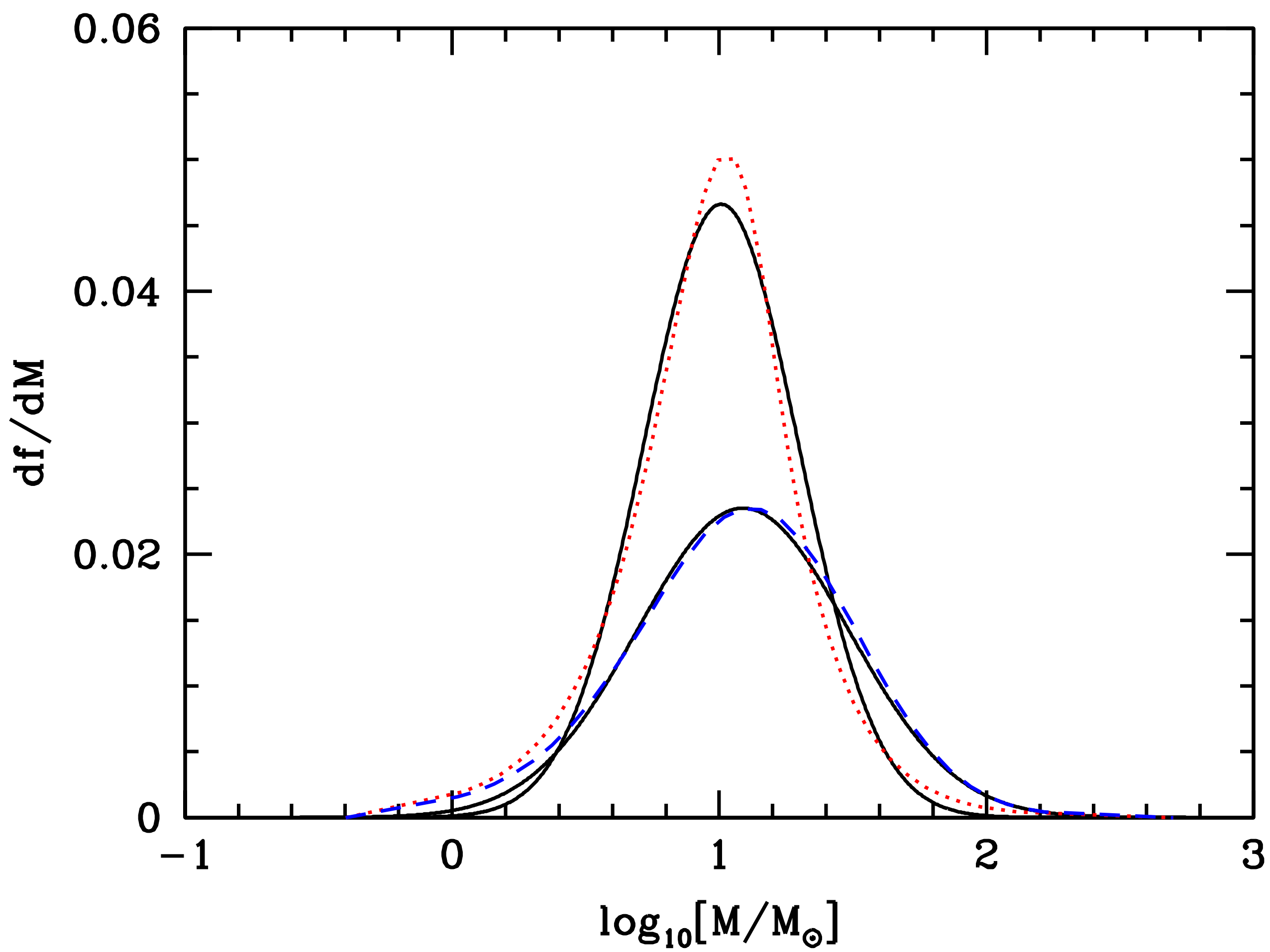



Yokoyama

## non delta-function primordial density perturbation power spectrum, taking into account critical collapse

Extended MFs produced by inflation models with finite width peak in power spectrum, often well approximated by a log-normal distribution: Green; Kannike et al.

$$\frac{\mathrm{d}n}{\mathrm{d}M} \propto \exp\left[-\frac{\log(M/M_{\rm c})}{2\sigma^2}\right]$$




axion-like curvaton

running mass inflation

# Summary

Primordial Black Holes can form in the early Universe, for instance from the collapse of large density perturbations during radiation domination.

A non-negligible number of PBHs will only be produced if the amplitude of the fluctuations is ~4 orders of magnitude larger on small scales than on cosmological scales.

This can be achieved in inflation models (e.g. with a feature in the potential or multiple fields). However fine-tuning is required.

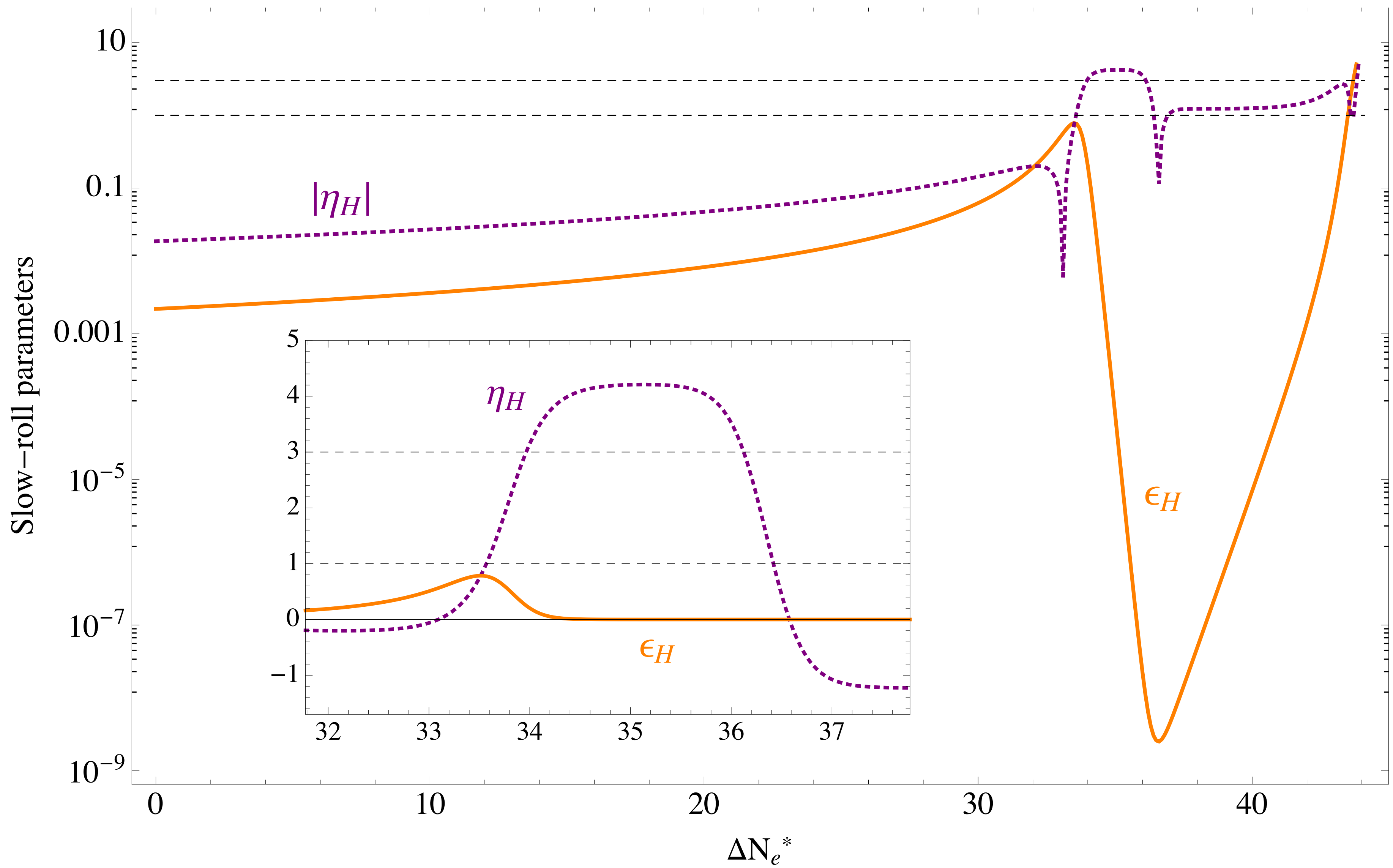
PBHs are expected to have an extended mass function (due to critical collapse and also width of primordial power spectrum).

Some aspects of PBH abundance and distribution (e.g. effect of non-gaussianity on abundance, clustering, mergers) are not yet accurately studied.

## Lecture 2: observational constraints on the PBH abundance

Back-up slides

# Hubble slow roll parameters for potential with inflection point from Ballesteros & Taoso



PBHs can also form from collapse of density perturbations during matter domination.

In this case regions must be sufficiently spherically symmetric Yu, Khlopov & Polnarev; Harada et al. and $\beta(M) \approx 0.02\sigma^{13/2}(M)$ .